A

TREATISE

ON

PRACTICAL MENSURATION

IN

Eight Parts;

CONTAINING

The most approved Methods of drawing Geometrical Figures; Mensuration of Superficies, Land Surveying, Mensuration of Solids; the Use of the Carpenter's Rule, Timber Measure, in which is shewn the Method of measuring and valuing standing Timber; Artificers' Works, illustrated by the Dimensions and Contents of a House; Mensuration of Haystacks, Drains, Canals, Marlpits, Ponds, Mill-dams, Embankments, Quarries, Coal-heaps, and Clay-heaps; Conic Sections and their Solids; the most useful Problems in Gauging, according to the New Imperial Measures; and a Dictionary of the Terms used in Architecture.

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And Author of "A Treatise on Practical Arithmetic;" "A Complete
Treatise on Practical Land Surveying;" "A Treatise on Practical
Gauging;" "Keys to the Arithmetic, Mensuration, and
Gauging;" "An Introduction to Parsing, adapted
to Murray's Grammar;" &c. &c.

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YORK:

Printed by Thomas Wilson and Sons, High-Ousegate;
FOR LONGMAN, ORME, BROWN, GREEN, AND LONGMANS,
LONDON; AND FOR WILSON AND SONS, YORK.

1838.

Price Six Shillings, Bound.

TO

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OF

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AS AN ACKNOWLEDGMENT OF GRATITUDE [illegible] NUMEROUS FAVOURS RECEIVED,

AND AS

A Token of Esteem for His general Knowledge of Scientific Subjects,

THIS TREATISE

ON

Practical Mensuration,

In all its Departments,

IS

MOST HUMBLY AND RESPECTFULLY DEDICATED,

BY HIS MUCH OBLIGED,

AND MOST OBEDIENT SERVANT,

A. NESBIT.

Entered at Stationers' Hall.

PREFACE.

VARIOUS have been the conjectures concerning the origin of Geometry or Mensuration, but as it is a Science of general utility, there can, I think, be little doubt that its existence is nearly coeval with the creation of Man. Indeed I can see no reasonable objection why we may not attribute its invention to our first parent Adam, especially as we are informed in Holy Writ, that his son Cain built a city; to do which, it is evident, would require some knowledge of a measuring unit, which is the first principle of Mensuration. By the same infallible testimony, we find that the Arts and Sciences were cultivated to a considerable extent long before the Flood. Jubal was the father of all such as handled the harp and organ; and Tubal-cain, an instructor of every artificer in brass and iron. It is also more than probable that Noah was well acquainted with the Art of Mensuration, as practised in his day; for it does not appear that he found any difficulty in building the Ark, which consisted of three stories, according to certain dimensions given him by the Lord of the Universe.

Diodorus, Herodotus, and Strabo, are of opinion that the Science of Mensuration had its rise among the Egyptians; whom they represent as constrained, on account of the removal or defacing of the land-marks by the annual inundation of the Nile, to devise some method of ascertaining the ancient boundaries, after the waters had retired. By Josephus, however, the invention is ascribed to the Hebrews. This writer assents that the A[redacted] Sciences, for which Egypt was so long famous, were carried [redacted] that country, by the Patriarch Abraham, from [redacted] of the Chaldees;* but as Egypt was peopled b[redacted]e descendants of Ham, is it not more probable that they had the rudiments of their Sciences originally from their father?

Be this as it may, it is well known that Egypt was, for many ages, the mother and nurse of the Arts and Sciences. From this country they were conveyed into Greece by Thales, about 600 years before the Christian Æra. This Philosopher, after travelling into Egypt, and studying, under its sages, Astronomy, Geometry, and other branches of the Mathematics, returned to his own country, and employed himself in communicating the knowledge which he had acquired.

The great utility of Mensuration, without which it is impossible rightly to conduct the affairs of civilized life, induced many of the most celebrated Philosophers and Mathematicians of antiquity to study its principles, and to Thales, Anaxagoras, Pythagoras, Socrates, Plato, Apollonius, Philo, Ptolemy, Aristotle, Euclid, Archimedes, &c. we are indebted for many substantial improvements. The moderns, likewise, have not been less solicitous to enrich this Science, than the ancients, accordingly, Huygens, Wallis, Gregory, Halley, Euler, Leibnitz, the Bernoulles, Vieta, Metius, Van Ceulen, Barrow, Newton, Sharp, Machin, Moss, Leadbetter, Simpson, Emerson, Holliday, Fletcher, Robertson, Hutton, Bonnycastle, Keith, Beckett, &c. have greatly improved it by their labours.

After so many eminent men have written more or less upon this Science, it may perhaps be thought presumption in me to attempt to add any thing to its stores; but as I can say, without arrogance, that I have had considerable experience in the Practical Part of Mensuration, in all its Departments, I am persuaded that this work will be found to contain many things both new and useful.

With regard to the Rules, indeed, nothing new can be expected; as they are to be found, with very little variation, in every modern Treatise on Mensuration; but the Questions, which amount to five hundred and eighty, are almost wholly new ones; and a great number of them have been made from actual admeasurements. I have likewise given copious directions for taking dimensions, which are certainly forms a very essential part of Mensuration, for if the dimensions be improperly taken, the results must, of course, be incorrect.

The work is divided into Eight Parts; and some of these Parts are again subdivided into Sections.

Part the First contains Practical Geometry, and a few of the most useful Geometrical Theorems, most of which are employed in solving Questions in this Work. The Theorems are not demonstrated; but references are given to the elements of Euclid, Simpson, and Emerson, where their demonstrations may be found.

Part the Second contains Mensuration of Superficies. In the last Problem of this Part, I have given the invaluable Rule for finding the areas of curvilineal figures, by means of equi-distant ordinates. This Rule was first demonstrated by the illustrious Newton; but it is to Mr. Thomas Moss that we owe its present simplicity.

Part the Third is divided into two Sections. In the first are given the Methods of surveying and planning single Fields, Woods, Roads, and Rivers, and also Rules and Directions for Parting off, and Dividing Land. It likewise exhibits four of the most approved Methods of surveying large Estates, illustrated by three distinct Plans, and an engraven Field-Book.

Sometimes one of these Methods claims the preference, and sometimes another, according to the different forms of Estates, but they are all approved of by our best Land Surveyors; and are more accurate and practical than any others, that have come to my knowledge

This Section will be found to contain every thing that is necessary for persons, in general, to know of Surveying. In order to become a complete Surveyor, it is requisite not only to study Works written professedly on the subject, but also to have a considerable portion of *Field-practice,* under the direction of an able Tutor.

The second Section contains a collection of useful Questions concerning Superficial Mensuration, which will serve to exercise the ingenuity of the Learner; and to prove his knowledge of the Theorems and Rules given in the first three Parts.

Part the Fourth is divided into four Sections. The first contains the Mensuration of Solids, the second, the Description and Use of the Carpenter's Rule; and the third, Timber Measure.

The last Problem of this Section contains Rules and Directions for measuring and valuing standing Timber; many of which were never before published.

In this part of the Work, I have been assisted by Mr. Joseph Webster, of Farnley, near Leeds; who has been many years very extensively employed, as a valuer of timber, by the Earl of Cardigan, Lord Mexborough, &c. &c.

In order to render this Problem as useful as possible, I have given a description of Timber Trees, and pointed out the purposes for which their wood is best adapted, for it is impossible to become a valuer of timber without being made acquainted with the properties of trees.

The fourth Section contains Miscellaneous Questions concerning the Mensuration of Solids.

Part the Fifth treats of the Method of measuring the Works of Artificers; viz. Bricklayers, Masons, Carpenters and Joiners, Slaters and Tilers, Plasterers, Painters, Glaziers, Plumbers, and Pavers.

The directions in this Part, for taking the dimensions, making the deductions, &c. will be found to be very copious. For some of these I am indebted to Mr. Benjamin Jackson, *senior,* an able and experienced Architect in Leeds; and to Mr. Joseph Brooke, Teacher of the Mathematics, at Wortley, near Leeds, who has had much experience in measuring the Works of Artificers.

This Part is concluded with a general Illustration, containing the dimensions of a House, and exhibiting the methods of ruling the Book, entering the dimensions with the contents; and forming the Bills for workmanship and materials.

Part the Sixth treats of the Mensuration of Hay-stacks, Drains, Canals, Marlpits, Embankments, Ponds, Mill-dams, Quarries, Coal-heaps, Clay-heaps, and other irregular figures, by means of equi-distant, parallel sections, founded upon the method of equi-distant ordinates

This method of finding the contents of irregular figures is, pointed out by Dr. Hutton, in his valuable Treatise on Mensuration, octavo,* page 375; but Mr. Joseph Beckett appears to have been the first who has applied it with any degree of success

This Part also contains the method of measuring the circular Ponds made upon the Wolds in Yorkshire This was communicated to me by the Rev. W. Putsey, Master of the Classical, Commercial, and Mathematical Academy, Pickering

In order to give the young Reader an idea of the great improvements made in Agriculture and Commerce, by means of Drains and Canals; and also to make him acquainted with some of the stupendous Works which have been accomplished by the ingenuity, perseverance, and combined efforts of men; I have concluded this Part with a description of a few of the principal Canals in England, Scotland, France, and China; and with an account of some of the chief Drainages which have been executed in the counties of York and Lincoln.

Part the Seventh treats of Conic Sections and their Solids. It also contains a few of the leading properties of the Ellipse, the Parabola, and the Hyperbola. Those who desire more information on this subject, may consult Simpson's, Emerson's, and Hutton's Conic Sections.

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* Price 18s in boards.

Part the Eighth displays the method of gauging all kinds of open vessels used by Maltsters, Brewers, Distillers, Wine Merchants, Victuallers, &c. &c.

In this Part I have applied the method of equi-distant ordinates or sections, to the gauging of vessels whose sides are curved; such as coppers, stills, &c. &c. This I have not seen in any other Work.

I have also given the process of gauging and inching a guile-tun, according to the method practised in the Excise. Malt-gauging, and Cask-gauging are likewise treated of, in this Part; and it is concluded with a few Miscellaneous Examples.

Those who intend to become Candidates, for the Excise, ought to read Works written expressly on the subject of Gauging: but it is hoped that every thing will be found here which is requisite for persons in general to know of that Science.

In order to render the Treatise as useful as possible, I have given a Dictionary containing an explanation of the most general terms made use of in Architecture This will be found a valuable addition by Bricklayers, Masons, Joiners, and every other person concerned in measuring or building.

The Work is brought to a conclusion by a number of questions to be answered verbally by the Pupil; for nothing will tend more to make him an adept in Mensuration, than committing the Definitions and Rules to Memory.

Nearly the whole of the Rules upon which Mensuration depends, require Algebra or Fluxions to demonstrate them; but as these Sciences are too abstruse and sublime to be comprehended by the generality of those who are concerned in Mensuration, it has been thought advisable to give the Rules without the Demonstrations. Those who desire to penetrate the deepest recesses of scientific knowledge, are referred to Simpson's, Emerson's, and Holliday's Fluxions, and to Doctor Hutton's Mensuration, in one or other of which Works, all the Rules given in this Treatise are demonstrated.

The Demonstrations would also have swelled the Work very considerably, both in size and price; but by withholding them, space has been obtained for a greater body of useful matter, on the subject of Mensuration, than is to be found in any other Work, of equal size, with which I am acquainted.

I should be wanting in respect, if I concluded this Preface without paying a tribute to the memory of that profound and indefatigable Mathematician, the late Mr John Ryley of Leeds;—a man who has enriched almost every periodical, mathematical publication, for nearly half a century, with problems and demonstrations in science, which have struck many of his contemporaries with unmixed admiration of his genius and attainments. To him I submitted the Plan of this Work, which, I am happy to say, received his entire approbation; and I flatter myself that it will be generally approved by the Public.

It now only remains for me to solicit the indulgence of the candid Reader; for in solving so large a number of new questions as this Work contains, it is almost impossible to avoid errors. These, however, it is hoped will be found to be few, and of little moment; for much care has been employed in working the questions, and correcting the press; and I may add, that neither labour nor expense has been withheld in order to produce a Work of general Utility.

ANTHONY NESBIT.

MANCHESTER, *April*, 1838.

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TO THE SECOND EDITION.

WITH heart-felt gratitude the Author embraces the present opportunity of returning his unfeigned thanks to his Friends and the Public; but particularly to Masters of Seminaries, for the unparalleled patronage with which they have honoured this Work.

He also wishes to express his warmest acknowledgments to those Teachers and kind Friends who have signified either verbally or by letter, their good opinion of it. To have succeeded in gaining the approbation of Men of Knowledge and Science, cannot but be gratifying to the Author; and is a pleasing recompense for the great labour which he has bestowed upon it.

In consequence of the extensive manner in which Mensuration, in all its departments, was treated in the first Edition, the present cannot boast of many improvements. Such notes and observations, however, as the Author thought necessary, have been added in various parts of the work. The fifth Problem in Part the Fifth, on measuring Saloons, has been further elucidated, and two more questions given, and in the second Problem of Part the Sixth, the method of measuring Hay-mows has been described.

In the third Problem of the same Part, the Rule for finding the areas of figures by means of equi-distant ordinates, has been simplified, and is consequently better adapted to the capacities of Learners, than that given in the twenty-third Problem of Part the Second.

These additions and improvements, will, at least, render this as acceptable to the Public as the former Edition; and as the flattering sale of that fully proves the Work has been duly appreciated, the Author may, perhaps, reasonably indulge a hope that in its improved state, it will be still more widely circulated, more extensively read, and prove still more useful in disseminating the principles, and elucidating the practice of the inestimable Science of Mensuration.

A. NESBIT.

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TO THE FOURTH EDITION.

THE Author has carefully revised and corrected this Edition, and made, in different parts of the Work, several important and useful Additions and Improvements.

Part the Eighth, on Gauging, has been entirely rewritten, and much improved by the addition of New Matter, and all the Rules and Examples have been adapted to the New Imperial Standard Gallon and Bushel.

The Author has calculated Three New Tables, which exhibit the number of Cubic Inches in the different denominations of Wine and Spirit Measure, Ale and Beer Measure, and Corn or Dry Measure, according to the New Imperial Standard Gallon and Bushel; and also given Rules and Directions for reducing the Old English, Irish, and Scotch Measures to the New Imperial Measures, and *vice versâ.*

He has also shewn the method of finding the New Imperial Multipliers, Divisors, and Gauge-Points; and given a Table in which they are all arranged in a convenient order.

He has likewise described the New Imperial Sliding Rule, and Diagonal Rod, given *particular* and *general* Rules for Cask Gauging; and also calculated a New Table of Multipliers, by which the Content of any Cask may be easily found, in Imperial Gallons.

These Alterations and Additions will be found very great improvements to the Work; and will tend to secure and increase that popularity and patronage which it has already obtained among Teachers and the Public, as it now contains such a body of Practical Information, on Mensuration in all its Departments, as cannot be found in any other Work on the same Subject.

A. NESBIT.

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TO THE FIFTH EDITION.

THIS Edition has been carefully revised and corrected, and several new and useful Rules have been subjoined to Problems 4, 5, 6, 7, 8, and 10, Part the Fourth, for finding the perpendicular Altitudes of *intended new* Vessels, such as Cisterns, Couches, Tuns, Vats, &c &c., when we know their Capacities and horizontal Dimensions.

In Part the Sixth, we have described the methods of finding the Solidities and Weights of Clay-Heaps, which form no inconsiderable department of Mensuration, for, in Lancashire, Yorkshire, Staffordshire, and many other Parts of England, Clay, in immense quantities, is dug out of Pits, and sold by the cubic Yard, or by the Ton, for the purposes of making Bricks, Tiles, Earthen Ware, &c. &c

These Additions and Improvements will be found to increase the value and general utility of the Work; and will render it still more deserving of the Patronage of Teachers and the Public

A. NESBIT.

MANCHESTER, *June*, 1830

P. S. A KEY to the Mensuration, containing Solutions to all the Questions which are not solved in that Work, has been published for the use of Teachers and private Students. This will be found of great advantage, as it will render it quite unnecessary for the Tutor to inspect every operation of his Pupil; and will enable those who may not have had an opportunity of acquiring a knowledge of Mensuration in their Youth, and who cannot now have the advantage of a living Instructor, to pursue their private studies with ultimate success, if they apply themselves with diligence and perseverance.

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A TREATISE

ON

PRACTICAL MENSURATION.

PART I.

Definitions, Problems, and Theorems, in Geometry.

GEOMETRY originally signified the Art of measuring the Earth, or any distance or dimensions upon or within it; but it is now used for the Science of Quantity, Extension, or Magnitude, abstractedly considered.

GEOMETRICAL DEFINITIONS.

1. A point is considered as having neither length, breadth, nor thickness.

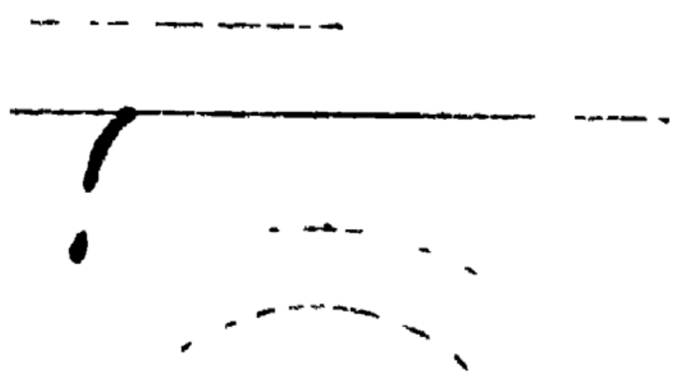
2. A line has length, but is considered as having neither breadth nor thickness.

3. Lines are either right, curved, or parallel.

4. A right or straight line lies wholly in the same direction between its extremities, and is the shortest distance between two points.

5. A curved line continually changes its direction between its extremities.

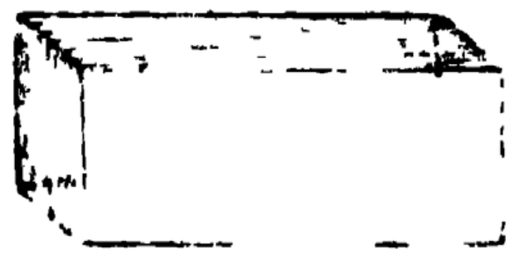
6. Parallel lines always remain at the same distance from each other, though infinitely produced.



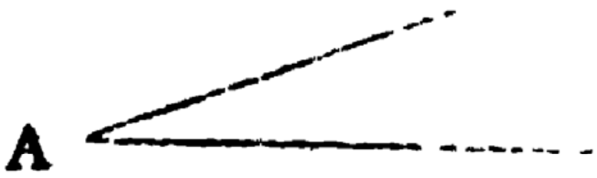
7. A superficies has length and breadth, but is considered as having no thickness.

8. A superficies may be contained within one curved line, but cannot be contained within fewer than three straight lines.

9. A solid is a figure of three dimensions, namely, length, breadth, and thickness.

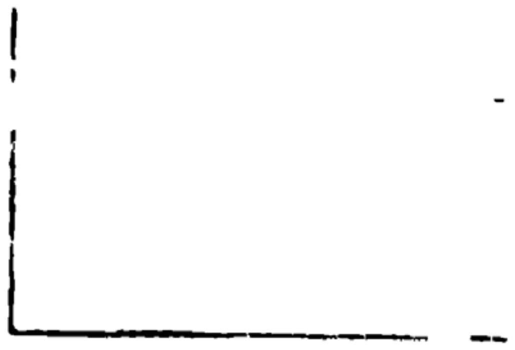


10. An angle is the inclination or opening of two lines, having different directions, and meeting in a point, which is called the angular point, as at A; and when three letters are used, the middle one denotes that point.

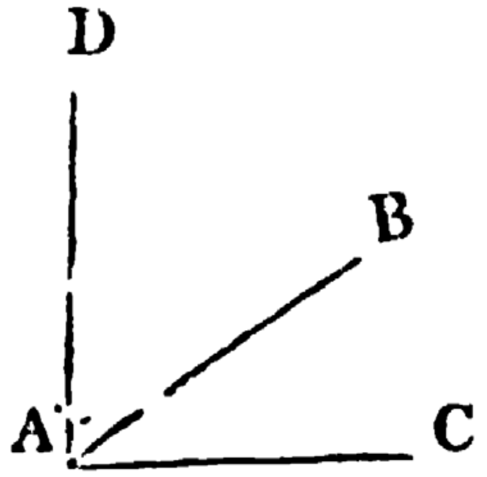


11. Angles are of three kinds, viz. right, acute, and obtuse.

12. A right angle is made by two right lines which are perpendicular to each other.

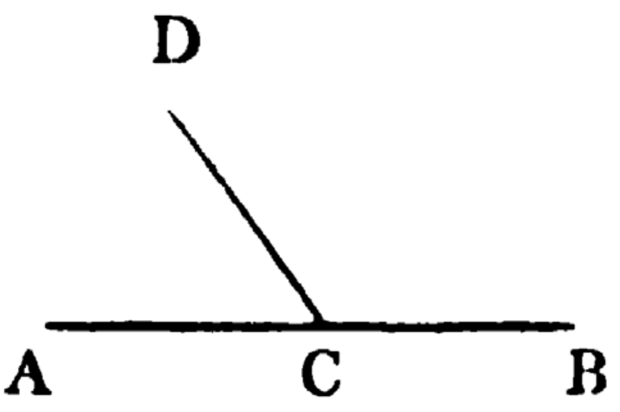


13. An acute angle is less than a right angle, as CAB.



14. The complement of an angle is what it wants to complete a right angle, as the angle DAB, is the complement of the angle CAB.

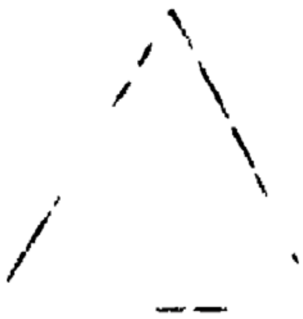
15. An obtuse angle is greater than a right angle, as BCD.



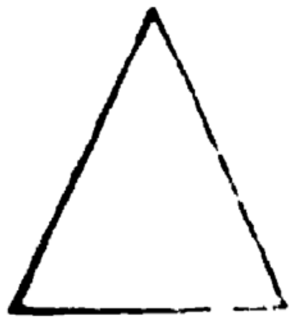
16. The supplement of an angle is what it wants of two right angles, as the angle ACD, is the supplement of the angle BCD.

17. A triangle is a figure or superficies bounded by three right lines, and admits of three varieties, viz. equilateral, isosceles, and scalene.

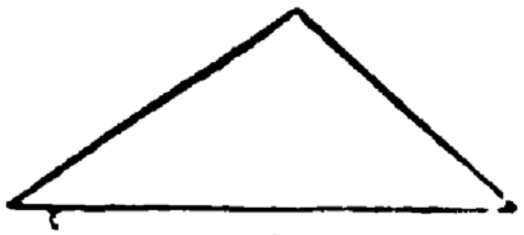
18. An equilateral triangle has all its sides equal.



19. An isosceles triangle has only two of its sides equal.

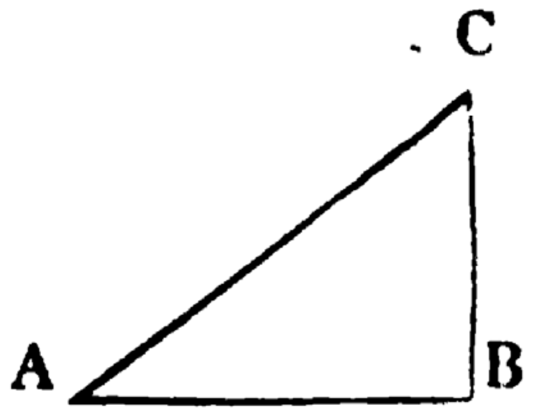


20. A scalene triangle has all its sides unequal.

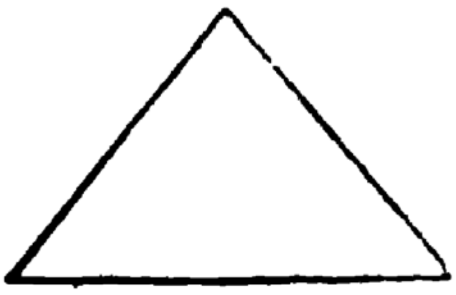


21. Triangles are also right-angled, acute-angled, and obtuse-angled.

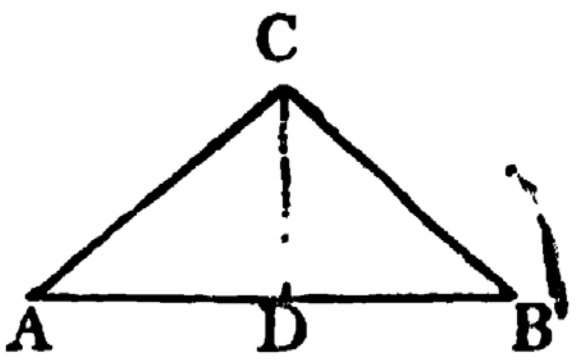
22. A right-angled triangle has one right-angle, the side opposite to which is called the hypothenuse, the other two being termed legs, or one the perpendicular, and the other the base; thus AC is the hypothenuse, BC the perpendicular, and AB the base



23. An acute-angled triangle has all its angles acute.



24. An obtuse-angled triangle has one of its angles obtuse, as ACB.

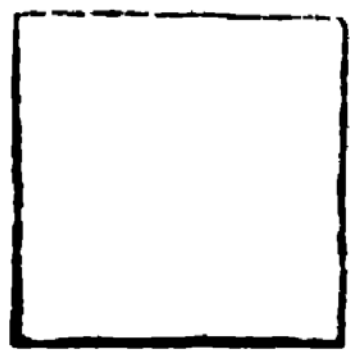


25. The base of any figure is that side upon which it is supposed to stand, or upon which a perpendicular is let fall from the *vertex* or opposite angle; and the altitude of a figure is its perpendicular height. In the last figure, AB is the base, and CD the perpendicular.

26. A figure of four sides and angles is denominated a quadrangle or quadrilateral figure.

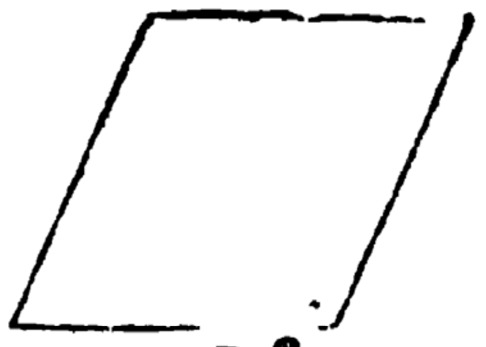
27. A parallelogram is a quadrilateral figure, having its opposite sides parallel and equal, and admits of four varieties, viz. the square, the rectangle, the rhombus, and the rhomboid.

28. A square is an equilateral parallelogram, having all its angles right angles.



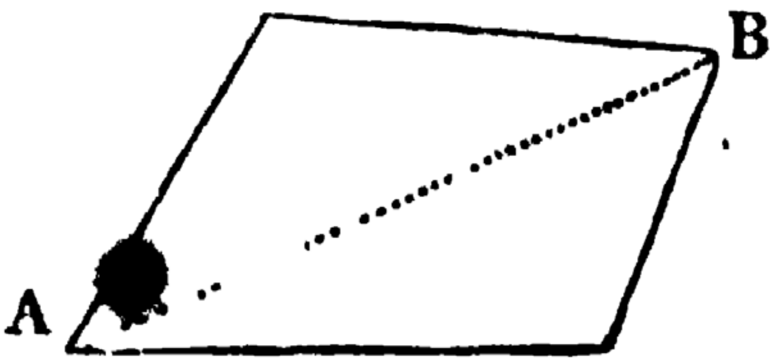
29. A rectangle is a parallelogram, having its opposite sides equal, and all its angles right angles.

30. A rhombus is an equilateral parallelogram, having its opposite angles equal; two of which are acute, and two obtuse, which, in a regular rhombus, are 60 and 120 degrees.



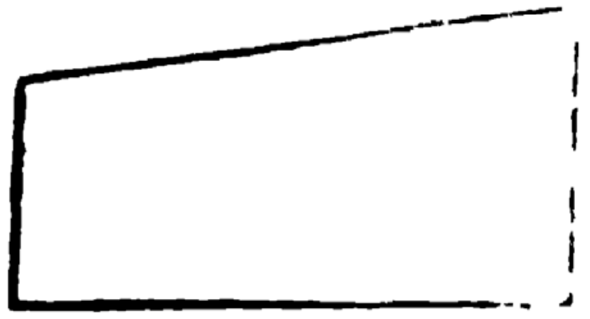
31. A rhomboid or rhomboides is a parallelogram, having its opposite sides and angles equal; two of its angles being acute, and two obtuse; and when the figure is regular, the angles are 60 and 120 degrees.

32. A trapezium is a quadrilateral figure, whose opposite sides are not parallel to each other.



33. A diagonal is a right line joining the opposite angles of a quadrilateral figure, as AB.

34. A trapezoid is a quadrilateral figure, having two of its opposite sides parallel.

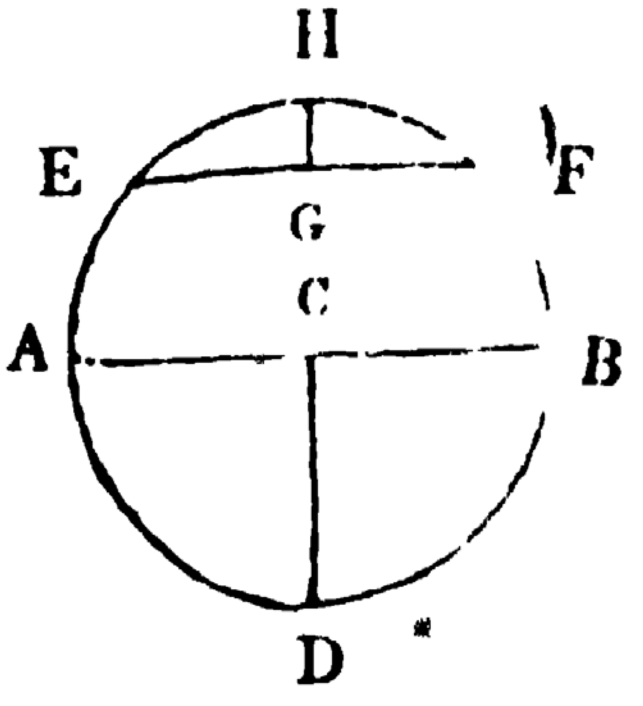


35. Plain figures having more than four sides, are generally called polygons; and receive their particular denominations from the number of their sides or angles.

36. A pentagon is a polygon of five sides; a hexagon of six; a heptagon of seven; an octagon of eight; a nonagon of nine; a decagon of ten; an undecagon of eleven, and a duodecagon of twelve.

37. A regular polygon has all its sides and angles equal; when they are unequal, the polygon is irregular.

38. A circle is a plane figure, bounded by a curved line, called the circumference, which is every where equi-distant from a certain point within it, called the centre.



39. The diameter of a circle is a right line drawn through the centre, and terminating in the circumference on each side, as AB.

40. The radius of a circle is half the diameter, or it is a right line drawn from the centre to the circumference, as CD.

41. An arc of a circle is any part of the circumference, as the arc EHF.

42. A chord is a right line joining the extremities of an arc, as the line EF; and the versed sine is part of the diameter cut off by the chord, as GH.

43. A segment is any part of a circle, bounded by an arc and its chord.

44. A semicircle is half of a circle, or a segment cut off by the diameter, as ADB.

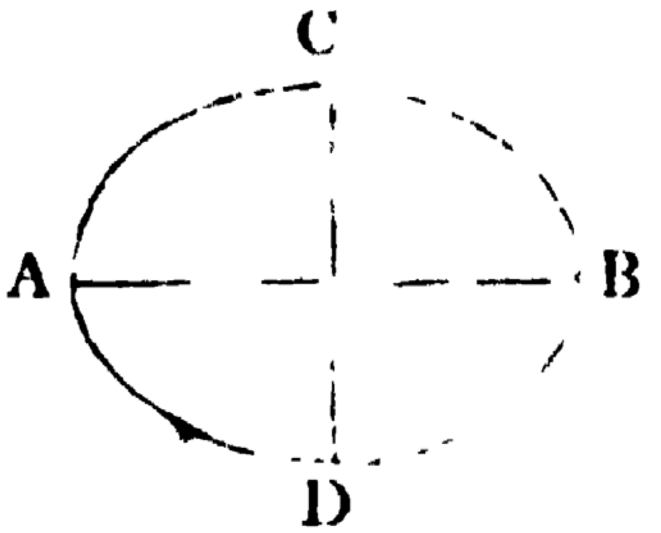
45. A quadrant is the fourth part of a circle, as ADC.

46. A sector is any part of a circle, bounded by an arc and two radii.

47. The circumference of every circle is supposed to be divided into 360 equal parts, called degrees; each degree into 60 equal parts, called minutes, and each minute into 60 equal parts, called seconds.

48. The arc of a quadrant contains 90 degrees, which is the measure of a right angle.

49. An ellipse is a plane figure bounded by a curved line, called the circumference; but as the figure is not a circle, it is described from two points in the longest diameter, called the foci, or focuses.

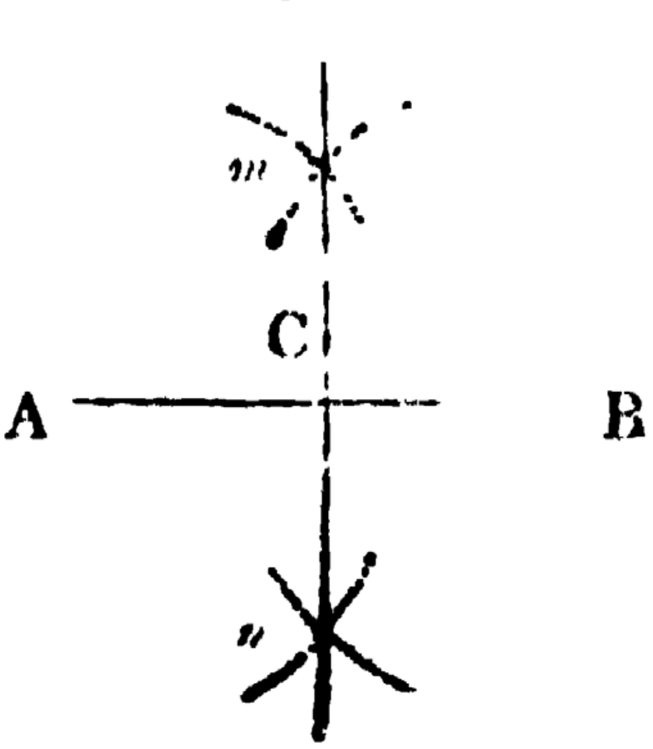


50. The longest diameter that can be drawn within an ellipse, is called the transverse diameter, as AB; and the shortest, the conjugate diameter, as CD. Sometimes these diameters are termed axes.

GEOMETRICAL PROBLEMS

PROBLEM I.

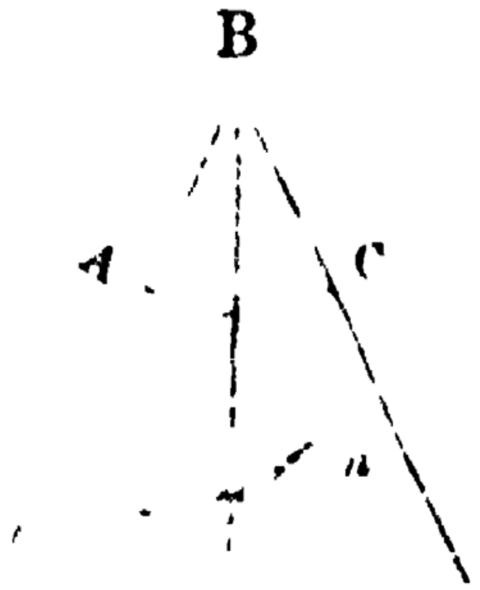
To bisect a given line AB



From A and B as centres, with any radius, greater than half AB, in your compasses, describe arcs cutting each other in *m* and *n*. Draw the line *m*C*n*, and it will bisect AB in C.

PROBLEM II.

To bisect a given angle ABC.

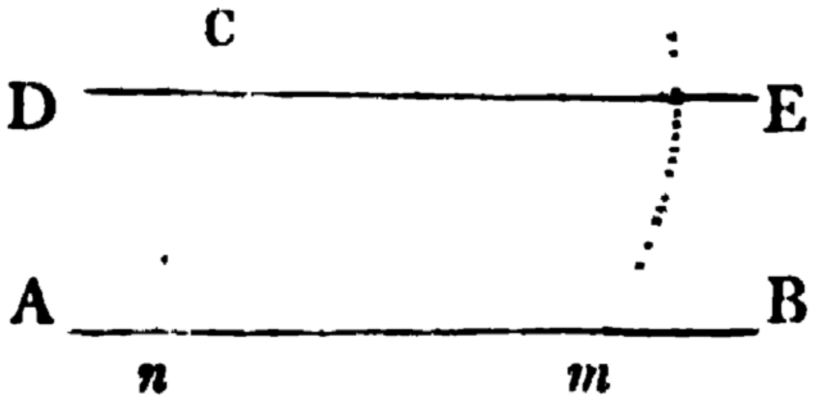


From the point B, with any radius, describe the arc AC. From A and C, with the same, or any other radius, make the intersection *m*. Draw the line B*m*, and it will bisect the angle ABC, as required.

PROBLEM III.

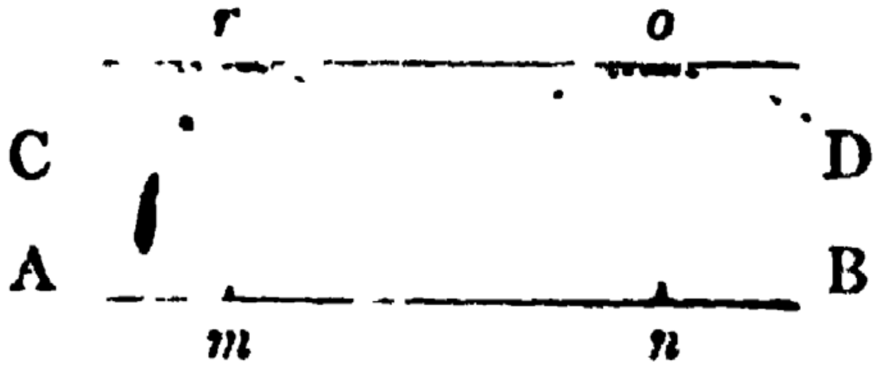
To draw a line parallel to a given line AB.

Case 1. When the parallel line is to pass through a given point C.



From any point *m*, in the line AB, with the radius *m*C, describe the arc C*n*. From the centre C, with the same radius, describe the arc *mr*. Take the distance C*n* in the compasses, and apply it from *m* to *r*. Through C and *r* draw the line DE, and it will be the parallel required.

CASE 2. When the parallel lines is to be at a given distance from AB.



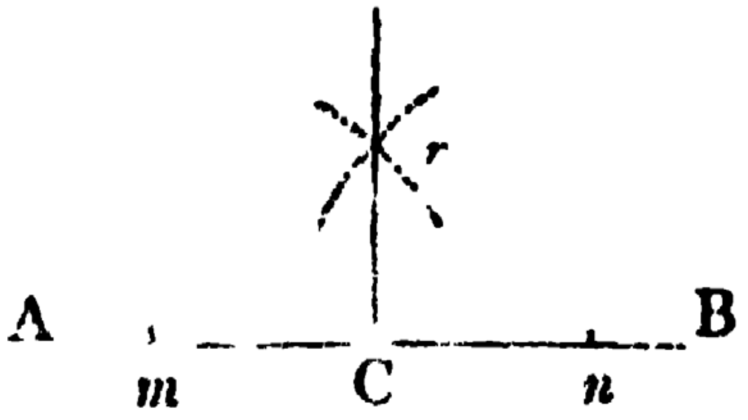
From any two points *m* and *n*, in the line AB, with a radius equal to the given distance, describe the arcs *r* and *o*. Draw the line CD, to touch these arcs, without cutting them, and it will be the parallel required.

NOTE.—This Problem may be more easily performed by means of a parallel ruler, which may also be used to advantage in several operations in Practical Geometry.

PROBLEM IV.

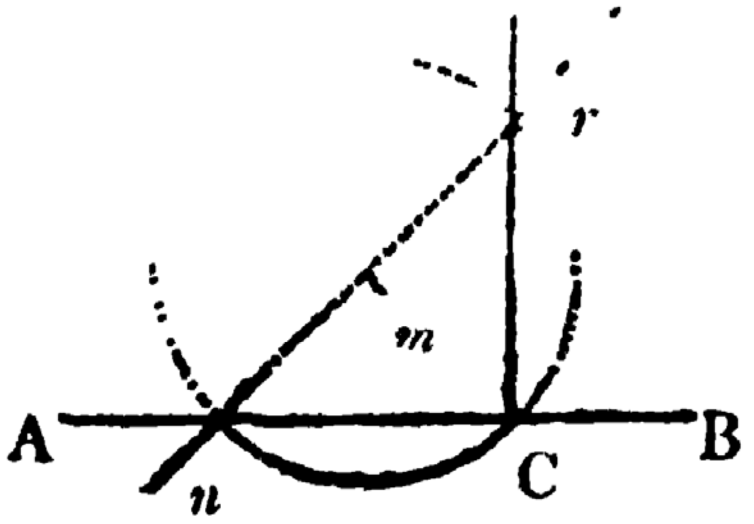
To erect a perpendicular from a given point C, in a given line AB.

CASE 1. When the point is near the middle of the line.



On each side of the point C, take any two equal distances, C*m* and C*n*. From *m* and *n*, as centres, with any radius greater than C*m* or C*n*, describe two arcs cutting each other in *r*. Draw the line C*r*, and it will be the perpendicular required.

CASE 2. When the point C is at or near the end of the given line.

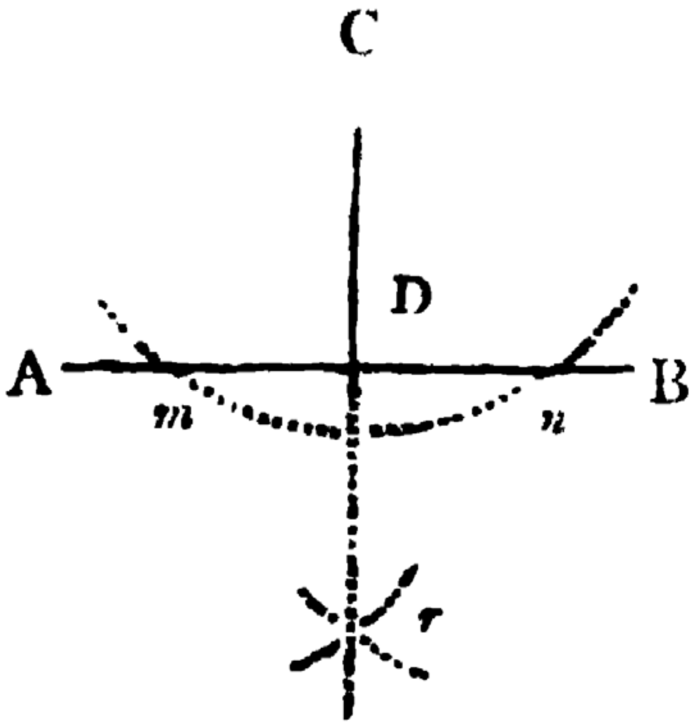


From any point *m*, as a centre, with the radius or distance C*m*, describe the arc *n*C*r*, cutting the given line in *n* and C. Through *n* and *m* draw a line cutting the arc in *r*. Draw the line C*r*, and it will be the perpendicular required.

PROBLEM V.

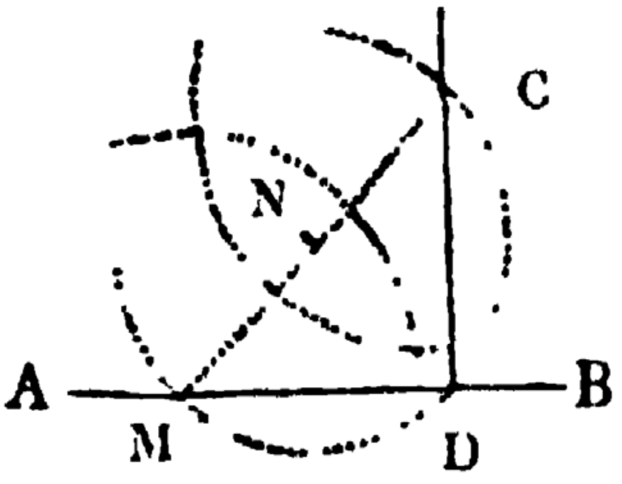
From a given point C, *to let fall a perpendicular upon a given line* AB.

CASE 1. When the point is nearly opposite to the middle of the line.



With C as a centre, and any radius a little exceeding the distance of the given line, describe an arc cutting AB, in *m* and *n*. With the centres *m* and *n*, and the same, or any radius exceeding half their distance, describe arcs intersecting each other in *r*. Draw the line C*r*; and CD will be the perpendicular required.

Case 2. When the given point C is nearly opposite to the end of the given line.

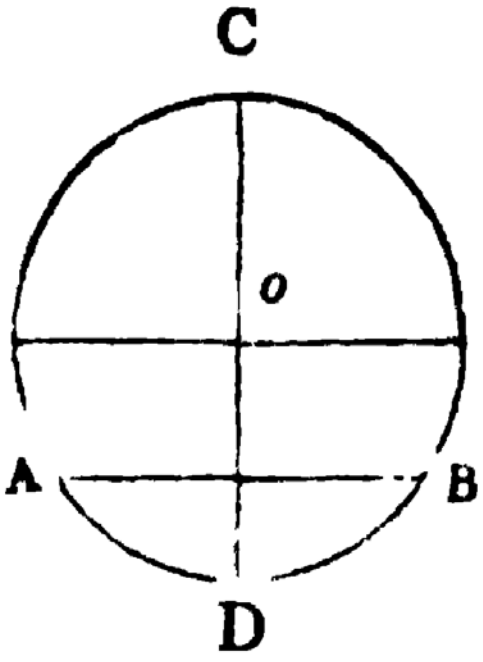


From C draw the line CM to meet AB, in any point M. Bisect the line CM in the point N, and with the centre N, and radius CN or MN, describe an arc cutting AB in D. Draw the line CD, and it will be the perpendicular required.

Note.—Perpendiculars may be more easily raised or let fall by means of a square, or of a feather-edged scale or ruler, with a line across it perpendicularly to its edges.

PROBLEM VI.

To find the centre of a given circle, or one already described.

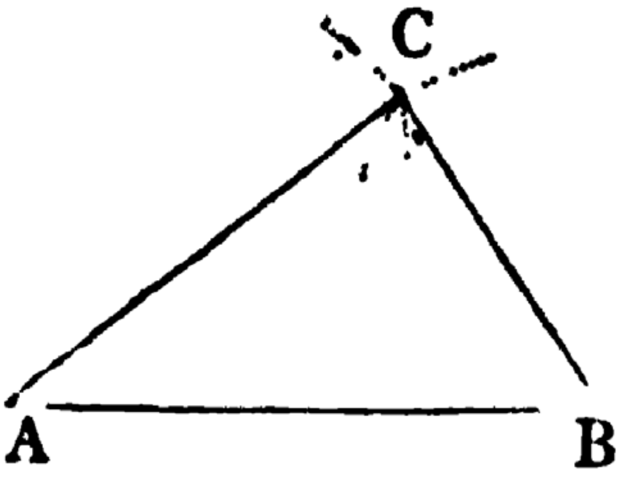


Draw any chord AB; and bisect it perpendicularly with CD, which will be a diameter. Bisect CD in the point *o*, which will be the centre required.

PROBLEM VII.

To make a triangle with three given lines, any two of which, taken together, must be greater than the third. (Euclid I. 22.)

Let the given lines be AB = 12, AC = 10, and BC = 8.



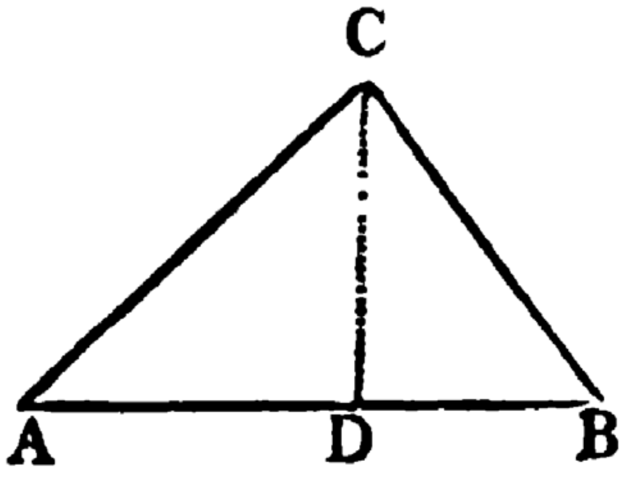
From any scale of equal parts (which is to be understood as employed likewise in the following Problems) lay off the base AB. With the centre A, and radius AC, describe an arc. With the centre B, and radius BC, describe another arc, cutting the former in C. Draw the lines AC and BC, and the triangle will be completed.

NOTE.—A trapezium may be constructed in the same manner; having the four sides and one of the diagonals.

PROBLEM VIII.

Having given the base, the perpendicular, and the place of the perpendicular upon the base, to construct a triangle.

Let the base AB = 12, the perpendicular CD = 6, and the distance AD = 7.



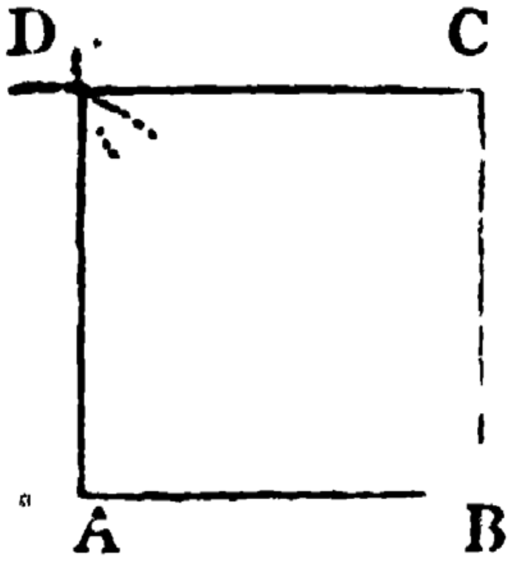
Make AB equal to 12, and AD equal to 7. At D erect the perpendicular DC, which make equal to 6. Join AC and BC, and the figure will be completed.

NOTE.—A trapezium may be constructed in a similar manner, by having one of the diagonals, the two perpendiculars let fall thereon from the opposite angles, and the places of those perpendiculars upon the diagonal; and a trapezoid may be constructed by drawing the two parallel sides perpendicularly to their base at a given distance.

PROBLEM IX.

To describe a square, whose side shall be equal to a given line.

Let the given line AB = 8.

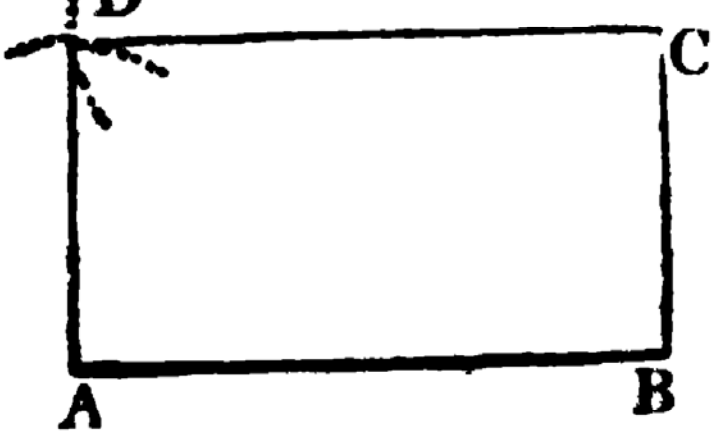


Upon one extremity B, of the given line, erect the perpendicular BC, which make equal to AB. With A and C as centres, and the radius AB, describe arcs cutting each other in D. Join AD and CD, and the square will be completed.

PROBLEM X.

To describe a rectangular parallelogram, whose length and breadth shall be equal to two given lines.

Let the length AB = 12, and the breadth BC = 6.

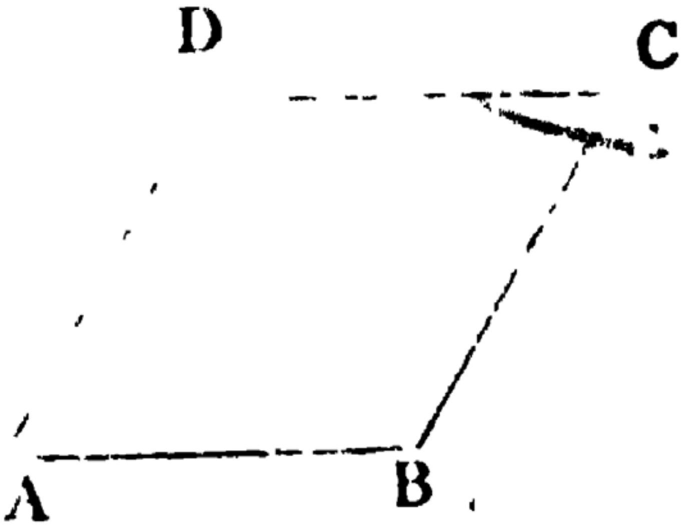


At B erect the perpendicular BC, which make equal to 6. With A as a centre, and the radius BC, describe an arc; and with C as a centre, and the radius AB, describe another arc, cutting the former in D. Draw the lines AD and CD, and the rectangle will be completed.

PROBLEM XI.

Upon a given right line to construct a regular rhombus.

Let the given line AB = 8.

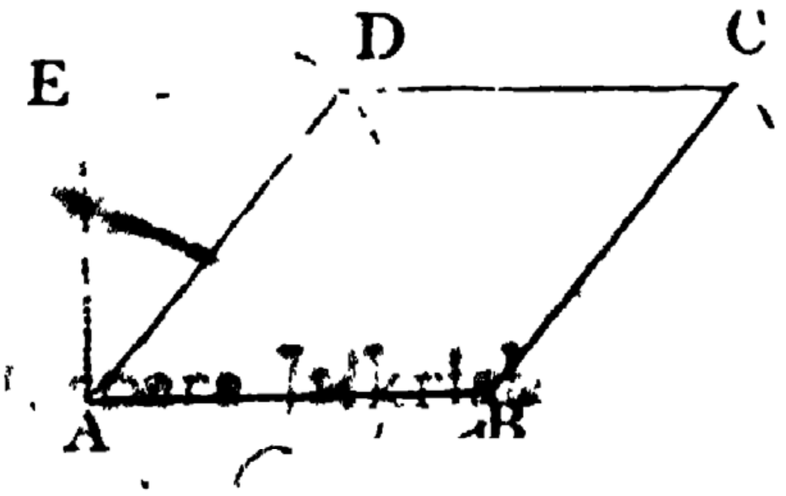


Draw the line AB equal to 8. With A and B as centres, and the radius AB, describe arcs cutting each other in D; then with B and D as centres, and the same radius, make the intersection C. Draw the lines AD, DC, and BC, and the rhombus will be completed.

PROBLEM XII.

To construct an irregular rhombus, having given its side and perpendicular height.

Let the side = 8, and the perpendicular = 6.

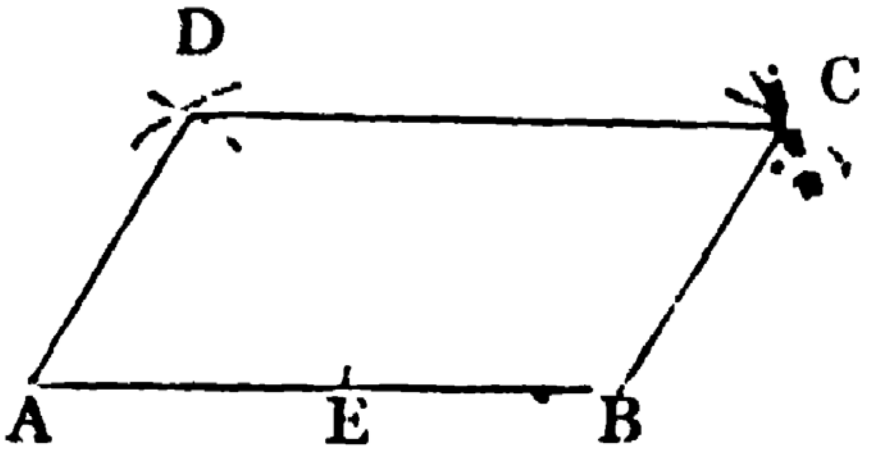


Draw AB equal to 8; at A erect the perpendicular AE, which make equal to 6; and draw EC parallel to AB. With the radius AB, and A as a centre, make the intersection D; and with the same radius, and B as a centre, make the intersection C. Join AD, DC, and CB, and the figure will be completed.

PROBLEM XIII.

Having any two right lines given, to construct a regular rhomboid.

Let the given lines be AB = 12, and BC = 6.

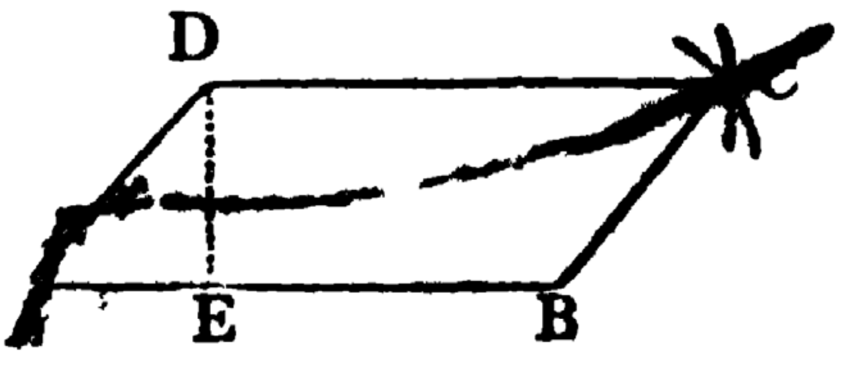


Draw the line AB equal to 12. Take in your compasses the line BC, and lay it from A to E. With A and E as centres, and the radius AE, make the intersection D. Then with B as a centre, and the same radius, describe an arc; and with D as a centre, and the radius AB, describe another arc, cutting the former in C. Draw the lines AD, DC, and BC, and the rhomboid will be completed.

PROBLEM XIV.

Having given the base, the perpendicular, and the place of the perpendicular upon the base, to construct an irregular rhomboid.

Let the base AB = 15, the perpendicular ED = 6, and the distance AE = 5.



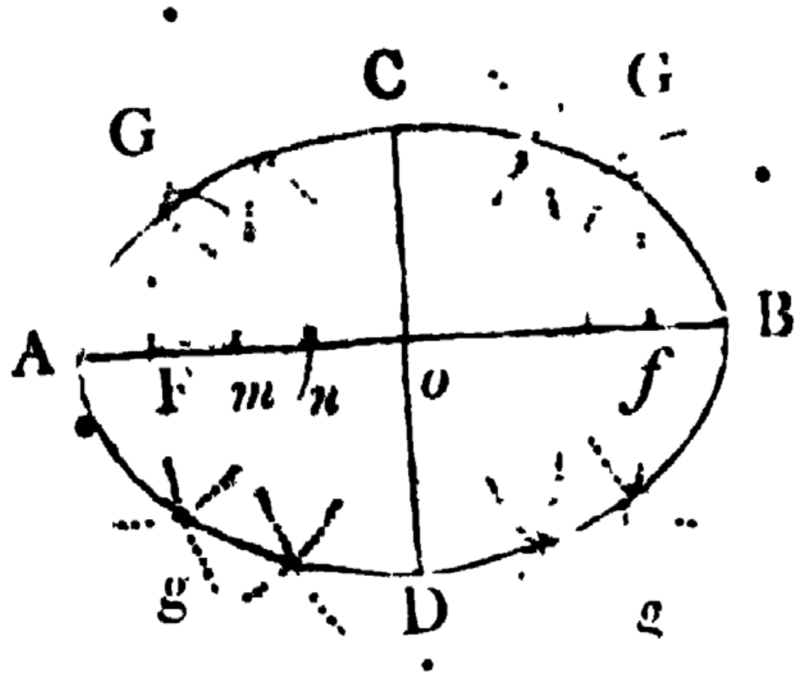
Make AB equal to 15, and AE equal to 5. At E erect the perpendicular ED, which make equal to 6; and join AD. With the radius AB, and D as a centre, describe an arc; and with B as a centre, and the radius AD, describe another arc, cutting the former in C. Draw the lines DC and BC, and the figure will be completed.

NOTE.—The sum of all the interior angles of any quadrilateral figure, is equal to four right angles.

PROBLEM XV.

Having the transverse and conjugate diameters given, to construct an ellipse.

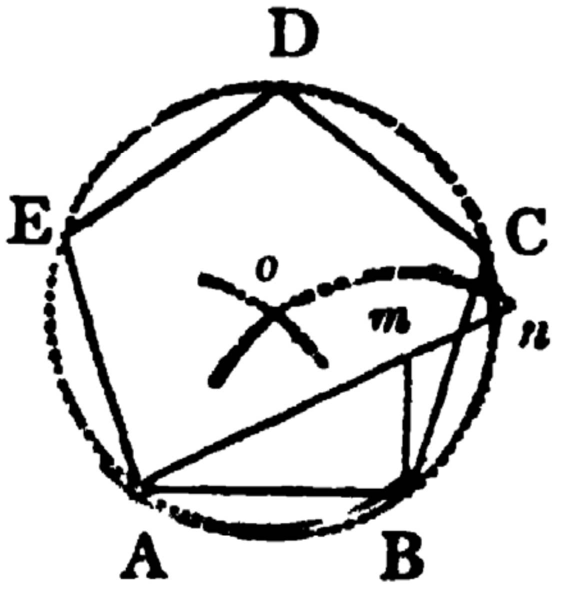
Let the transverse diameter AB = 14, and the conjugate diameter CD = 8.



Draw the two diameters to bisect each other perpendicularly in the centre *o*. With the radius A*o*, and the centre D, intersect AB in F and *f*: these two points will be the foci of the ellipse. Take any point *m*, in the transverse diameter, and with F and *f* as centres, and the radius A*m*, describe the arcs G, G, *g*, *g*. Then with the same centres, and the radius B*m*, describe arcs cutting the former in the points G, G, *g*, *g*; thus you will have four points in the circumference of the ellipse. Again, take a second point *n*, in the transverse diameter, and proceeding as before, you will determine other four points. By the same method you may determine as many more as you please; through all of which, with a steady hand, draw the circumference of the ellipse. (See another method of construction on page 315.)

PROBLEM XVI.

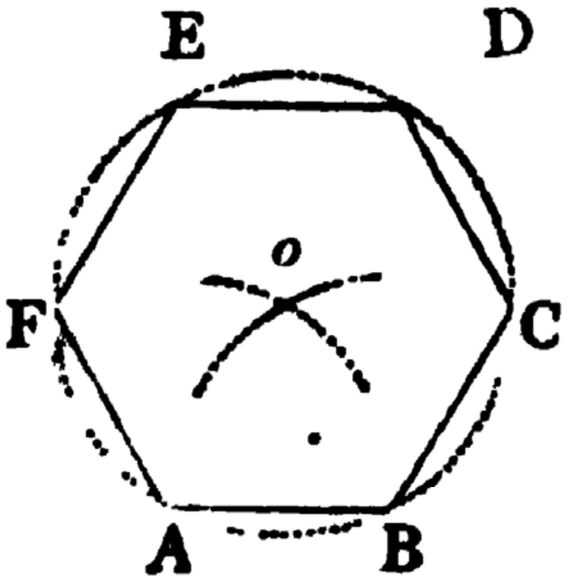
Upon a given line AB, *to make a regular pentagon.*



Make B*m* perpendicular to AB, and equal to half of it. Draw A*m*, and produce it till *mn* be equal to B*m*. With the radius B*n*, and A and B as centres, describe arcs intersecting each other in *o*, which will be the centre of the circumscribing circle. From the point *o*, with the same radius, describe the circle ABCDE; and apply the line AB five times to the circumference, marking the angular points, which connect with right lines, and the figure will be completed.

PROBLEM XVII.

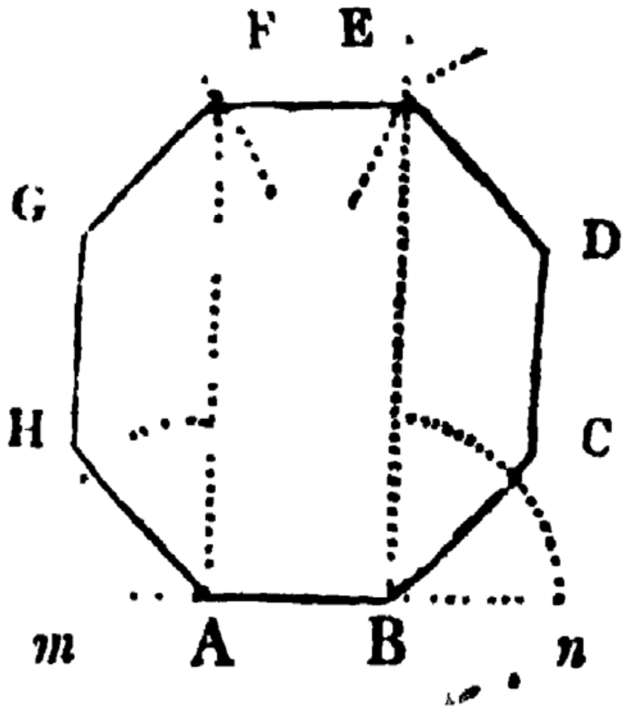
Upon a given line AB, *to make a regular hexagon.*



With A and B as centres, and the radius AB, describe arcs intersecting each other in *o*; and with *o* as a centre, and the same radius, describe the circle ABCDEF. Apply the line AB six times to the circumference, and it will form the hexagon required.

PROBLEM XVIII.

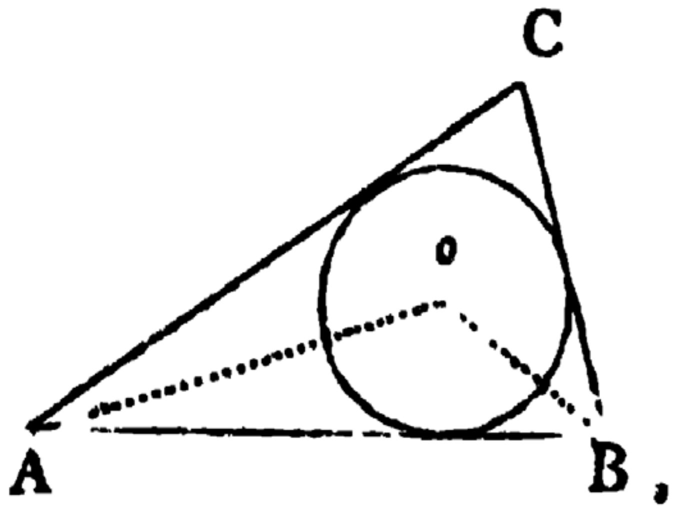
Upon a given line AB, *to construct a regular octagon.*



On the extremities of the given line AB, erect the indefinite perpendiculars AF and BE. Produce the line AB, both ways to *m* and *n*; and with the lines AH and BC, each equal to AB, bisect the angles *m*AF and *n*BE. Draw CD and HG parallel to AF or BE, and each equal to AB. With D and G as centres, and the radius AB, describe arcs intersecting AF and BE, in the points F and E. Join GF, FE, and ED, and the figure will be completed.

PROBLEM XIX.

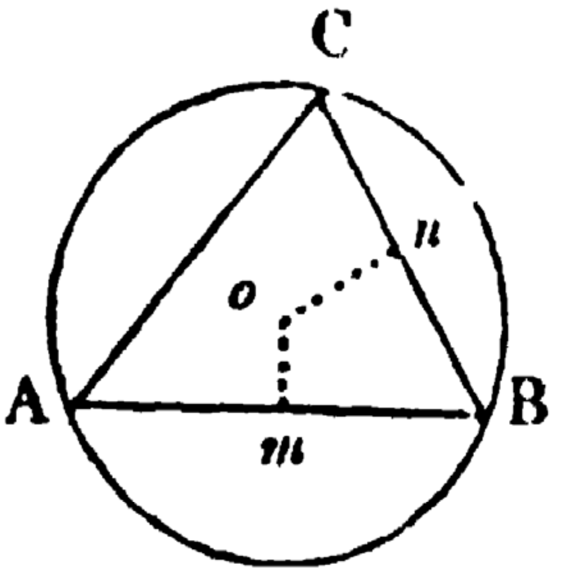
In a given triangle ABC, *to inscribe a circle.*



Bisect the angles A and B, with the lines Ao, Bo, and o will be the centre of the required circle; and its radius will be the nearest distance to any one of the sides; hence the circle may be described.

PROBLEM XX.

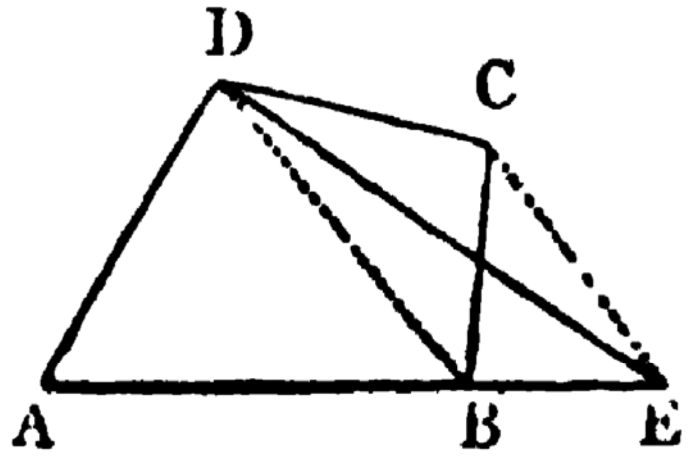
About a given triangle ABC, *to circumscribe a circle, or to describe the circumference of a circle through three given points* A, B, C.



Bisect the sides AB, BC, with the perpendiculars *mo* and *no*; and *o* will be the centre of the circle, and its radius will be A*o*, B*o*, or C*o*.

PROBLEM XXI.

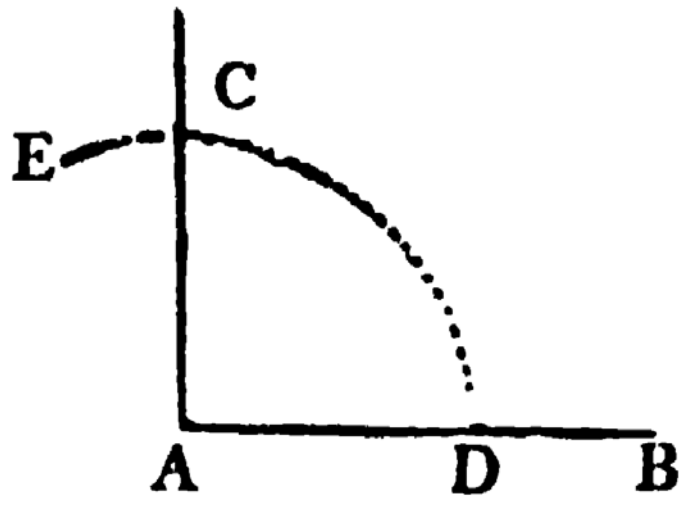
To make a triangle equal to a given trapezium ABCD



Draw the diagonal DB, and parallel to it draw CE, meeting AB produced in E. Join the points DE; so shall the triangle ADE be equal to the trapezium ABCD.

PROBLEM XXII.

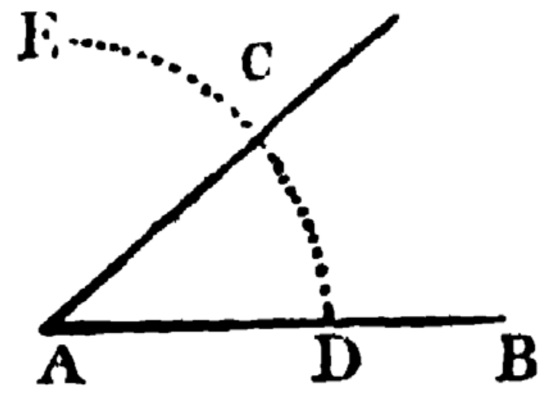
To make a right angle by the line of chords on the plane scale.



Draw the unlimited line AB; then take in your compasses 60° from the line of chords, and with A as a centre, describe the arc ED. Take 90° from the same scale, and set off that extent from D to C. Draw the line AC; and CAD will be the angle required.

PROBLEM XXIII.

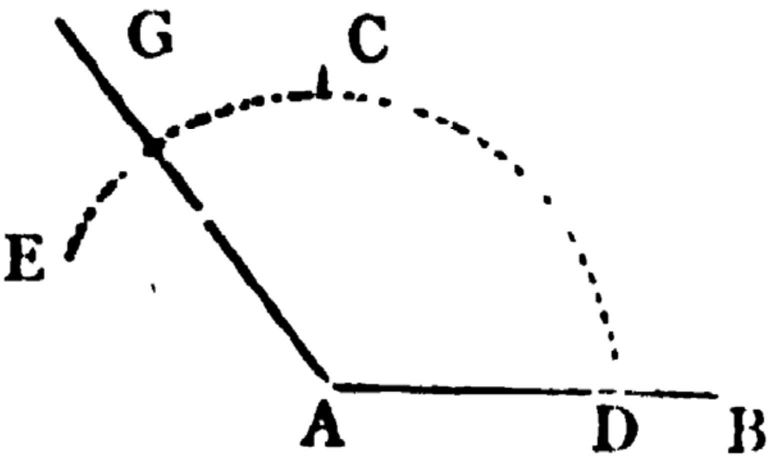
To make an acute angle that shall contain any number of degrees; suppose 35° 30′.



Draw the unlimited line AB; then take 60° in your compasses, and with A as a centre, describe the arc ED. Set off the angle 35° 30′, from D to C. Draw the line AC; and CAD will be the angle required.

PROBLEM XXIV.

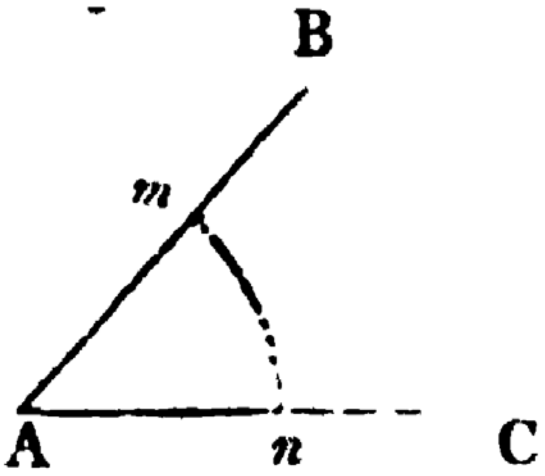
To make an obtuse angle that shall contain any number of degrees; suppose 128° 35′.



Draw the indefinite line AB; and with the chord of 60° in your compasses describe the arc DE. Set off 90° from D to C; and from C to G set off the excess above 90°, which is 38° 35′. Draw the line AG, and GAD will be the required angle.

PROBLEM XXV.

To find the number of degrees contained in any given angle BAC.

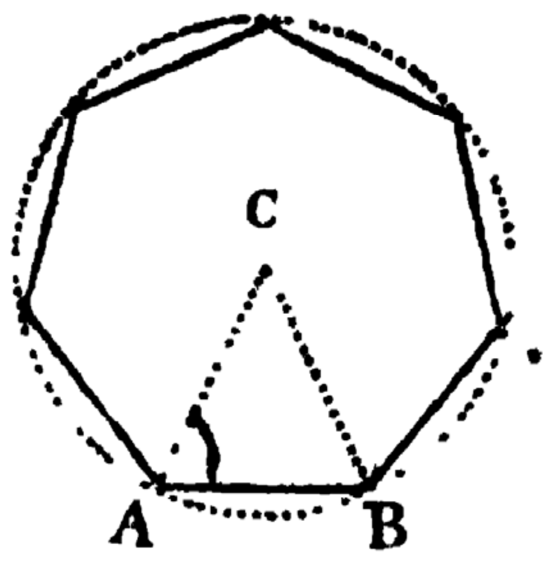


With the chord of 60, and A as a centre, describe the arc *mn*. Take the distance *mn* in your compasses, and apply it to the line of chords; and it will shew the number of degrees required.

NOTE.—Angles may be more expeditiously laid down or measured, by means of a semi-circle of brass, called "a Protractor," the arc of which is divided into 180 degrees.

PROBLEM XXVI.

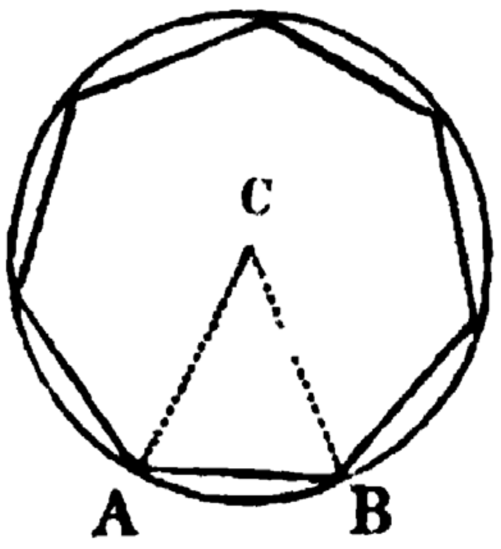
Upon a given line AB, *to make a regular polygon of any proposed number of sides.*



Divide 360° by the number of sides, subtract the quotient from 180°, and divide the difference by two. Make the angles ABC and BAC each equal to the quotient last found; and the point of intersection C, will be the centre of the circumscribing circle. With the radius AC or BC, describe the circle; and apply the chord AB to the circumference the proposed number of times, and it will form the polygon required.

PROBLEM XXVII.

In any given circle to inscribe a regular polygon of any proposed number of sides; or to divide the circumference into any number of equal parts.



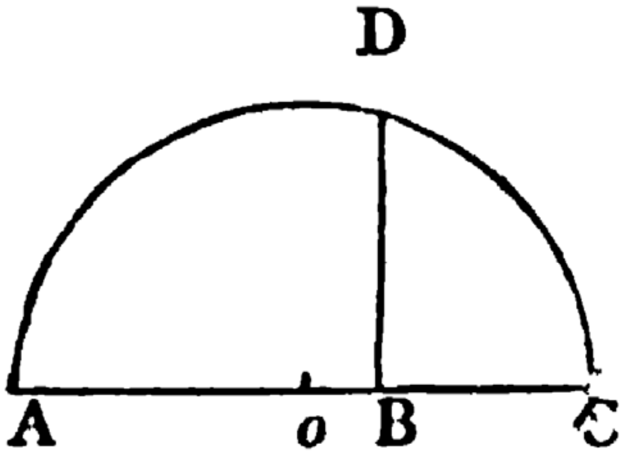
Divide 360° by the number of sides, and make the angle ACB, at the centre, equal to the number of degrees contained in the quotient; and the arc AB will be one of the equal parts of the circumference; hence the polygon may be formed.

Note.—The sum of all the interior angles of any polygon, whether regular or irregular, is equal to twice as many right angles, wanting four, as the figure has sides.

PROBLEM XXVIII.

To find a mean proportional between two given lines.

Let the given lines be AB = 32, and BC = 18.

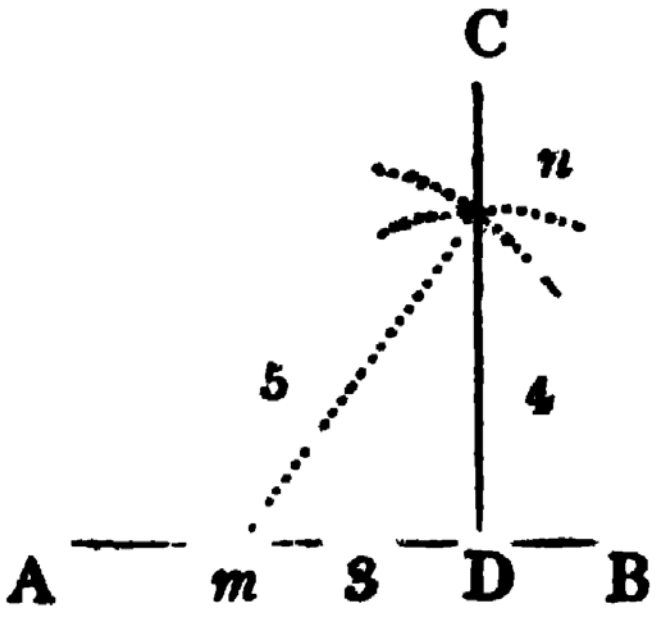


Make AC = 50, the sum of the given lines; and with the radius Ao = 25, and o as a centre, describe the semicircle ADC. From the point B, erect the perpendicular BD, and it will measure 24, the mean proportional sought.

Note.—A mean proportional between any two numbers may also be found, by multiplying them together, and extracting the square root of their product.

PROBLEM XXIX.

To raise a perpendicular from any point D, in a given line AB, by a scale of equal parts.

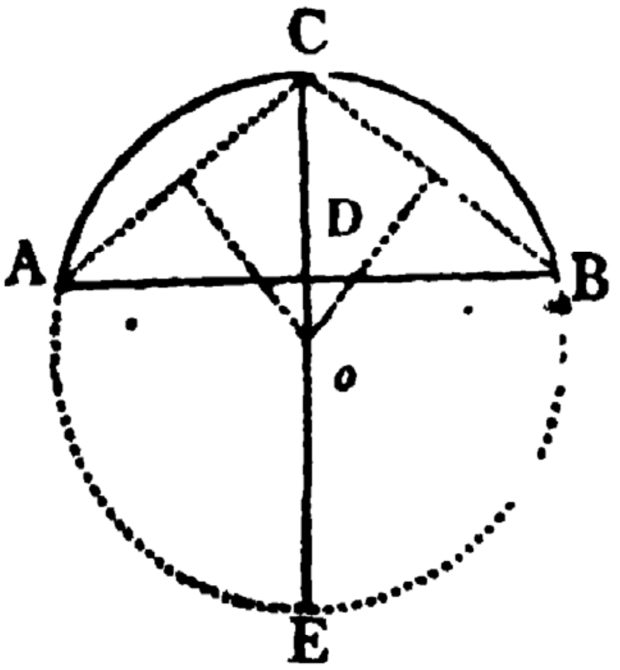


Make D*m* = 3; and from the points D and *m*, with the distances 4 and 5, describe arcs intersecting each other in *n*. From D, through the point *n*, draw the line DC, and it will be the perpendicular required.

NOTE.—This Problem may be performed by any other numbers in the same proportion; but 3, 4, and 5 are the least whole numbers that will form a right-angled-triangle.

PROBLEM XXX.

Given the span or chord line, and height or versed sine of the arch of a bridge or cellar, to find the radius of the circle that will strike the arch.



Draw the unlimited line CE; and take the versed sine from a scale of equal parts, and set it from C to D. Through the point D draw the line AB perpendicularly to CE, and make AD and BD each equal to half the given chord; also draw the chords AC, BC. Bisect either of these chords perpendicularly, with a line meeting CE in *o*, which will be the centre of the circle; hence the arch ACB may be described. Or, divide the square of half the chord by the versed sine; to the quotient add the versed sine, and the sum will be the diameter of the circle.

NOTE.—This Problem is extremely useful to Joiners in striking circular arcs, forming centres for bridges, cellars, &c. and also to Masons and Bricklayers, in describing circular pediments and other ornamental arches.

GEOMETRICAL THEOREMS,

THE

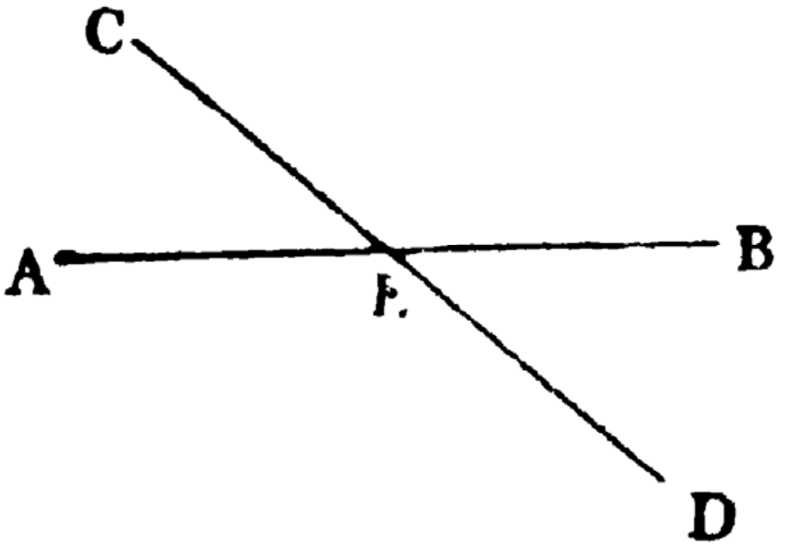
DEMONSTRATIONS

OF WHICH

May be seen in the Elements of Euclid, Simpson, and Emerson.

THEOREM I.

If two straight lines AB, CD, cut each other in the point E, the angle AEC will be equal to the angle DEB; and CEB to AED. (*Euclid* I. 15. *Simp.* I. 7 *Em.* I. 2.)



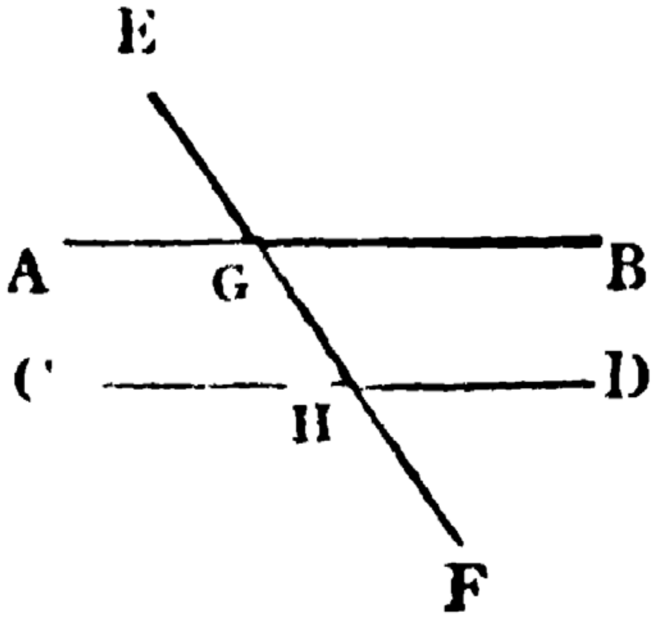
THEOREM II.

The greatest side of every triangle is opposite to the greatest angle. (*Euc.* I. 18. *Simp.* I. 13. *Em.* II. 4.)

THEOREM III.

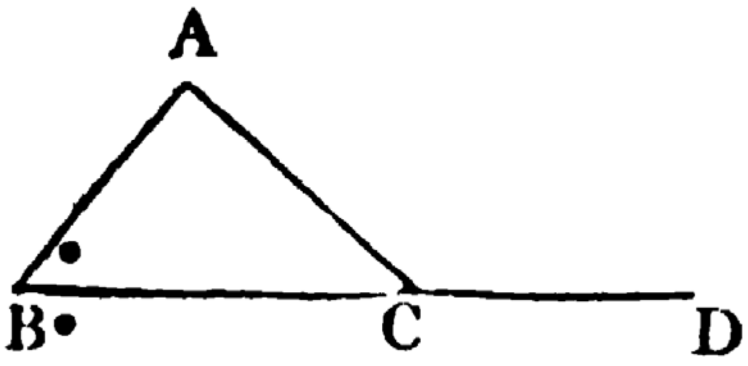
Let the right line EF fall upon the parallel right lines AB, CD; the alternate angles AGH, GHD are equal to each other; and the exterior angle EGB is equal to the interior and opposite, upon the same side

GHD; and the two interior angles BGH, GHD, upon the same side, are together equal to two right angles. (*Euc.* I. 29. *Simp.* I. 7. *Em.* I. 4.)



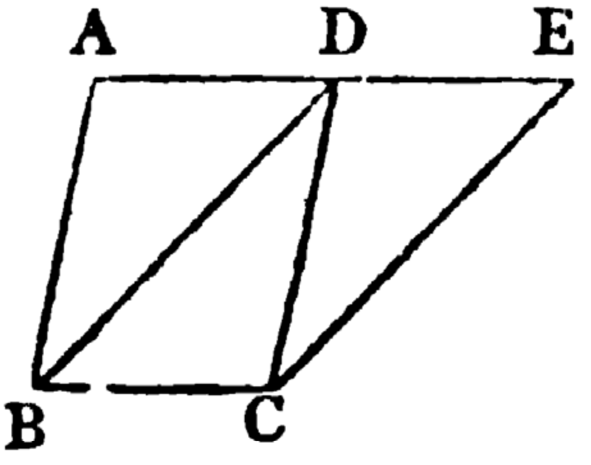
THEOREM IV.

Let ABC be a triangle, and let one of its sides BC be produced to D; the exterior angle ACD is equal to the two interior and opposite angles CAB, ABC; also the three interior angles of every triangle are together equal to two right angles. (*Euc.* I. 32. *Simp.* I. 9 & 10. *Em.* II. 1 & 2.)



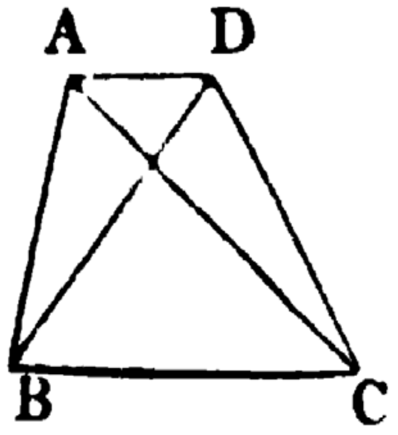
THEOREM V.

Let the parallelograms ABCD, DBCE be upon the same base BC, and between the same parallels AE, BC; the parallelogram ABCD is equal to the parallelogram DBCE. (*Euc.* I. 35. *Simp.* II. 2. *Em.* III. 6.)



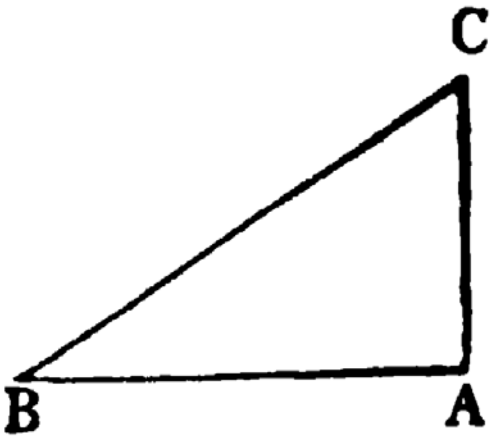
THEOREM VI.

Let the triangles ABC, DBC be upon the same base BC, and between the same parallels AD, BC; the triangle ABC is equal to the triangle DBC. (*Euc.* I. 37. *Simp.* II. 2. *Em.* II. 10.)



THEOREM VII.

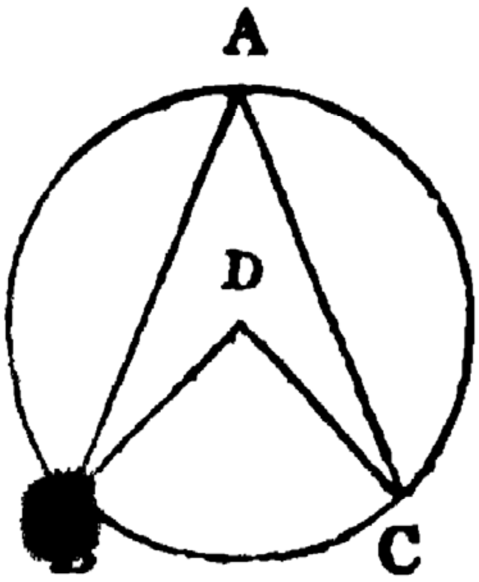
Let ABC be a right-angled triangle, having the right angle BAC; the square of the side BC is equal to the sum of the squares of the sides AB, AC. (*Euc.* I. 47. *Simp.* II. 8. *Em.* II. 21.)



NOTE.—Pythagoras, who was born about 2400 years ago, discovered this celebrated and useful Theorem; in consequence of which, it is said, he offered a hecatomb to the gods.

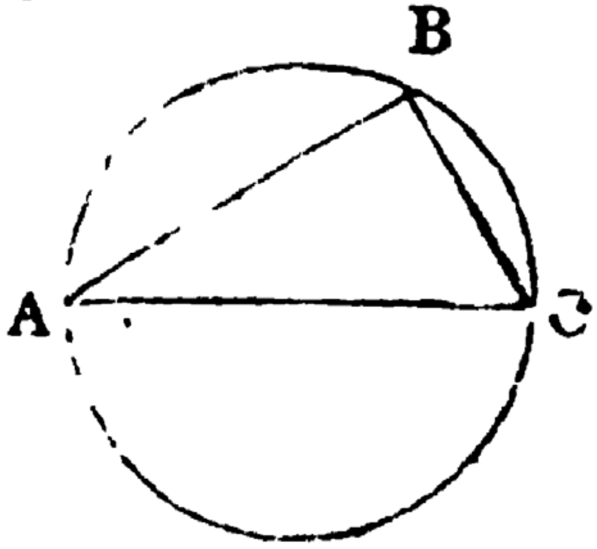
THEOREM VIII.

Let ABC be a circle, and BDC an angle at the centre, and BAC an angle at the circumference, which have the same arc BC for their base; the angle BDC is double of the angle BAC. (*Euc.* III. 20. *Simp.* III. 10. *Em.* IV. 12.)



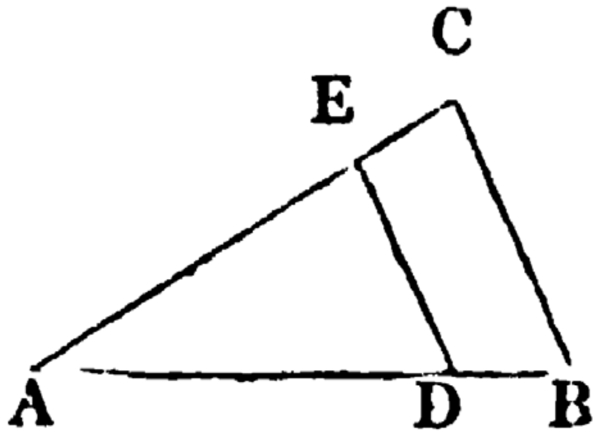
THEOREM IX.

Let ABC be a semicircle ; then the angle ABC in that semicircle, is a right-angle. (*Euc.* III. 31. *Simp.* III. 13. *Em.* VI. 14.)



THEOREM X.

Let DE be drawn parallel to BC, one of the sides of the triangle ABC ; then BD is to DA, as CE to EA. (*Euc.* VI. 2. *Simp.* IV. 12. *Em.* II. 12.)



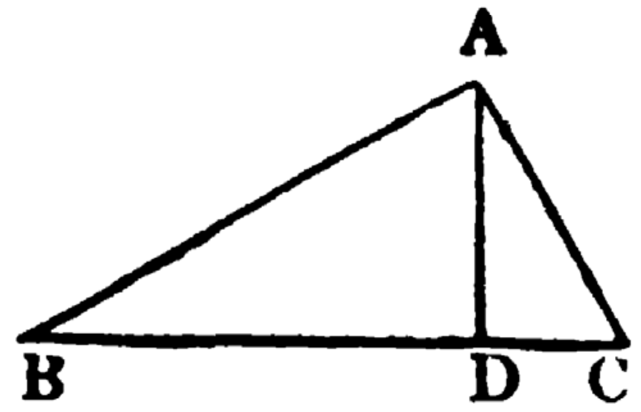
THEOREM XI.

In the preceding figure, DE being parallel to BC, the triangles ABC, ADE are equi-angular or similar ;

therefore, AB is to BC, as AD to DE; and AB is to AC, as AD to AE. (*Euc.* VI. 4. *Simp.* IV. 12. *Em.* II. 13.)

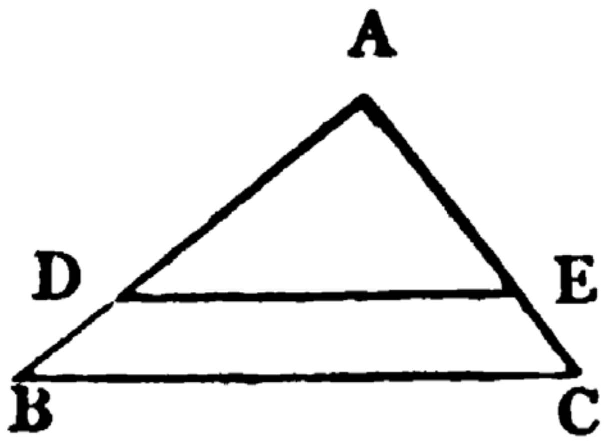
THEOREM XII.

Let ABC be a right-angled triangle, having the right angle BAC; and from the point [illegible] let AD be drawn perpendicularly to the base BC; the triangles ABD, ADC are similar to the whole triangle ABC, and to each other. Also the perpendicular AD is a mean proportional between the segments of the base; and each of the sides is a mean proportional between the base and its segment adjacent to that side; therefore, BD is to DA, as DA to DC; BC is to BA, as BA to BD; and BC is to CA, as CA to CD. (*Euc.* VI. 8. *Simp.* IV. 19. *Em.* VI. 17.)



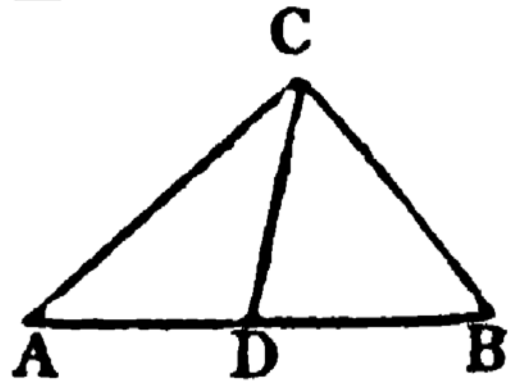
THEOREM XIII.

BC, ADE, be similar triangles, having the angle A common to both; then the triangle ABC is to the triangle ADE, as the square of BC to the square of DE. That is, similar triangles are to one another in the duplicate ratio of their homologous sides. (*Euc.* VI. 19. *Simp.* IV. 24. *Em.* II. 18.)



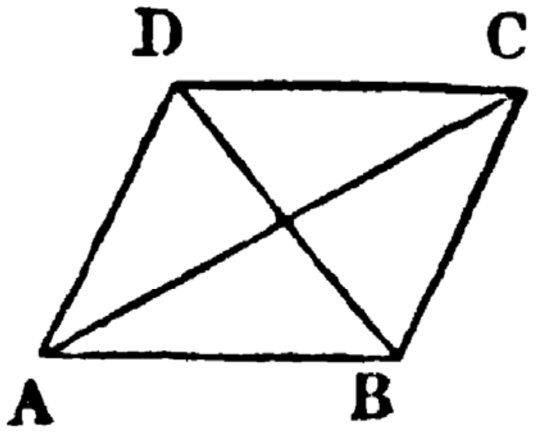
THEOREM XIV.

In any triangle ABC, double the square of a line CD, drawn from the vertex to the middle of the base AB, together with double the square of half the base AD or BD, is equal to the sum of the squares of the other sides AC, BC. (*Simp.* II. 11. *Em.* II. 28.)



THEOREM XV.

In any parallelogram ABCD, the sum of the squares of the two diagonals AC, BD, is equal to the sum of the squares of all the four sides of the parallelogram. (*Simp.* II. 12. *Em.* III. 9.)



THEOREM XVI.

All similar figures are in proportion to each other as the squares of their homologous sides. (*Simp.* IV. 26. *Em.* III. 20.)

THEOREM XVII.

The circumference of circles, and the arcs and chords of similar segments, are in proportion to each other, as the radii or diameters of the circles. (*Em.* IV. 8 & 9.)

THEOREM XVIII.

Circles are to each other as the squares of their radii, diameters, or circumferences. (*Em.* IV. 35.)

THEOREM XIX.

Similar polygons described in circles, are to each other, as the circles in which they are inscribed; or as the squares of the diameters of those circles. (*Em.* IV. 36.)

THEOREM XX.

All similar solids are to each other, as the cubes of their like dimensions. (*Em.* VI. 24.)

EXPLANATION

OF THE

Principal Mathematical Characters.

The sign or character = (called equality) denotes that the respective quantities, between which it is placed, are equal; as 4 poles = 22 yards = 1 chain = 100 links.

The sign + (called *plus*, or more) signifies that the numbers between which it is placed, are to be added together; as 9 + 6 (read 9 *plus* 6) = 15. Geometrical lines are generally represented by capital letters; then AB + CD, signifies that the line AB is to be added to the line CD.

The sign — (called *minus*, or less) denotes that the quantity which it precedes, is to be subtracted; as 15 — 6 (read 15 *minus* 6) = 9. In geometrical lines also, AB — CD, signifies that line CD is to be subtracted from the line AB.

The sign × denotes that the numbers, between which it is placed, are to be multiplied together; as 5×3 (read 5 multiplied by 3) $= 15$.

The sign ÷ signifies division; as $15 \div 3$ (read 15 divided by 3) $= 5$. Numbers placed like a vulgar fraction, also denote division; the upper number being the dividend, and the lower the divisor; as $\frac{15}{3} = 5$.

The signs : :: : (called proportionals) denote proportionality; as $2 : 5 :: 6 : 15$, signifying that the number 2 bears the same proportion to 5, as 6 does to 15: or, in other words, as 2 is to 5, so is 6 to 15.

The sign $\overline{\qquad\qquad}|$ (called *vinculum*) is used to connect several quantities together; as $\overline{\overline{9 + 3}| - 6}| \times 2 = \overline{12 - 6}| \times 2 = 6 \times 2 = 12$.

The sign ^{2}, placed above a quantity, represents the square of that quantity; as $\overline{5 + 3}|^2 = 8^2 = 8$ $= 64$.

The sign ^{3}, placed above a quantity, denotes the cube of that quantity; as $\overline{\overline{9 + 3}| - 8}|^3 = \overline{12 - 8}|$ $= 4^3 = 4 \times 4 \times 4 = 64$.

The sign $\overset{2}{\surd}$ or $\surd$, placed before a quantity, denotes the square root of that quantity; as $\sqrt{9 \times 4} = \sqrt{36} = 6$.

The sign $\overset{3}{\surd}$, placed before a quantity, represents the cube root of that quantity; as $\sqrt[3]{\overline{6 \times 4 \times 3}| - 8} = \sqrt[3]{\overline{24 \times 3}| - 8} = \sqrt[3]{72 - 8} = \sqrt[3]{64} = 4$.

PART II.

MENSURATION OF SUPERFICIES.

THE area of any plane figure is its superficial content, or the measurement of its surface, without any regard to thickness.

The dimensions of figures are taken in lineal measure. Sometimes they are taken in inches and tenths, sometimes in feet, inches, and parts; sometimes in feet, tenths, and hundredths; and sometimes in chains and links, and the area of any figure is estimated by the number of square inches, square feet, square chains, &c. contained in that figure.

---

1 Note. A measuring tape, (usually called a *box and tape*), divided into feet and inches on one side, and links on the other, is admirably calculated for taking dimensions. Tapes are of various lengths, but those of four poles, or twenty-two yards, are most useful.

2 In practice, the dimensions may generally be commodiously entered upon a rough sketch of the figure.

3 The following tables of *lineal* and *square* measures, ought to be well understood by the learner, before he proceeds further.

A TABLE OF LINEAL MEASURE.

Inches.	*Link.*	*Foot.*	*Yard.*	*Pole or Perch.*	*Chain.*	*Furlongs.*	*Mile.*
7.92=	1						
12	1.5151=	1					
36	4.5454	3=	1				
198	25	16.5	5.5=	1			
792	100	66	22	4=	1		
7920	1000	660	220	40	10=	1	
63360	8000	5280	1760	320	80	8=	1

Note.—Two yards make one fathom, and seven yards one rood of fencing or ditching.

A TABLE OF SQUARE MEASURE

SQUARE *Inches.*	SQUARE *Link.*	SQUARE *Foot.*	SQUARE *Yard.*	SQUARE *Perch.*	SQUARE *Chain.*	SQUARE *Rood.*	SQUARE *Acre.*	SQUARE *Mile.*
62.7264=	1							
144	2.2956=	1						
1296	20.6611	9=	1					
39204	625	272.25	30.25=	1				
627264	10000	4356	484	16=	1			
1568160	25000	10890	1210	40	2.5=	1		
6272640	100000	43560	· 4840	160	10	4=	1	
4014489600	64000000	27878400	3097600	102400	6400	2560	640=	1

NOTE.—Forty-nine square yards make one rood of digging.

PROBLEM I.

To find the area of a square.

RULE.

Multiply the length of one of its sides by itself, and the product will be the area.

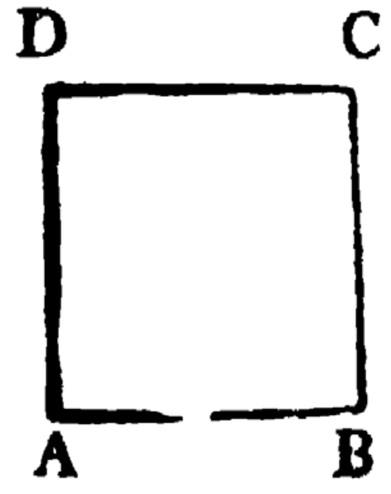
Note 1. The side of a square may be found by extracting the square root of its area.

2. When the area of a figure is found by duodecimals, it is generally said to be in feet, inches, parts, &c., but it is evident that it is in feet, twelfths, inches, &c , because 144 squares inches, and not 12, make a square foot In this work, I have, however, conformed to the general practice

3. The learner should carefully *examine*, *work over*, and *put down* all the solutions given in this book, in order that he may the better comprehend the different rules. The definitions and Rules should also be committed to memory.

EXAMPLES.

1. What is the area of the square ABCD, the side of which is 8 feet 6 inches?



By Decimals.

FEET.
8.5
8.5
425
680
72.25 *feet.*
12
3.00 *inches.*

Answer 72 *feet* 3 *inches.*

By Duodecimals.

FEET.	INCHES.
8	6
8	6
68	0
4	3
72	3 *Answer.*

2. Required the area of a square floor, whose side is 18 feet 9 inches.

Here 18 *feet* 9 *inches* = 18.75 ; *and* 18.75 × 18.75 = 351.5625 *feet* = 351 *feet*, 6 *in*. 9 *parts*, *the area required.*

3 What is the area of a square table, the side of which measures 6 feet 8 inches? *Ans.* 44 *ft.* 5 *in.* 4 *pa.*

4. The side of a square court-yard measures 85 feet 3 inches; what will it cost paving at 2*s.* 9*d.* per yard?
Ans. £111. 0*s.* $7\frac{3}{4}$*d.*

5 The base of the largest Egyptian pyramid is a square, whose side is 693 feet; how many acres of ground does it cover? *Ans.* 11*a* 0*r.* 4*p.*

6. What is the side of a square whose area is 132496 square feet? *Ans* 364 *feet.*

7. Required the side of a square garden that cost £3 18*s.* $1\frac{1}{2}$*d.* trenching at $1\frac{1}{2}$*d.* per square yard.
Ans. 25 *yards.*

Note.—The pyramids of Egypt are some of the most ancient structures in the world, their antiquity extending beyond the records of all history, and their original uses being entirely unknown. Herodotus, who wrote about 2200 years ago, speaks with as much uncertainty concerning the time of their building as we do at present. Some have conjectured that these majestic monuments were built by the children of Israel, during their bondage in Egypt, and designed as sepulchres for the Egyptian kings. Should this supposition be correct, they must have stood about 3300 years. The perpendicular height of the largest of these stupendous structures is about [illegible] feet, but if measured obliquely, it is 700 feet from the base to the termination at the top.

PROBLEM II.

To find the area of a rectangle.

RULE.

Multiply the length by the breadth, and the product will be the area.

Note 1. If the length and breadth be in inches, their product must be divided by 144, in order to obtain the area in square feet; but if one dimension be in feet, and the other in inches, the product divided by 12, will give the area in square feet. This observation, however, is not applicable, when the area is found duodecimally.

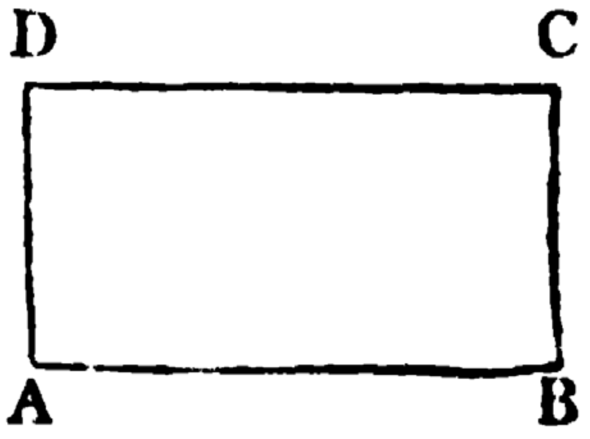
2. If the area of a rectangle be divided by one of its sides, the quotient will be the other side; but the area must first be reduced to the same denomination as the given side.

3 Acres, roods, and perches may be reduced into square links, by multiplying the whole quantity, in perches, by 625, the number of square links in a square perch.

4. In taking dimensions with a chain, it is usual to set down the number of links. In this case, five figures must be cut off, as decimals, towards the right-hand of the area, and those on the left will express the number of acres. The decimals must then be reduced into roods and perches, by multiplying them successively by 4, and by 40, cutting off five figures as before, in each product.

EXAMPLES.

1. Required the area of the rectangle ABCD, whose length AB is 12.75 chains or 1275 links, and breadth BC 6.75 chains or 675 links.



Links.
1275
675

6375
8925
7650

8.60625
4

2.42500
40

17.00000 *Ans.* 8*a.* 2*r.* 17*p.*

2. A door measures 6 feet 3 inches, by 3 feet 6 inches; what did it cost at 1*s.* 8*d.* per square foot?
Ans. £1. 16*s.* 5½*d.*

3. The length of a mahogany tea-table is 45 inches, and its breadth 42 inches; what is its area in square feet?
Ans. 13.125 *feet.*

4. How many square feet are contained in a rectangular fir deal, whose length is 18 feet, and breadth 11 inches?
Ans. 16½ *feet.*

5. Required the area of a granary floor, whose length is 45 feet 6 inches, and breadth 20 feet 10 inches.

Ans. 947 *feet*, 11 *in.*

6. How many yards of carpet, 3 quarters wide, will cover a room that measures 28 feet 6 inches, by 18 feet 9 inches? *Ans.* 79 *yds.* 6 *in.*

7. The area of a rectangular field is 14*a.* 2*r.* 11*p.*; what is its length, its breadth being 925 links?

Ans. 1575 *links.*

8. A rectangular allotment upon a common, cost £78. 1*s.* 10½*d.* digging and levelling, at £7. 10*s.* per acre, what will be the expense of fencing it half round at 5*s.* 6*d.* per rood, its length being 1225 links?

Ans. £17. 18*s.* 8*d.*

PROBLEM III.

To find the area of a rhombus or rhomboides.

RULE.

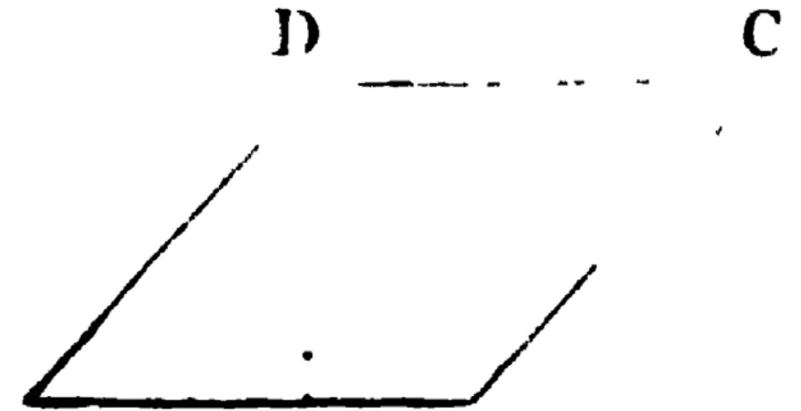
Multiply the length of the base by the perpendicular breadth, and the product will be the area.

Note 1. If the area of a rhombus or rhomboides be divided by one of its dimensions, the quotient will be the other dimension.

2. The area of a parallelogram may be found by multiplying the product of any two of its sides by the natural sine of their included angle.

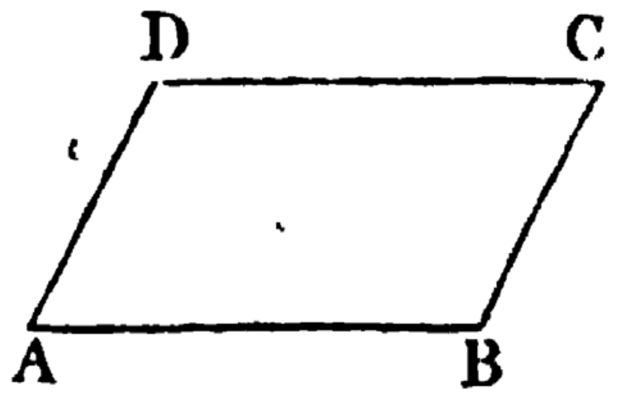
EXAMPLES.

1. Required the area of the irregular rhombus ABCD, whose base AB is 12 feet 10 inches, and perpendicular DE 9 feet 8 inches. (*See Prob. XII. Part I.*)



Here 12 *ft.* 10 *in.* = 154 *inches*, and 9 *ft.* 8 *in.* = 116 *inches*; *then* $\frac{154 \times 116}{144} = \frac{17864}{144} = 124.05555$ *feet* = 124 *ft.* 0 *in.* 8 *pa.* the area required.

2. What is the area of the regular rhomboides ABCD, whose side AB is 2185, and AD 1426 links?



By Note 2, we have 2185 × 1426 × .8660254 (*the natural sine of* 60°, *the angle* BAD) = 2698370 *sq. links* = 26*a*. 3*r*. 37*p*. *the area required.*

3. How many square yards of paving are there in a court-yard in the form of a rhombus, whose base measures 265 feet 9 inches, and perpendicular, which rises at the distance of 100 feet from the end of the base, 246 feet 3 inches? *Ans.* 7271.21527 *yards.*

4. Measuring along the base of a field in the form of a rhomboides, I found the perpendicular to rise at 678, and its length 1264 links; the remainder of the base measured 2435 links; what is the area of the field?

Ans. 39*a*. 1*r*. 15¾*p*.

[illegible] A window contains 36 panes, each in the form of a rhombus, whose base measures 11¾ inches, and perpendicular 8¼ inches; what will it cost glazing at 1*s*. 11¾*d*. per square foot? *Ans.* £2. 7*s*. 11½*d*.

6. A grass-plot, in a gentleman's pleasure-ground, cost £3. 14*s*. 1*d*. making, at 4*d*. per square yard; what is the length of the base, the perpendicular being 40 feet, and the figure a rhombus? *Ans.* 50 *feet.*

PROBLEM IV.

To find the area of a triangle, when the base and perpendicular are given.

RULE.

Multiply the base by the perpendicular, and half the product will be the area.

Or, multiply the base by half the perpendicular; or the perpendicular by half the base, and the product will be the area.

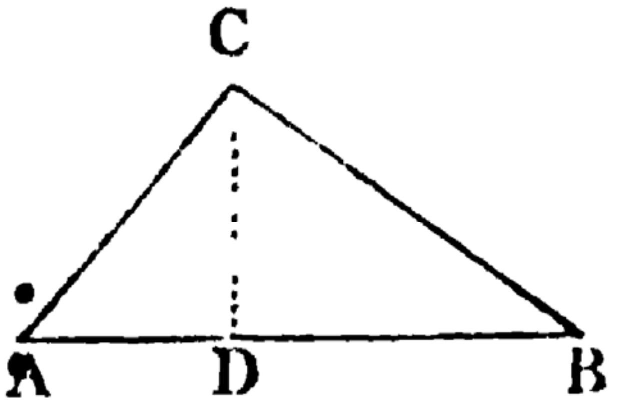
Note 1. If double the area of a triangle be divided by one of its dimensions, the quotient will be the other dimension.

2. When the base, the perpendicular, and the place of the perpendicular upon the base are given, the triangle may be truly constructed, and its other two sides measured by the scale used in the construction.

3. In order to find the true place of the perpendicular, in taking the dimensions of a triangle, or any other figure, it will be necessary to use an instrument called a "cross, or cross-staff," which may be made in the following manner. Procure a piece of board, either of sycamore, box, or mahogany, out of which form the quadrant of a circle, of about four inches radius. In it, with a fine saw, make two grooves at right angles to each other, parallel to the radii, and about half an inch deep. Through the centre of the quadrant make a circular hole, large enough to receive a staff of about three quarters of an inch in diameter. The cross or quadrant must rest and turn upon a shoulder made at a convenient distance from the top of the staff, and it may be prevented from coming off, by means of a screw passing through a hollow cylinder of wood or brass, made to fit the staff, immediately above the cross. The staff must be piked with iron at the bottom, in order to enter the ground readily. It may be about eight feet in length, and if it be divided into feet and inches, or into links, it will be found very useful in taking dimensions. Crosses may be constructed in different ways, but the cross described above I find to be most convenient.

EXAMPLES.

1. Required the area of the triangle ABC, whose base AB measures 21.68 feet, the perpendicular DC 9.46 feet, and AD 8.26 feet.



FEET.

21.68
9.46

13008
8672
19512

2)205.0928

102.5464

2. What is the area of a triangular field, the base of which measures 3568 links, the perpendicular 1589 links, and the distance between one end of the base and the place of the perpendicular 1495 links?

Ans. 28*a.* 1*r.* 15½*p.*

3. Required the area of the gable end of a house, the base or distance between the eaves being 22 feet 5 inches, and the perpendicular from the ridge to the middle of the base, 9 feet 4 inches. *Ans.* 104*ft.* 7*in.* 4*pa.*

4. After measuring along the base of a triangle 895 links, I found the place of the perpendicular, and the perpendicular itself = 994 links; the whole base measured 1958 links; what is the area of the triangle?

Ans. 9*a.* 2*r.* 37*p.*

5. How many square yards of slating are there in the hipped roof of a square building; the base or length of the eaves from hip to hip being 23 feet 9 inches, and the distance between the middle of the base and the vertex of the roof 9 feet 6 inches? *Ans.* 50*yds.* 1*ft.* 3*in.*

6. The area of a triangle is 6 acres 2 roods and 8 perches, and its perpendicular measures 826 links; what will be the expense of making a ditch, the whole length of its base, at 2*s.* 6*d.* per rood? *Ans.* £6 4*s.* 7¼*d.*

PROBLEM V.

To find the area of a triangle, the three sides only of which are given.

RULE.

From half the sum of the three sides subtract each side severally; multiply the half sum and the three remainders continually together; and the square root of the last product will be the area of the triangle.

Note 1. If a triangle be accurately laid down, from a pretty large scale of equal parts, the perpendicular may be measured, and the area found by the last Problem.

2. If the rectangle of any two sides of a triangle be multiplied by the natural sine of their included angle, the product will be double the area of the triangle; consequently, if double the area of a triangle be divided by the rectangle of any two of its sides, the quotient will be the natural sine of their included angle.

3. If the area of a triangle be divided by half the sum of the sides, the quotient will be the radius of the inscribed circle.

4. Many calculations in Arithmetic, Mensuration, &c. may be greatly facilitated by the assistance of logarithms; and it is matter of surprise that their use is not more generally taught in public schools.

The method of working by logarithms is extremely simple, and may be applied to almost every kind of calculation, yet the advantages of this great discovery have hitherto been chiefly enjoyed by those only who have leisure and inclination to cultivate the higher branches of the Mathematics.

The following are the most useful Rules on this subject, they, however, apply only to numbers greater than unity, for if the numbers be less than unity, the indices of their logarithms will be negative, which requires a little more ingenuity in the management.

Multiplication by Logarithms.

RULE.

Add the logarithms of all the factors together, and their sum will be the logarithm of the product.

Division by Logarithms.

RULE.

From the logarithm of the dividend, subtract the logarithm of the divisor, and the remainder will be the logarithm of the quotient.

Rule of Three by Logarithms.

RULE.

Add the logarithms of the second and third terms together, and from their sum subtract the logarithm of the first term, and the remainder will be the logarithm of the fourth term.

Involution, or Raising of Powers.

RULE.

Multiply the logarithm of the given number, by the index of the proposed power, and the product will be the logarithm of the power sought.

Evolution, or Extraction of Roots.

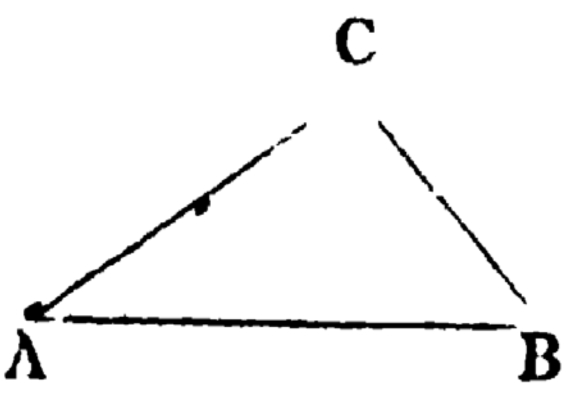
RULE.

Divide the logarithm of the given number, by the index of the proposed root, and the quotient will be the logarithm of the root required.

Further directions on this head appear to be unnecessary, as it is presumed that few Teachers are without a treatise on logarithms: it may, however, be observed, that Dr. Hutton's Mathematical Tables are considered the most useful.

EXAMPLES.

1. What is the area of the triangle ABC, the side AB measuring 25, AC 20, and BC 15 chains?



Here $\frac{25 + 20 + 15}{2} = \frac{60}{2} = 30$ = *half the sum of the sides; then* $30 - 25 = 5$, *the first remainder;* $30 -$

20 = 10, the second remainder; and 30 − 15 = 15, the third remainder; whence $\sqrt{30 \times 5 \times 10 \times 15} = \sqrt{22500}$ *= 150 square chains = 15 acres, the area required.*

By Logarithms.

The log. of . . . 30 . . . . . = 1.4771213
——— . . . 5 . . . . . = 0.6989700
——— . . . 10 . . . . . = 1.0000000
——— . . . 15 . . . . . = 1.1760913

Divide by the index of the root . . 2)4.3521826

The quotient is the log. of 150, the area 2.1760913

2. Required the area of a grass-plot in the form of an equilateral triangle, whose side is 36 feet.

Ans. 561 18446 *feet.*

3. What is the area of the gable end of a house forming an isosceles triangle, whose base is 25, and each of its other equal sides 20 feet? *Ans.* 195 15618 *feet.*

4. What is the area of a triangular field, whose three sides measure 2564, 2345, and 2139 links?

Ans. 23*a.* 2*r.* 0½*p*

5. The three sides of a triangular fish-pond measure 293, 239, and 185 yards; what did the ground which it occupies cost, at £185. per acre?

Ans. £843. 7*s.* 8*d.*

PROBLEM VI.

Any two sides of a right-angled triangle being given, to find the third side.

RULE.

1. *When the two legs are given, to find the hypothenuse.*

Add the square of one of the legs to the square of the other, and the square root of the sum will be the hypothenuse.

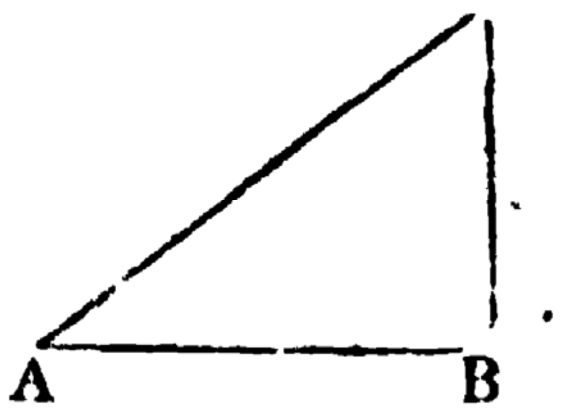
2. *When the hypothenuse and one of the legs are given, to find the other leg.*

From the square of the hypothenuse subtract the square of the given leg, and the square root of the remainder will be the required leg. *(See Theorem 7.)*

Note.—When the area of a right-angled triangle and the hypothenuse are given, the legs may be found by the following *General Rule.*—To the square of the hypothenuse add four times the area of the triangle, and the square root of this number will be the sum of the legs. From the square of the hypothenuse take four times the area of the triangle, and the square root of the remainder will be the difference of the legs. Add half the difference of the legs to half their sum, and you will obtain the greater leg, but if half the difference of the legs be taken from half their sum, the remainder will be the less leg.

EXAMPLES.

1. In the right-angled triangle ABC, are given the base AB = 36, and the perpendicular BC = 27, to find the hypothenuse AC.



Here $36^2 + 27^2 = 1296 + 729 = 2025$; *and* $\sqrt{2025} = 45$, *the hypothenuse* AC.

2. If the hypothenuse AC be 60, and the perpendicular BC 36; what is the base AB?

Here $60^2 - 36^2 = 3600 - 1296 = 2304$; *and* $\sqrt{2304} = 48$, *the base* AB.

3. Required the length of a scaling ladder to reach the top of a wall whose height is 33 feet; the breadth of the moat before it being 44 feet. *Ans.* 55 *feet.*

4. What must be the length of a shore, which, strutting 10 feet 9 inches from the upright of a building, will support a jamb 18 feet 6 inches from the ground? *Ans.* 21 *ft.* 4 *in.* 9 *pa.*

5. The distance from the ridge to the eaves of a building is 15 feet, and the perpendicular height of the gable end 9 feet; what is the breadth of the building? *Ans.* 24 *feet.*

6. A ladder 46 feet in length, being placed in a street, reached a window 26 feet from the ground, on

one side; and by turning it over, without removing the bottom, it reached another window 35 feet high, on the other side; what is the breadth of the street?

Ans. 67·79695 *feet.*

7. A castle wall there was, whose height was found
To be 100 feet from th' top to th' ground;
Against the wall a ladder stood upright,
Of the same length the castle was in height:
A waggish youngster did the ladder slide
(The bottom of it) 10 feet from the side:
Now I would know how far the top did fall,
By pulling out the ladder from the wall?

Ans. 6 *inches, nearly.*

PROBLEM VII.

To find the area of a trapezium.

RULE.

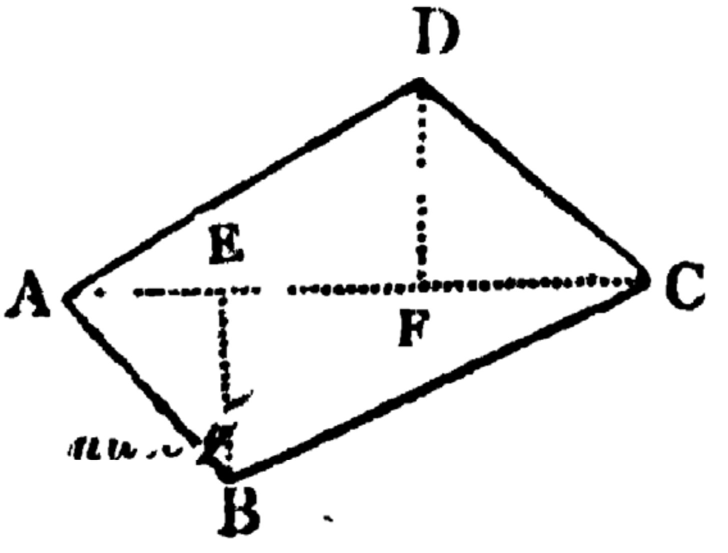
Multiply the sum of the two perpendiculars by the diagonal upon which they fall, from the opposite angles, and half the product will be the area.

Or, divide the trapezium into two triangles, in the most convenient manner; and the sum of their areas found by Problem IV, or Problem V, will be the area required.

Note.—If the trapezium can be inscribed in a circle, that is, if the sum of any two of its opposite angles be equal to 160 degrees, its area may be found as follows: From half the sum of the four sides subtract each side severally; then multiply the four remainders continually together, and the square root of the last product will be the area.

EXAMPLES.

1. It is required to lay down the trapezium ABCD, and find its area; AE being 10.26 feet, AF 25 34 feet, AC 40.18 feet, DF 14.32 feet, and BE 12.86 feet.



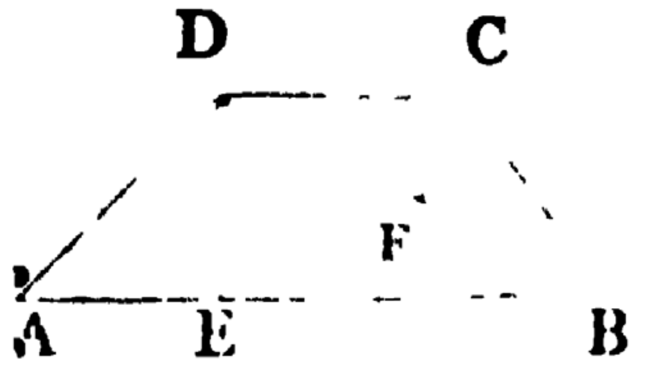
Here $\overline{14.32 + \text{[illegible]}6} \times 40.18 \doteq 27.18 \times 40.18 =$ 1092.0924; and $\frac{109\text{[illegible]}924}{2} = 546.0462$ feet, the area required.

By Logarithms.

The log. of	27.18 is .	1.4342495
The log. of	40.18 is .	1.6040099
Their sum	is .	3.0382594
The log. of	2 is .	0 3010300
The diff. is the log. of 546.0462 the area .		2.7372294

2. Lay down the following trapezium, and find its area; AE measuring 1125, AB 3243, DE 1168, DF 1216, D[illegible] 2418, and CF 610 links.

Note.—This figure is divided into two triangles, because two perpendiculars cannot be taken upon either of the diagonals.



Here $\overline{3243 \times 1168} \div 2 = 3787824 \div 2 = 1893912$, *the area of the triangle* ABD.

Also, $\overline{2418 \times 610} \div 2 = 1474980 \div 2 = 737490$, *the area of the triangle* BCD.

Then 1893912 + 737490 = 2631402 *square links* = 26*a*. 1*r*. 10*p*. *the area of the trapezium* ABCD.

3. How many square yards of paving are there in a trapezium whose diagonal is found to measure 126 feet 3 inches, and perpendiculars 58 feet 6 inches, and 65 feet 9 inches? *Ans.* 871.47569 *yards.*

4. In taking the dimensions of a trapezium, I found the first perpendicular to rise at 508, and to measure

835 links; the second at 1865, and to measure 9[illegible] links; the whole diagonal measured 2543 links; what is the area of the trapezium? *Ans.* 22*a.* 1*r.* 0*p.*

5. Lay down a trapezium, and find its area from the following dimensions; namely, the side AB measures 345, BC 156, CD 323, DA 192, and the diagonal AC 438 feet. *Ans.* 52330.33406 *feet.*

6. The sides of a trapezium, two of whose opposite angles are together equal to 180 degrees, measure 30, 32½, 35, and 37½ feet; what is its area? *Ans.* 1131 *ft.* 2 *in.* 9 *pa.*

PROBLEM VIII.

To find the area of a trapezoid.

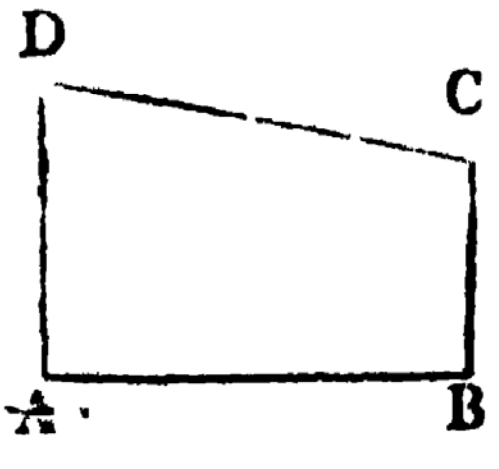
RULE.

Multiply the sum of the parallel sides by the perpendicular distance between them, and half the product will be the area.

Or, half the sum of the sides multiplied by their distance will give the area.

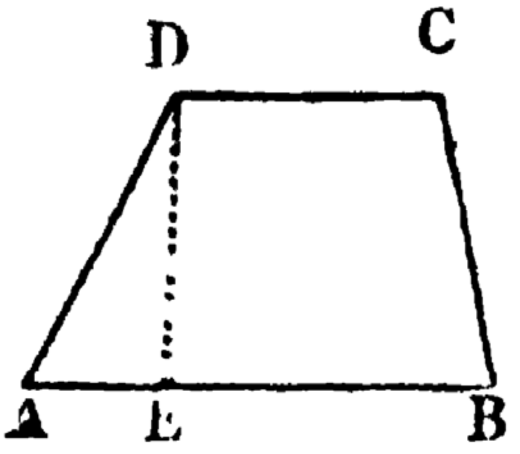
EXAMPLES.

1. What is the area of the trapezoid ABCD, the parallel sides AD and BC of which, are 25 and 18; and AB, the perpendicular distance between them, 38 feet?



Here $\overline{25 + 18} \times 38 = 43 \times 38 = 1634$; *and* $\frac{1634}{2} = 817$ *feet, the area required.*

2. Required the area of the trapezoid ABCD, whose parallel sides AB and DC measure 46 feet 10 inches, and 28 feet 4 inches; DE, the perpendicular distance between them, 26 feet 9 inches; and AE 12 feet 6 inches.



Here 46 *feet* 10 *in.* $= 46.83333$, *and* 28 *feet* 4 *inches* $= 28.83333$; *then* $\overline{46.83333 + 28.83333} \times 26.75 = 75.16666 \times 26.75 = 2010.708155$, *half of which is* $= 1005.3540775$ *feet, the area required.*

3. The parallel sides of a piece of ground measure 856 and 684 links, and their perpendicular distance 985 links, what is its area? *Ans.* 7*a.* 2*r.* $13\frac{1}{2}$*p.*

4. If the parallel sides of a garden be 65 feet 6 inches, and 49 feet 3 inches, and their perpendicular distance 56 feet 9 inches; what did it cost, at £325, 10*s* per acre? *Ans.* £24. 6*s.* $7\frac{1}{4}$*d.*

5. The hipped roof of a square building is flat at the top; the length of the eaves, from hip to hip, is 54 feet 6 inches, the side of the square at the top, is 30 feet 9 inches; and the nearest distance from the top to the eaves, is 18 feet 3 inches; how many square yards of slating are contained in the four sides of the roof? *Ans.* 345.73611 *yards.*

PROBLEM IX.

To find the area of an irregular polygon of any number of sides.

RULE.

Divide the figure into triangles and trapeziums in the most convenient manner; and find the area of each separately; then the sum of these areas will be the area of the polygon.

Note.—In calculating the content of an irregular polygon, it is sometimes more eligible to find the double area of each figure into which it is divided, and half the sum of these double areas, will be the area of the whole polygon.

EXAMPLES.

1. It is required to lay down the irregular figure ABCDEFGA, and to find its area from the following dimensions.

Diagonals.

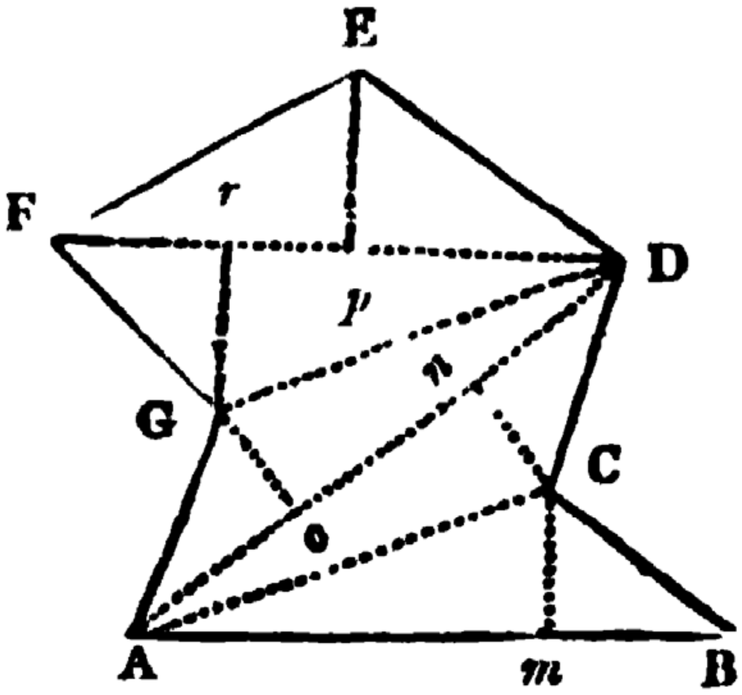
FEET.

$Am = 25$
$AB = 35.5$
$Ao = 12$
$An = 24.8$
$AD = 36$
$Dp = 16$
$Dr = 23.5$
$DF = 34$

Perpendiculars.

FEET.

$Cm = 8$
$Go = 7.5$
$Cn = 8.6$
$Ep = 10.4$
$Gr = 9.2$



CONSTRUCTION. Draw the line AB, which make $= 35.5$; and lay off 25 from A to m, at which point erect the perpendicular Cm = 8; join AC and BC, and you will have the triangle ABC.

With C as a centre, and the radius Cn describe an arc; and with A as a centre, and the radius An, describe another arc cutting the former in n. Through n draw the diagonal AD = 36, upon which lay off Ao = 12. At o erect the perpendicular Go = 7.5; join CD, DG, and GA, and the trapezium ACDG will be completed.

The trapezium DEFG may be constructed in a similar manner.

CALCULATION. *Here* $35.5 \times 8 = 284$, *double the area of the triangle* ABC.

Again, $\overline{7.5 + 8.6} \times 36 = 16.1 \times 36 = 579.6$, *double the area of the trapezium* ACDG.

Also, $\overline{10.4 + 9.2} \times 34 = 19.6 \times 34 = 666.4$, *double the area of the trapezium* DEFG.

$$\textit{Then, } \frac{284 + 579.6 + 666.4}{2} = \frac{1530}{2} = 765 \textit{ feet,}$$

the area of the irregular polygon required.

2. It is required to lay down a pentangular field, and find its annual value at £2. 5s. per acre; the first side measuring 926, the second 536, the third 835, the fourth 628, and the fifth 587 links; and the diagonal from the first angle to the third 1194, and that from the third to the fifth 1223 links. *Ans.* £18. 10*s.* 7½*d.*

Note.—This field is divided into three triangles, the areas of which may be found by Problem V.

PROBLEM X.

Given the side of a regular polygon, to find the radius of its inscribed or circumscribing circle.

RULE.

Multiply the given side by the number standing opposite to the name of the polygon, in the third, or fourth column of the following table, as the case requires; and the product will be the radius of the inscribed or circumscribing circle.

I. TABLE OF POLYGONS, &c.

No. of SIDES.	NAMES.	*Rad. of the Inscribed Circle.*	*Rad. of the Circums. Circle.*
3	Trigon, or equil. triangle	0.2886751	0.5773503
4	Tetragon, or square .	0.5000000	0.7071068
5	Pentagon . . .	0.6881910	0.8506508
6	Hexagon . . .	0.8660254	1.0000000
7	Heptagon . . .	1.0382617	1.1523825
8	Octagon . . .	1.2071068	1.3065630
9	Nonagon . . .	1.3737387	1.4619022
10	Decagon . . .	1.5388418	1.6180340
11	Undecagon . . .	1.7028437	1.7747329
12	Duodecagon . . .	1.8660254	1.9318516

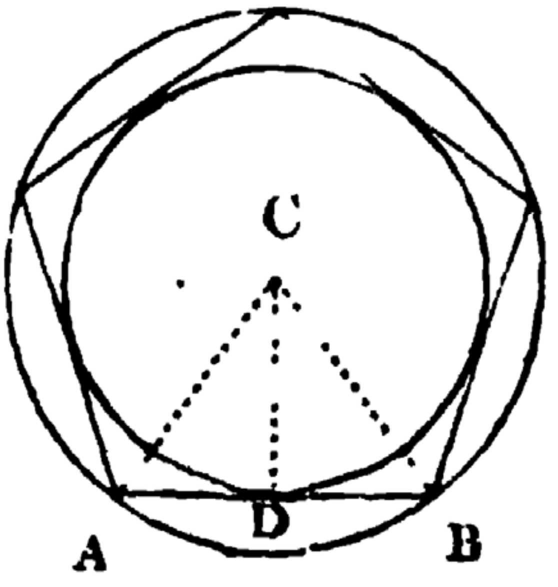
Note 1. If the radius of a circle be given, the side of any inscribed polygon may be found by dividing the given radius by the number standing opposite to the name of the polygon, in the fourth column of the preceding table.

2. By the assistance of this Problem, any regular polygon, whose side is given, may be easily constructed in the following manner. Find the radius of the circumscribing circle, and to the circumference of this circle apply the given side the proposed number of times, and you will have the polygon required. Or if the radius of a circle be given, find the side of the inscribed polygon, with which proceed as before.

3. The perpendicular let fall from the centre of a regular polygon upon one of its sides is equal to the radius of the inscribed circle, and the sum of the sides is equal to the product of one side multiplied by the number of sides.

EXAMPLES.

1. The side of a regular pentagon is 21.75; what are the radii of the inscribed and circumscribing circles?



Here .688191 × 21.75 = 14.96815425 = DC, *the radius of the inscribed circle; and* .8506508 × 21.75 = 18.5016549 = AC, *the radius of the circumscribing circle.*

2. If the side of a regular heptagon be 25.25; what is the radius of the circumscribing circle?

Ans. 29.097658.

3. If the side of an octagonal grass-plot, in a gentleman's pleasure-ground, measure 86 feet 10 inches; what will be the expense of making a gravel-walk from the middle of one of its sides to the middle of the opposite side, at $2\frac{1}{2}d.$ per yard, lineal measure?

Ans. 14*s.* $6\frac{1}{2}d.$

4. If the radius of a circle be 65 feet; what is the sum of the sides of its inscribed nonagon?

Ans. 400.16356 *feet.*

PROBLEM XI.

To find the area of a regular polygon.

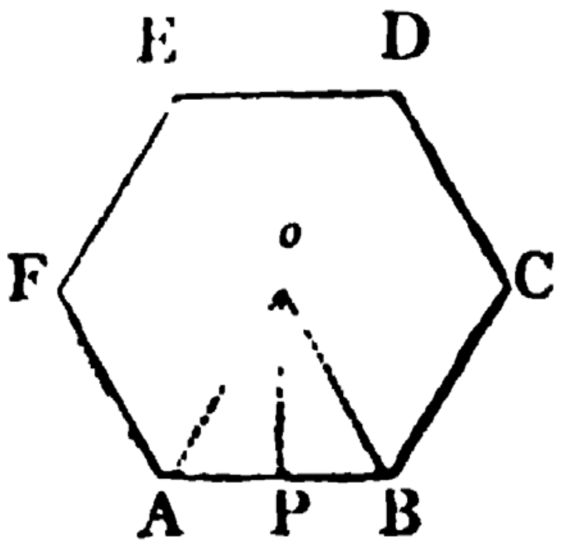
RULE.

Multiply the sum of the sides, or perimeter of the polygon, by the perpendicular demitted from its centre to one of the sides, and half the product will be the area.

Note.—If double the area of a regular polygon be divided by the perpendicular, the quotient will be the sum of the sides.

EXAMPLES.

1. Required the area of the regular hexagon ABCDEF, whose side AB is 20 feet 6 inches, and perpendicular P*o* is 17 feet 9 inches.



Here $20.5 \times 6 \times 17.75 = 2183.25$; *and* $\frac{2183.25}{2}$ $= 1091.625$ *feet* = 1091 *ft.* 7 *in.* 6 *pa. the area required.*

2. What is the area of a court-yard in the form of a regular pentagon, whose side measures 92 feet 6 inches, and perpendicular 63 feet 8 inches?

Ans. 14722 91666 *feet.*

3. Required the area of a heptagonal stone, whose side measures 8 feet 9 inches, and perpendicular 9 feet.

Ans. 275 *ft* 7 *in.* 6 *pa.*

4. What will the floor of an octagonal summer-house cost paving with black and white marble, at 1s. 6*d.* per square foot, the side of which measures 9 feet 6 inches, and the nearest distance from one of its sides, to the opposite side 22 feet 11 inches?

Ans. £97. 19*s.* 4½*d.*

5. A hexagonal piece of ground, in a gentleman's park, cost £29. 10*s.* 5¾*d.* planting with trees, at £5. 10*s.* per acre; and a gravel-walk leading from the middle of one of its sides to the middle of the opposite side, cost £2. 3*s.* 3¾*d.* making, at 3*d.* per yard, lineal measure; what was the expense of fencing the perimeter of the polygon, at 6*s.* 6*d.* per rood?

Ans. £27. 17*s.*

PROBLEM XII.

To find the area of a regular polygon, when the side only is given.

RULE.

Multiply the square of the given side by the number or area standing opposite to the name of the polygon, in the following Table, and the product will be the area. (This Rule is founded on Theorem XIII.)

II. TABLE OF POLYGONS, &c.

No. of SIDES.	NAMES	*Multiplier* or AREAS.	*Angle* ACB.	*Angle* DAC.
3	Trigon . . .	0.4330127	120°	30°
4	Tetragon . . .	1.0000000	90°	45°
5	Pentagon . . .	1.7204774	72°	54°
6	Hexagon . . .	2.5980762	60°	60°
7	Heptagon .	3.6339124	$51\frac{3}{7}$°	$64\frac{2}{7}$°
8	Octagon . . .	4.8284271	45°	$67\frac{1}{2}$°
9	Nonagon . . .	6.1818242	40°	70°
10	Decagon . . .	7.6942088	36°	72°
11	Undecagon . .	9.3656399	$32\frac{8}{11}$°	$73\frac{7}{11}$°
12	Duodecagon . .	11.1961524	30°	75°

Note 1.—If the area of a polygon be divided by the number standing opposite to its name, in the foregoing table, the quotient will be the square of the polygon's side.

2. The multipliers in the table of polygons, Problem X. are the radii of the inscribed and circumscribing circles, when the side of the polygon is unity, or 1, and may be found by trigonometry, in the following manner. Divide 360 degrees by the number of sides, and the quotient will be the angle ACB at the centre of the polygon, half of which will be the angle ACD; then say, as the nat. sine of the angle ACD is to AD, so is the nat. co-sine of the angle ACD to CD, the radius of the inscribed circle, or the perpendicular of the polygon; and, as the nat. sine of ACD is to AD, so is the radius (1) to AC, the radius of the circumscribing circle.

The multipliers in the last table, are the areas of their respective polygons, when the side is 1, and may be found thus. Multiply the perpendicular or number in column the third, table 1, by .5, half of AB, and the product will be the area of the triangle ACB, which being multiplied by the number of sides, we obtain the area or multiplier in table 2.

Or if the nat. tangent of the angle DAC be multiplied by the number of sides, one fourth of the product will be the multiplier. (See the figure, Problem X.)

EXAMPLES.

1. If the side of a pentagon be 8 feet 4 inches; what is its area?

Here 8 *feet* 4 *inches* = $8\frac{1}{3}$ *feet* = $\frac{25}{3}$, *and* $\frac{25}{3} \times \frac{25}{3} = \frac{625}{9}$, *the square of the side; then* $1.7204774 \times \frac{625}{9} = \frac{1075.298375}{9} = 119.4775972$ *feet, the area required.*

2. What is the area of the base of a hexagonal stone pillar, whose side measures 1 foot 6 inches? *Ans.* 5.84567 *feet.*

3. If the side of an octagonal brick pillar measure 1 foot 5 inches; what is the area of its base? *Ans.* 9 *ft.* 8 *in.* 3 *pa.*

4. Required the area of a decagon whose side measures 25 feet 9 inches. *Ans.* 5101 *ft.* 8 *in.* 10 *pa.*

5. A gardener wishes to make a hexagonal grass-plot that shall contain 260 square yards; what must be the length of its side? *Ans.* 10 *yards.*

PROBLEM XIII.

The diameter of a circle being given, to find the circumference; or, the circumference being given, to find the diameter.

RULE I.

As 7 is to 22, so is the diameter to the circumference, or, as 22 is to 7, so is the circumference to the diameter

RULE II.

As 113 is to 355, so is the diameter to the circumference: or, as 355 is to 113, so is the circumference to the diameter.

RULE III.

Multiply the diameter by 3.1416, and the product will be the circumference: or, divide the circumference by 3.1416, and the quotient will be the diameter.

Note 1.—There is no figure that affords a greater variety of useful properties than the circle, nor is there any that contains so large an area within the same perimeter.

The ratio of the diameter of a circle to its circumference has never yet been exactly determined, although this celebrated Problem, called the *squaring of the circle*, has engaged the attention and exercised the abilities of the ablest mathematicians, both ancient and modern. But though the relation between the diameter and circumference cannot be exactly defined in known numbers; yet approximating ratios have been determined, sufficiently correct for practical purposes

Archimedes, a native of Syracuse, who flourished about 200 years before the Christian æra, after attempting in vain to determine the true ratio of the diameter to the circumference, found it to be nearly as 7 to 22

The proportion given by Vieta, a Frenchman, and Metius, a Dutchman, about the end of the 16th century, is as 113 to 355, which is rather more accurate than the former. and is a very commodious ratio, for being reduced into decimals, it agrees with the truth to the sixth figure inclusively.

The first, however, who ascertained this ratio to any great degree of exactness, was Ludolph Van Ceulen, a Dutchman. He found that if the diameter of a circle be 1, the circumference will be 3.14159265358979323846264338327950288 nearly, which is true to 36 places of decimals. This was thought so extraordinary a performance, that the numbers were cut on his *tomb-stone*, in St Peter's church-yard, at Leyden.

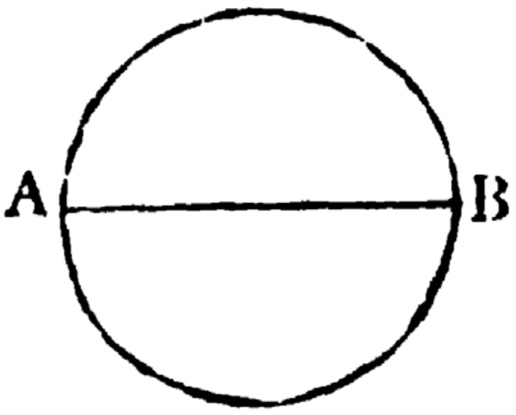
Since the invention of fluxions, by the illustrious Sir Isaac Newton, the squaring of the circle has become more easy, and the late ingenious Mr. Abraham Sharp, of Little Horton, near Bradford, in Yorkshire, has not only confirmed Ceulen's ratio, but extended it to 72 places of decimals.

Mr John Machin, professor of Astronomy in Gresham College, London, has also given us a quadrature of the circle, which is true to 100 places of figures, and even this has been extended, by the French mathematicians, to 128.

[2] The first Rule is the proportion of Archimedes, the second that of Vieta and Metius, and the third is an abridgement of Van Ceulen's ratio. This Rule is not quite so accurate as the second, but is most commonly used, as being most convenient, and in most cases, correct enough for practice.

EXAMPLES.

1. If the diameter AB of a circle be 12; what is the circumference?



By Rule I.

As 7 : 22 :: 12 : $\frac{264}{7}$ = 37.714285, *the circumference required.*

By Rule II.

As 113 : 355 :: 12 : 37.699115, *the circumference required.*

By Rule III.

Here 3.1416 × 12 = 37.6992, *the circumference required.*

2. If the circumference of a circle be 45; what is the diameter?

By Rule I.

As 22 : 7 :: 45 : 14.318181, *the diameter required.*

By Rule II.

As 355 : 113 :: 45 : 14.323943, *the diameter required.*

By Rule III.

Here $\frac{45}{3.1416}$ = 14.323911, *the diameter required.*

3. If the diameter of a well be 3 feet 9 inches; what is its circumference? *Ans.* 11 *ft.* 9 *in.* 4 *pa.*

4. The diameter of a circular plantation is 100 yards; what did it cost fencing round, at 6*s.* 9*d.* per rood? *Ans.* £15. 2*s.* 11¼*d.*

5. What is the diameter of a stone column whose circumference measures 9 feet 6 inches? *Ans.* 3 *ft.* 0 *in.* 3 *pa.*

6. The circumference of the earth is 25000 miles; what is its diameter, supposing it a perfect sphere? *Ans.* 7957.72854 *miles.*

7. The diameter of the sun is 883220 miles; what is his circumference? *Ans.* 2774723.952 *miles.*

8. The circumference of the moon is 6850 miles; what is her diameter? *Ans.* 2180.41762 *miles.*

9. The diameter of Venus is 7680 miles; what is her circumference? *Ans.* 21127.488 *miles.*

Note.—Those who are desirous of making themselves acquainted with the method of finding the distances of the sun, moon, and planets from the earth, and also their diameters, are referred to Martin's Trigonometry, Vol. I. page 208. Ferguson's Astronomy, page 100. and Bonnycastle's Astronomy, page 277.

PROBLEM XIV.

To find the length of any arc of a circle.

RULE I.

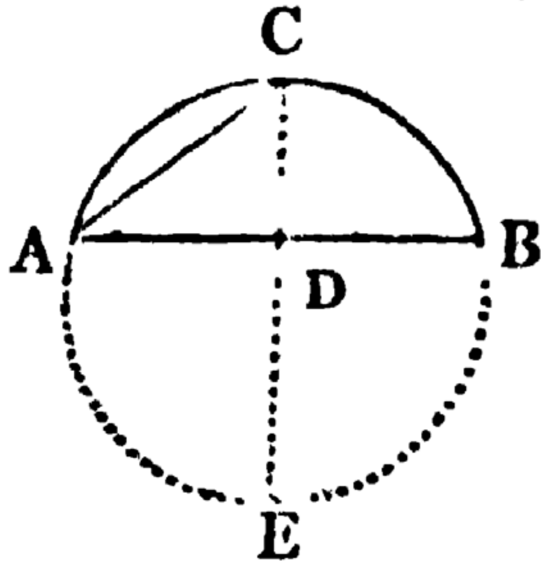
From 8 times the chord of half the arc subtract the chord of the whole arc, and ⅓ of the remainder will be the length of the arc, *nearly.*

Note 1. Half the chord of the whole arc, the chord of half the arc, and the versed sine are sides of a right-angle triangle; any two of which being given, the third may be found by Problem VI.

2. The difference of the diameters of any two circles multiplied by 3.1416, will give the difference of their circumferences, and vice versa.

EXAMPLES.

1. The chord AB of the whole arc is 24, and the versed sine CD 9; what is the length of the arc ACB?



Here $12^2+9^2=144+81=225$; *and* $\sqrt{225}=15$ $=$ AC, *the chord of half the arc*; *then* $\frac{15\times 8-24}{3}=\frac{120-24}{3}=\frac{96}{3}=32$, *the length of the arc required.*

2. The chord of the whole arc is 45, and the chord of half the arc is 25.5; what is the length of the arc?
Ans. 53.

3. The chord of half the arc is 21.25, and the versed sine 10; what is the length of the arc?
Ans. 44.1666.

4. The chord of the whole arc is 30, and the versed sine 8; what is the length of the arc? *Ans.* $35\frac{1}{3}$.

RULE II.

Divide the square of half the chord by the versed sine; to the quotient add the versed sine, and the sum will be the diameter.

Subtract $\frac{41}{50}$ of the versed sine from the diameter; divide $\frac{2}{3}$ of the versed sine by the remainder; and to the quotient last found, add 1; then this sum being multiplied by the chord of the whole arc, will give the length of the arc, *nearly*.

Note.—When great accuracy is required, the second Rule should be used, as the first gives the length of the arc too little; though near enough for most cases in practice.

EXAMPLES.

1. The chord AB is 24, and the versed sine CD 9; what is the length of the arc ACB?

Here $\frac{12^2}{9}+9=\frac{144}{9}+9=16+9=25=$ *the diameter* CE; *and* $9\times\frac{2}{3}\div\left(25-\frac{9\times 41}{50}\right)=6\div\overline{25-7.38}=6\div 17.62=.34052$; *then* $\overline{1+.34052}\times 24=1.34052\times 24=32.17248$, *the length of the arc required.*

2. The upper part or head of a window is the segment of a circle, whose chord line measures 6 feet 9 inches, and versed sine 2 feet 6 inches; what is the length of the arch? *Ans.* 8 *ft.* 11 *in.* 11 *pa.*

3. What is the length of the circular arch of a bridge, the span of which is 15 feet 6 inches, and height above the top of the piers, 6 feet 9 inches?

Ans. 22 *ft.* 4 *in.* 9 *pa.*

4. If the span of the circular roof of a cellar be 21 feet 9 inches; what is the length of the arch, its height being 6 feet 6 inches? *Ans.* 26 *ft.* 7 *in.* 5 *pa.*

5. If the span of a circular pediment be 18 feet 6 inches; what is the length of the arch, its height above the top of the entablature being 4 feet 6 inches?

Ans. 21 *ft.* 3 *in.* 7 *pa.*

RULE III.

As 180 is to the number of degrees in the arc, so is 3.1416 times the radius to its length. Or, multiply together the number of degrees in the arc, the radius, and the number .01745329; and the product will be the length of the arc.

Note.—The length of the arc of a semicircle, a quadrant, &c. may also be found by taking one-half, one-fourth, &c. of the whole circumference.

EXAMPLES.

1. Required the length of an arc of 46 degrees 35 minutes, the radius being 12 feet.

As 180° : 46° 35′ :: 3.1416 × 12 : 9.75641, *the length of the arc required.*

2. What is the length of the semicircular arch of a bridge, the span of which is 18 feet 6 inches?

Ans. 29 *ft.* 0 *in.* 8 *pa.*

3. The radius of a cart wheel, from the centre of the nave to the outside of the felloe, is 2 feet 3 inches; what is the length of one-sixth of the rim or circumference?

Ans. 2 *ft.* 4 *in.* 3 *pa.*

Note.—Those who wish to understand the principles upon which carriage wheels should be made so that the carriage may be drawn by the least power, may consult Marrat's Mechanics, Article 788.

PROBLEM XV.

To find the area of a circle.

RULES.

1. Multiply half the circumference by half the diameter, and the product will be the area. Or, divide the product of the whole circumference and diameter by 4, and the quotient will be the area.

2. Multiply the square of the diameter by .7854, and the product will be the area.

3. Multiply the square of the circumference by .07958, and the product will be the area.

Note 1. If the area of a circle be divided by .7854, the quotient will be the square of the diameter.

2. A circle may be considered as a regular polygon of an infinite number of sides, the perimeter of which being equal to the circumference, and the perpendicular equal to the radius: consequently by Problem XI. the area of a circle is equal to half the circumference multiplied by half the diameter. Also, the area of a circle whose diameter is 1, is .7854 nearly; and by Theo. 18, circles are to each other as the squares of their diameters; hence we derive the *second rule*. The area of a circle whose circumference is 1, is .07958, and the areas of circles are to each other as the squares of their circumferences: hence our *third rule*.

The following Rules will solve most of the useful Problems relating to the circle and its equal or inscribed square, &c.

Rule 1. The diameter of a circle multiplied by .8862269, will give the side of a square equal in area.

2. The circumference of a circle multiplied by .2820948, will give the side of a square equal in area.

3. The diameter of a circle multiplied by .7071068, will give the side of the inscribed square.

4. The circumference of a circle multiplied by .2250791, will give the side of the inscribed square.

5. The area of a circle multiplied by .6366197, and the square root of the product extracted, will give the side of the inscribed square.

6. The side of a square multiplied by 1.414214, will give the diameter of its circumscribing circle.

7. The side of a square multiplied by 4.442883, will give the circumference of its circumscribing circle.

8. The side of a square multiplied by 1.128379, will give the diameter of a circle equal in area.

9. The side of a square multiplied by 3.544908, will give the circumference of a circle equal in area.

Note—When the diameter of a circle is 1, its area is found to be .785398163 3974483096156608458198757?1, which is true to 36 places of decimals, but .7854 is sufficiently correct for all practical purposes.

EXAMPLES.

1. What is the area of a circle whose diameter is 106, and circumference 333 feet?

Here $\overline{333 \times 106} \div 4 = 35298 \div 4 = 8824\frac{1}{2}$ *feet, the area required.*

2. Required the area of the end or base of a roller, whose diameter is 2 feet 3 inches.

Here $2.25 \times 2.25 = 5.0625$, *the square of the diameter; and* $5.0625 \times .7854 = 3.97608$ *feet* = 3 *ft.* 11 *in.* 8 *pa. the area required.*

3. If the circumference of a cylindrical stone column be 7 feet 9 inches; what is the area of its base?

Ans. 4 *ft.* 9 *in* 4 *pa.*

4. The diameter of a cylindrical vessel is 3 feet 6 inches; what is the area of its bottom?

Ans. 9 *ft.* 7 *in.* 5 *pa.*

5. The diameter of a circular building at the iron foundry of Messrs. Fenton, Murray, and Wood, in Leeds, measures 73 feet 3 inches; how many square yards of paving are contained in the ground floor?

Ans. 468.23475 *yards.*

6. In the midst of a meadow well stored with grass,
I engag'd just one acre to tether my ass;
What length must the cord be, that he, feeding all round,
May not graze less or more than an acre of ground?

Ans. 39.25068 *yards.*

PROBLEM XVI.

To find the area of a sector of a circle.

RULE I.

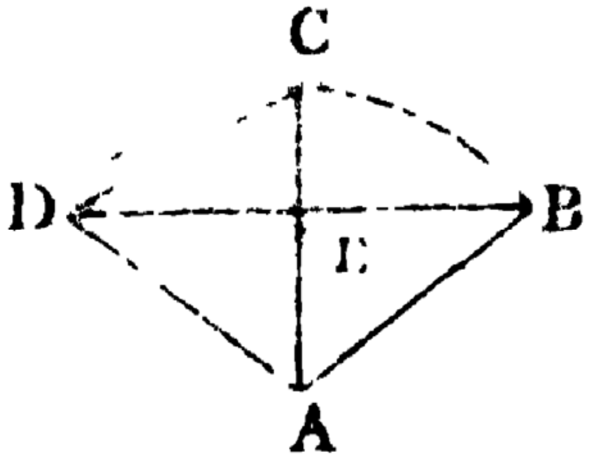
Multiply the length of the arc by the radius of the sector, and half the product will be the area.

Note 1. The length of the arc may be found by Problem XIV.

2. If the sector be greater than a semicircle, find the arc of the remaining sector, which subtract from the circumference of the whole circle, and the remainder will be the length of the arc required.

EXAMPLES.

1. The radius AD is 15, the chord BD of the whole arc 24, and the versed sine CE 6; what is the area of the sector ABCD?



Here $\sqrt{12^2+6^2} = \sqrt{144+36} = \sqrt{180} = 13.4164$ = DC, *the chord of half the arc; and* $\frac{13.4164\times 8-24}{3}$ $= \frac{107.3312-24}{3} = \frac{83.3312}{3} = 27.77706$, *the length of the arc; then* $\overline{27.77706\times 15}\div 2 = \frac{416.6559}{2} =$ 208.32795, *the area required.*

2. What is the area of the sector of a circle whose radius is 15, and the chord of the whole arc 18 feet?

Ans. 144 *ft.* 8 *in.* 10 *pa.*

3. If the radius of a sector be 12 feet 6 inches, and the length of the arc 16 feet, what is its area?

Ans. 100 *feet.*

4. The chord of half the arc is 42 feet 6 inches, and the versed sine 20 feet; what is the area of the sector?

Ans. 1994 *ft.* 4 *in.* 9 *pa.*

5. What is the area of a sector greater than a semicircle; the chord of the whole arc of the remaining sector being 72 feet, the chord of half the arc 45 feet, and the radius 37 feet 6 inches?

Ans. 2617 *ft.* 10 *in.* 6 *pa.*

RULE II.

As 360 is to the degrees in the arc of the sector, so is the area of the whole circle to the area of the sector.

Note 1. The area of a semicircle, a quadrant, &c. may be most easily found by taking one-half, one-fourth, &c of the area of the whole circle

2. The area of a circle may be found by multiplying the square of the radius by 3.1416; consequently, if the area be divided by 3.1416, the quotient will be the square of the radius

EXAMPLES.

1. The arc of a sector contains 35 degrees, and its radius is 45 feet; what is its area?

Here $45^2 \times 3.1416 = 2025 \times 3.1416 = 6361.74$, *the area of the whole circle; then as* $360^\circ : 35^\circ :: 6361.74 : 618.5025$ *feet* $= 618$ *ft.* 6 *in the area required.*

2. The radius of a sector is 25 feet 9 inches, and the length of its arc 218 degrees 54 minutes; what is its area? *Ans.* 1266 *ft.* 7 *in.* 6 *pa.*

3. Required the area of the semicircle, the radius of which is 18 feet 3 inches? *Ans.* 523 *ft.* 2 *in.* 1 *pa.*

4. What is the area of a quadrant whose radius is 27 feet 6 inches? *Ans.* 593 *ft.* 11 *in.* 6 *pa.*

5. What is the area of a sector whose radius is 50 feet, and the length of its arc 60 degrees? *Ans.* 1309 *feet.*

PROBLEM XVII.

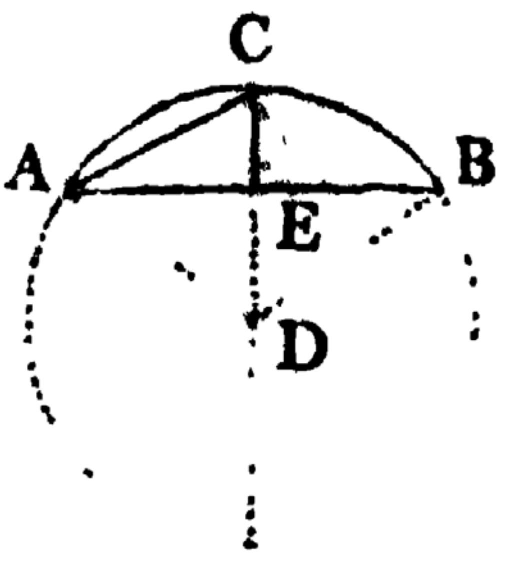
To find the area of the segment of a circle.

RULE I.

Find the area of the sector, having the same arc as the segment; also, find the area of the triangle formed by the chord of the segment and the radii of the sector; then the difference of these areas, when the segment is less than a semicircle, or their sum, when it is greater, will be the area of the segment.

EXAMPLES.

1. The radius AD is 25, and the chord AB of the whole arc 40; what is the area of the segment ABC?



Here $\sqrt{25^2 - 20^2} = \sqrt{625 - 400} = \sqrt{225} = 15$ = DE, *therefore the versed sine* CE = 10.

Again, $\sqrt{20^2 + 10^2} = \sqrt{400 + 100} = \sqrt{500} = 22.36068$ = AC, *the chord of half the arc;* *and*

$$\frac{22.36068 \times 8 - 40}{3} = \frac{178.88544 - 40}{3} = \frac{138.88544}{3}$$

= 46.295146, *the length of the arc* ACB; *then* $\overline{46.295146 \times 25} \div 2 = 1157.37865 \div 2 = 578.689325$, *the area of the sector* ADBC.

Now, $\frac{1}{2}$ AB × DE = 20 × 15 = 300, *the area of the triangle* ADB; *hence,* 578.689325 − 300 = 278.689325, *the area of the segment required.*

2. What is the area of the segment of a circle; the chord of the whole arc being 60 feet, and the chord of half the arc 37 feet 6 inches? *Ans.* 987 *ft.* 6 *in.*

3. The chord of the whole arc is 20, and the versed sine 5 feet, what is the area of the segment?

Ans. 69 *ft.* 8 *in.*

4. What is the area of a segment, the arc of which is a quadrant, whose radius is 24? *Ans.* 164.3904.

5. What is the area of a segment greater than a semicircle; the chord of the whole arc being 102 feet 6 inches, the chord of half the arc 100 feet, the chord of one quarter of the arc 57 feet 6 inches, and the diameter of the circle 116 feet 6 inches?

Ans. 8408.89399 *feet.*

6. Find the area of a segment whose arc contains 245 degrees 45 minutes; the diameter of the circle being 48 feet 9 inches. *Ans* 1545.03853 *feet.*

RULE II.

To two-thirds of the product of the chord and height of the segment, add the cube of the height divided by twice the chord, and the sum will be the area of the segment, nearly.

Note.—When the segment is greater than a semicircle, find the area of the remaining segment, which subtract from the area of the whole circle, and the remainder will be the area required.

EXAMPLES.

1. What is the area of a segment of a circle whose chord is 32, and height or versed sine 8?

Here $\overline{32 \times 8} \times \frac{2}{3} + \frac{8^3}{32 \times 2} = 256 \times \frac{2}{3} + \frac{512}{64}$ $= 170.66666 + 8 = 178.66666$, *the area required.*

2. The chord is 65, and versed sine 15; what is the area of the segment? *Ans.* 675.96153.

3. What is the area of a segment, greater than a semicircle, whose chord is 30, and height 20? *Ans.* 518.26171875.

RULE III.

Divide the height of the segment by the diameter, and find the quotient in the column of heights or versed sines, in the Table at the end of Part II.

Take out the corresponding *Area Seg.* which multiply by the square of the diameter, and the product will be the area of the segment.

Note 1. If the quotient of the height by the diameter do not terminate in three places of figures, without a fractional remainder, find the *Area Seg.* answering to the first three decimals of the quotient, subtract it from the next greater *Area Seg.*, multiply the remainder by the fractional part of the quotient, and the product will be the corresponding proportional part to be added to the first *Area Seg.*

This method ought to be used when accuracy is required, but for common purposes, the fractional remainder may be omitted.

2. When the area of a segment greater than a semicircle is required, subtract the quotient of the height by the diameter, from 1, find the *Area Seg.* corresponding to the remainder, which take from 785398, and the difference will be the *Area Seg.* answering to the quotient.

EXAMPLES.

1. What is the area of a segment whose chord is 32, the versed sine 8, and the diameter of the circle 40?

Here $\frac{8.0}{40} = .2$, *the quotient or tabular height; and the corresponding Area Seg. is* .111823; *hence*, $.111823 \times 40^2 = .111823 \times 1600 = 178.9168$, *the area of the segment required.*

2. What is the area of a segment whose height is 9, and the diameter of the circle 25?

Ans. 159.09375.

3. Find the area of a segment whose height is 25, and the diameter of the circle 55.

Ans. 1050.60065.

4. What is the area of a segment, greater than a semicircle, whose height is 66 feet, and the chord of the whole arc 60 feet 10 inches?

Ans. 4435 *ft.* 8 *in.* 11 *pa.*

5. The base of a stone column is the greater segment of a circle whose chord measures 2 feet, and versed sine 1 foot 4 inches; what is its area?

Ans. 2.304 *feet.*

PROBLEM XVIII.

To find the area of a circular zone, or the space included between any two parallel chords and their intercepted arcs.

RULE.

Find the area of that part of the zone forming the trapezoid ABCD, to which add twice the area of the segment AED, and the sum will be the area of the zone required. *(See the next figure.)*

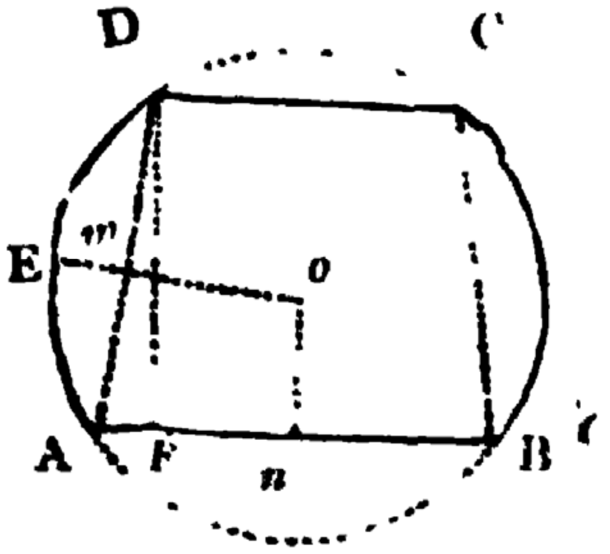
Note 1.—When great accuracy is required, the area of the segment should be found by Rule 3, Problem XVII.

2. The chord AD and versed sine L*m* may be found from the parallel sides AB, DC, and the perpendicular distance DF, by the help of Theorem [illegible], Part I.; much calculation may, however, be saved by measuring the chord and versed sine in taking the dimensions of the zone. (See the Key to the Mensuration, page 22.)

3. When the parallel sides and their perpendicular distance are given, the zone may be constructed in the following manner: draw the side AB; make AF equal to half the difference between AB and DC, and erect the perpendicular FD. From the point D, draw DC parallel to AB, and join AD. Bisect AB and AD with the perpendiculars *mo*, *no*, and *o* will be the centre of the circle of which the zone is a part; thus you will determine whether the centre of the circle falls within or without the zone.

EXAMPLES.

1. The greater side AB measures 40 feet, the less DC 30 feet, the perpendicular FD 35 feet, the chord AD 35 feet 3 inches, and the versed sine Em 7 feet 3 inches; what is the area of the zone ABCD?



By Problem VIII. we have $\overline{40+30} \times 35 = 70 \times 35 = 2450$; *and* $\frac{2450}{2} = 1225$, *the area of the trapezoid* ABCD.

Also, by Problem XVII. Rule 2, we have $\overline{35.25 \times 7.25} \times \frac{2}{3} + \frac{7.25^3}{35.25 \times 2} = 255.5625 \times \frac{2}{3} + \frac{381.078125}{70.5} = 170.375 + 5.40536 = 175.78036$, *the area of the segment* AED; *hence,* $1225 + \overline{175.78036 \times 2} = 1225 + 351.56072 = 1576.56072$ *feet* = 1576 *ft.* 6 *in.* 8 *pa. the area of the zone* ABCD, *as required.*

2. The greater side is 120 feet, the less 75 feet, the chord 39 feet 6 inches, and the versed sine 3 feet 3 inches, what is the area of the zone?

Ans 3337 *ft.* 4 *in.* 10 *pa.*

3. What is the area of a zone whose greater side measures 72 feet, the less 15 feet, and its breadth 19 feet 6 inches? *Ans.* 1201 *ft.* 11 *in.* 1 *pa.*

4. The greater side is 80, the less 60, and their distance 70, what is the area of the zone?

Ans. 6326.96.

PROBLEM XIX.

To find the area of a circular ring, or the space included between the circumferences of two concentric circles.

RULE.

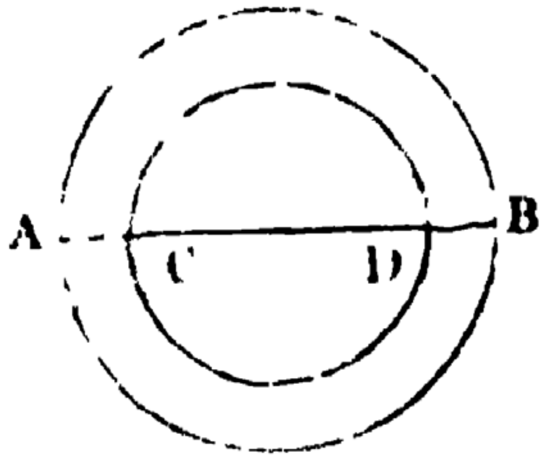
Multiply the sum of the diameters by their difference, and this product by .7854: and it will give the area required. Or, the difference of the areas of the two circles will be the area of the ring.

Note 1. The area of a circular ring may also be found by multiplying half the sum of the circumferences by half the difference of the diameters.

2. The area of part of a ring, or the segment of a sector, may be found by multiplying half the sum of the bounding arcs by the nearest distance between them.

EXAMPLES.

1. The diameters AB and CD are 30 and 20; what is the area of the circular ring.



Here $\overline{30 + 20} \times \overline{30 - 20} \times .7854 = 50 \times 10 \times .7854 = 500 \times .7854 = 392.7$, *the area of the ring required.*

2. The diameters of two concentric circles are 35 and 23; what is the area of the ring formed by the circumference of those circles? *Ans.* 546.6384.

3. The inner diameter of a circular building is 73 feet 3 inches, and the thickness of the wall 1 foot 9 inches; how many square feet of ground does the wall occupy? *Ans.* 412.335 *feet.*

4. What was the expense of making a moat round a circular island, at 2*s* 6*d*. per square yard; the diameter of the island being 525 feet, and the breadth of the moat 21 feet 6 inches? *Ans.* £512. 13*s*. 7¼*d*.

5. What is the area of the front of a circular arch, built with stones, each 3 feet 6 inches long, the length

of the upper bounding arc being 35 feet 3 inches, and the length of the lower 24 feet 9 inches? *Ans.* 105 *feet.*

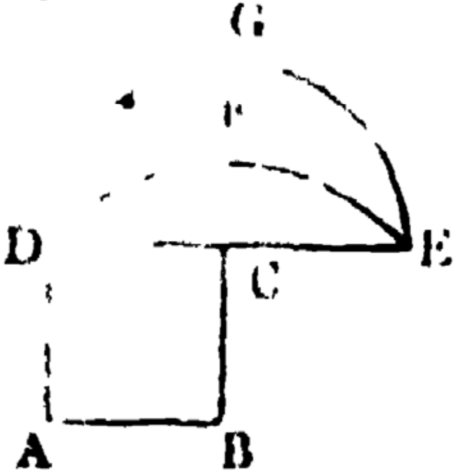
PROBLEM XX.

To find the area of a lune, or the space included between the intersecting arcs of two eccentric circles.

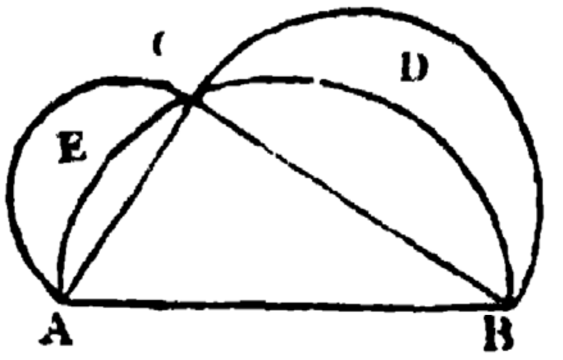
RULE.

Find the areas of the two segments forming the lune, and their difference will be the area required.

Note 1. If ABCD be a square, and circles be described from the points B and C, with the radii BD and CD, the area of the lune DGEFD will be equal to the area of the square ABCD.



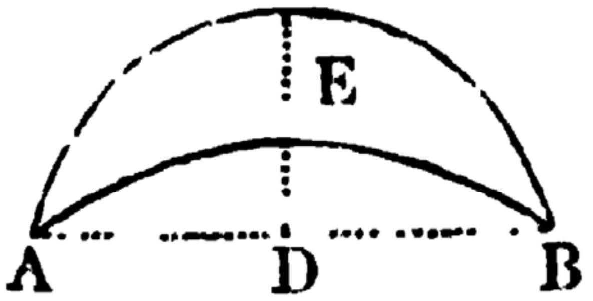
2. If ABC be a right-angled triangle, and semicircles be described on the three sides, as diameters, then will the area of the said triangle be equal to the sum of the areas of the two lunes D and E.



Several other curious properties of lunes may be seen in Dr. Hutton's Recreations, and Mathematical Dictionary.

EXAMPLES.

1. The length of the chord AB is 30, the height DC 12, and DE 5, what is the area of the lune ACBEA?



By Rule 2, *Problem XVII. we have* $\overline{30 \times 12} \times \frac{2}{3} + \frac{12^3}{30 \times 2} = 360 \times \frac{2}{3} + \frac{1728}{60} = 240 + 28.8 =$ 268.8, *the area of the segment* ACB.

Also, $\overline{30 \times 5} \times \frac{2}{3} + \frac{5^3}{30 \times 2} = 150 \times \frac{2}{3} + \frac{125}{60}$ $= 100 + 2.08333 = 102.08333$, *the area of the segment* AEB; *hence*, $268.8 - 102.08333 = 166.71667$, *the area of the lune required.*

2. The chord is 24, and the heights of the segments 9 and 4; what is the area of the lune? *Ans.* 93.8542.

3. The length of the chord is 48, and the versed sines of the segments 18 and 8; what is the area of the lune? *Ans.* 374.775.

Note. The answer to this question was obtained by the assistance of Rule 2, Problem XIV. and Rule 3, Problem XVII.

PROBLEM XXI.

To find the area of an ellipse.

RULE.

Multiply continually together the two diameters and the number .7854, and the product will be the area of the ellipse.

Note 1. The area of an elliptical ring, or the space included between the circumferences of two concentric similar or dissimilar ellipses may be found thus: from the product of the two diameters of the greater ellipse subtract the product of the two diameters of the less, multiply the remainder by .7854 and the product will be the area of the ring. Or, subtract the area of the less ellipse from that of the greater, and the remainder will be the area of the ring.

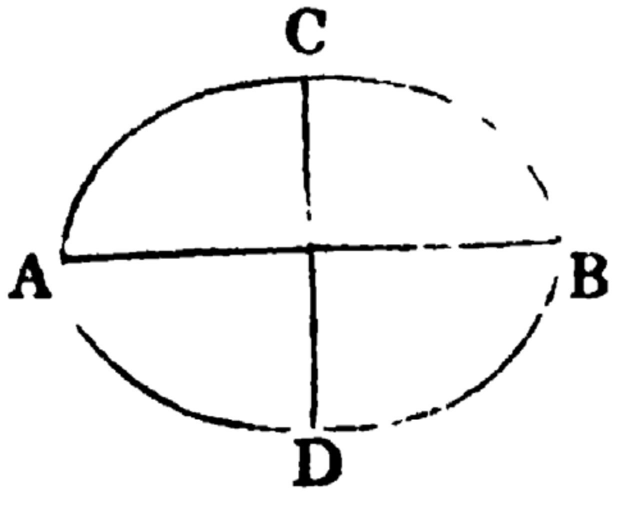
2. If the sum of the two diameters of an ellipse be multiplied by 1.5708, (half of 3.1416) the product will be the circumference, exact enough for most practical purposes.

3. To half the sum of the two diameters add the square root of half the sum of their squares, multiply the last sum by 1.5708, and the product will be the circumference, extremely near.

4. The ellipse is equal to a circle whose diameter is a mean proportional between the two axes; hence we obtain the above Rule.

EXAMPLES.

1. What is the area of the ellipse ABCD, whose transverse diameter AB is 34, and conjugate CD 25?



Here $34 \times 25 \times .7854 = 850 \times .7854 = 667.59$, *the area required.*

2. If the transverse diameter be 50 feet 9 inches, and the conjugate 35 feet 6 inches; what is the area of the ellipse? *Ans.* 1414 *ft.* 11 *in.* 11 *pa.*

3. The transverse diameter of an elliptical bath measures 25 feet, and the conjugate 15 feet; what was the expense of paving a walk round it, with Portland-stone, at 3*s.* 6*d.* per square foot; the breadth of the walk being 5 feet 6 inches? *Ans.* £77. 2*s.* 1½*d.*

4. The diameters of an elliptical piece of ground are 330 and 220 feet; how many quicks will plant the fence forming the circumference, supposing them to be set 5 inches asunder? *Ans.* 2073.

PROBLEM XXII.

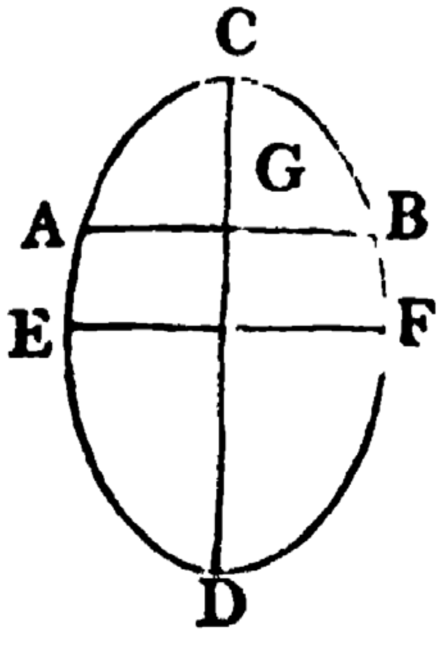
To find the area of an elliptical segment, the base of which is parallel to either of the diameters of the ellipse.

RULE.

Divide the height of the segment by that diameter of the ellipse of which it is a part, and find the *Area Seg.* answering to the quotient, in the Table at the end of Part II. Multiply the two diameters of the ellipse and the *Area Seg.* thus found, continually together, and the product will be the area required.

EXAMPLES.

1. What is the area of the elliptical segment ABC, cut off by the double ordinate AB; the height CG being 12, the diameter CD of the whole ellipse 40, and EF 25?



Here $\frac{12}{40} = .3$, *the tabular height; and the corresponding Area Seg. is* .198168; *then* .198168 × 25 × 40 = 198.168, *the area of the segment required.*

2. What is the area of the segment ADB, the height DG being 28, and the diameters of the ellipse 40 and 25? *Ans.* 587.23.

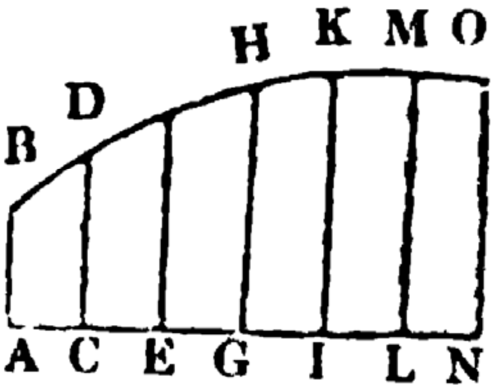
3. What are the areas of the two elliptical segments made by a double ordinate parallel to the conjugate diameter, at the distance of 9 from the centre of the ellipse; the diameters being 40 and 60?
Ans. 587.952, *and* 1297.008.

4. The diameters of an ellipse are 63 feet 9 inches, and 95 feet 3 inches, and the height of a segment, cut off by a line parallel to the transverse, is 25 feet 6 inches, what is its area? *Ans.* 1781 *ft.* 4 *in.* 8 *pa.*

PROBLEM XXIII.

To find the area of a segment of a circle, or any other curvilineal figure, by means of equidistant ordinates.

RULE.



If a right line AN be divided into any even number of equal parts AC, CE, EG, &c.; and at the points of division be erected perpendicular ordinates AB, CD, EF, &c., terminated by any curve BDF, &c.; and if A be put for the sum of the extreme or first and last ordinates AB, NO; B for the sum of the even ordinates CD, GH, LM, &c.; viz. the second, fourth, sixth, &c., and C for the sum of all the rest EF, IK, &c.; viz. the third, fifth, &c., or the odd ordinates, wanting the first and last: then the common distance AC, or CE, &c. of the ordinates, being multiplied by the sum arising from the addition of A, four times B, and two times C;

one-third of the product will be the area ABON, very nearly; that is, $\frac{A + 4B + 2C}{3} \times D =$ the area, putting $D = AC$, the common distance of the ordinates.

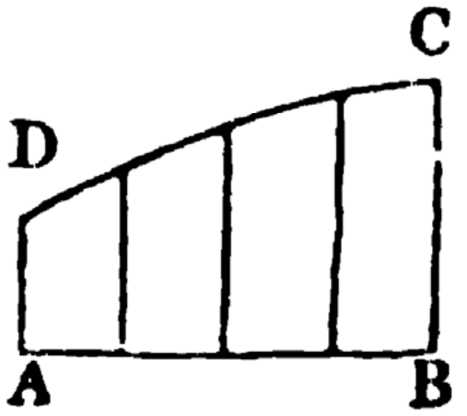
(See page 283, where the Rule is expressed in words.)

Note 1. When equidistant ordinates or perpendiculars cannot conveniently be taken, in consequence of the bends or corners of the boundary being at unequal distances, you may proceed as in Problem 4, Part III.

2. By the Rule given in this Problem, the contents of all solids, whether regular or irregular, may be found, by using the areas of the sections perpendicular to the axe, instead of the ordinates; and it is evident that the greater the number of ordinates or sections are used, the more accurately will the area or solidity be determined *(See the Scholium on page 274.)*

EXAMPLES.

1. It is required to find the area of the curved space ABCD, the lengths of the five equidistant ordinates being as follow; viz. the first or $AD = 8$, the second $= 10$, the third $= 12$, the fourth $= 14$, and the fifth or last $BC = 15$; and the length of the base $AB = 24$.



Here $A = 8 + 15 = 23$, $B = 10 + 14 = 24$, $C = 12$, *and* $D = \frac{24}{4} = 6$; *then* $\overline{A + 4B + 2C} \times \frac{1}{3} D = \overline{23 + 96 + 24} \times \frac{6}{3} = 143 \times 2 = 286$, *the area required.*

2. Given the lengths of seven equidistant ordinates of an irregular piece of ground, as follow; viz. 15, 19, 20, 23, 25, 30, and 33 feet; and the length of the base 72 feet; required the plan and area.

Ans. The area is 1704 *feet.*

A

TABLE

OF THE

Areas of the Segments of a Circle,

Whose Diameter is Unity, and supposed to be divided into 1000 equal Parts.

Height.	Area SEGMENT.	Height.	Area SEGMENT.	Height.	Area SEGMENT.
.001	.000042	.023	.004618	.045	.012554
.002	.000119	.024	004921	.046	012971
.003	.000219	.025	005230	.047	.013392
.004	.000337	.026	.005546	.048	.013818
.005	.000470	.027	.005867	.049	014247
.006	.000618	.028	.006194	050	.014681
.007	000779	.029	.006527	.051	.015119
.008	.000951	.030	.006865	.052	.015561
.009	.001135	.031	.007209	053	.016007
.010	.001329	032	.007558	.054	016457
.011	.001533	.033	.007913	055	.016911
.012	.001746	.034	008273	.056	017369
.013	.001968	.035	.008638	.057	.017831
.014	.002199	.036	.009008	.058	.018296
.015	.002438	.037	.009383	.059	.018766
.016	.002685	.038	.009763	060	.019239
.017	.002940	.039	.010148	.061	.019716
.018	.003202	.040	.010537	.062	.020196
.019	.003471	.041	.010931	.063	020680
.020	.003748	.042	.011330	.064	.021168
.021	.004031	.043	.011734	065	.021659
.022	.004322	.044	.012142	.066	.022154

The Areas of the Segments of a Circle.

Height.	*Area* SEGMENT.	*Height.*	*Area* SEGMENT.	*Height.*	*Area* SEGMENT.
.067	.022652	.100	.040875	.133	.062026
068	.023154	.101	.041476	.134	.062707
.069	.023659	.102	.042080	.135	.063389
.070	.024168	.103	042687	.136	.064074
.071	.024680	.104	.043296	.137	.064760
.072	.025195	.105	.043908	.138	.065449
.073	.025714	.106	.044522	.139	.066140
.074	026236	.107	045139	.140	.066833
.075	.026761	.108	.045759	.141	.067528
.076	.027289	.109	046381	.142	.068225
.077	.027821	.110	.047005	.143	.068924
.078	.028356	.111	047632	.144	.069625
.079	028894	.112	.048262	.145	.070328
.080	.029435	.113	.048894	.146	071033
.081	.029979	.114	049528	.147	.071741
.082	.030526	.115	050165	.148	.072450
.083	.031076	.116	050804	.149	.073161
.084	.031629	.117	051446	.150	073874
.085	.032186	.118	.052090	.151	.074589
.086	.032745	.119	.052736	.152	.075306
.087	.033307	.120	053385	.153	.076026
.088	.033872	.121	.054036	.154	076747
.089	.034441	.122	.054689	.155	.077469
.090	.035011	.123	.055345	.156	.078194
.091	.035585	.124	.056003	.157	.078921
.092	.036162	.125	.056663	.158	.079649
.093	.036741	.126	.057326	.159	.080380
.094	.037323	.127	.057991	.160	.081112
.095	.037909	.128	.058658	.161	.081846
.096	.038496	.129	.059327	.162	.082582
.097	.039087	.130	.059999	.163	.083320
.098	.039680	.131	.060672	.164	.084059
.099	.040276	.132	.061348	.165	.084801

The Areas of the Segments of a Circle.

Height.	*Area* SEGMENT.	*Height.*	*Area* SEGMENT.	*Height.*	*Area* SEGMENT.
.166	.085544	.198	.110226	.230	.136465
.167	086289	.199	.111024	231	.137307
.168	.087036	200	.111823	232	.138150
.169	.087785	.201	.112624	233	.138995
.170	088535	202	.113426	.234	139841
.171	.089287	203	.114230	.235	.140688
.172	090041	204	.115035	236	.141537
.173	090797	205	.115842	.237	.142387
174	.091555	206	.116650	238	143238
.175	.092313	.207	117460	.239	.144091
.176	.093074	.208	.118271	240	.144914
.177	093836	209	119083	.241	145799
.178	.094601	210	119897	242	.146655
.179	.095366	.211	.120712	.243	147512
180	096134	.212	.121529	.244	.148371
.181	096903	.213	.122347	.245	.149230
182	.097071	.214	.123167	246	150091
.183	.098417	215	.123988	.247	.150953
.184	.099221	.216	124810	248	.151816
.185	.099997	.217	125634	249	.152680
.186	.100774	.218	.126459	250	.153546
.187	.101553	219	.127285	251	.154412
.188	.102334	.220	.128113	.252	.155280
.189	.103116	.221	.128942	.253	.156149
190	.103900	.222	.129773	254	.157019
.191	.104685	.223	.130605	255	.157890
.192	.105472	.224	.131438	.256	.158762
.193	.106261	.225	132272	.257	.159636
.194	.107051	.226	.133108	.258	.160510
.195	.107842	.227	.133945	.259	.161386
.196	.108636	.228	.134784	.260	.162263
.197	.109430	.229	.135624	.261	.163140

The Areas of the Segments of a Circle.

Height.	*Area* SEGMENT.	*Height.*	*Area* SEGMENT.	*Height.*	*Area* SEGMENT.
.262	.164019	.294	.192684	.326	.222277
.263	.164899	.295	.193596	.327	.223215
.264	.165780	.296	.194509	.328	.224154
.265	.166663	.297	.195422	.329	.225093
.266	.167546	.298	.196337	.330	.226033
.267	.168430	.299	.197252	.331	.226974
.268	.169315	.300	.198168	.332	.227915
.269	.170202	.301	.199085	.333	.228858
.270	.171089	.302	.200003	.334	.229801
.271	.171978	.303	.200922	.335	.230745
.272	.172867	.304	.201841	.336	.231689
.273	.173758	.305	.202761	.337	.232634
.274	.174649	.306	.203683	.338	.233580
.275	.175542	.307	204605	.339	.234526
.276	.176435	.308	.205527	.340	.235473
.277	.177330	.309	.206451	341	.236421
.278	.178225	.310	207376	.342	.237369
.279	.179122	.311	.208301	.343	.238318
280	.180019	.312	.209227	.344	.239268
.281	.180918	.313	.210154	.345	.240218
.282	.181817	.314	.211082	.346	.241169
.283	.182718	.315	.212011	.347	.242121
.284	.183619	.316	.212940	.348	.243074
.285	.184521	.317	.213871	.349	.244026
.286	.185425	.318	.214802	.350	.244980
.287	.186329	.319	.215733	.351	.245934
.288	.187234	.320	.216666	.352	.246889
.289	.188140	.321	.217599	.353	.247845
.290	.189047	.322	.218533	.354	.248801
.291	.189955	.323	.219468	.355	.249757
.292	.190864	.324	.220404	.356	.250715
.293	.191775	.325	.221340	.357	.251673

The Areas of the Segments of a Circle.

Height.	*Area* SEGMENT	*Height.*	*Area* SEGMENT.	*Height.*	*Area* SEGMENT.
.358	.252631	.390	.283592	.422	.315016
.359	.253590	.391	.284568	.423	.316004
.360	.254550	.392	.285544	.424	.316992
.361	.255510	393	.286521	.425	.317981
.362	.256471	.394	.287498	.426	.318970
.363	.257433	.395	.288476	.427	.319959
.364	.258395	.396	.289453	.428	320948
.365	.259357	.397	.290432	.429	.321938
.366	.260320	.398	.291411	.430	.322928
367	.261284	.399	.292390	.431	.323918
.368	.262248	400	.293369	.432	.324909
.369	.263213	.401	294349	.433	.325900
.370	.264178	.402	.295330	.434	.326892
.371	.265144	.403	.296311	.435	.327882
.372	.266111	.404	.297292	.436	.328874
.373	.267078	.405	.298273	.437	.329866
374	268045	.406	.299255	.438	.330858
.375	.269013	407	.300238	.439	.331850
376	.269982	.408	.301220	.440	.332843
.377	.270951	.409	.302203	.441	.333836
.378	.271920	.410	.303187	442	.334829
.379	.272890	411	.304171	.443	.335822
.380	.273861	.412	.305155	.444	.336816
.381	.274832	.413	.306140	.445	.337810
.382	.275803	414	.307125	.446	.338804
.383	276775	415	.308110	.447	.339798
.384	.277748	.416	.309095	.448	.340793
.385	.278721	.417	.310081	.449	.341787
.386	.279694	.418	.311068	.450	.342782
.387	.280668	419	.312054	.451	.343777
.388	.281642	.420	.313041	.452	.344772
.389	.282617	.421	.314029	.453	.345768

The Areas of the Segments of a Circle.

Height.	Area SEGMENT.	Height.	Area SEGMENT.	Height.	Area SEGMENT.
.454	.346764	.470	.362717	.486	.378701
.455	.347759	.471	.363715	.487	.379700
.456	.348755	.472	.364713	.488	.380700
.457	.349752	.473	.365712	.489	.381699
.458	.350748	.474	.366710	.490	.382699
.459	.351745	.475	.367709	.491	.383699
.460	.352742	.476	.368708	.492	.384699
.461	.353739	.477	.369707	.493	.385699
.462	.354736	.478	.370706	.494	.386699
.463	.355732	.479	.371705	.495	.387699
.464	.356730	.480	.372704	.496	.388699
.465	.357727	.481	.373703	.497	.389699
.466	.358725	.482	.374702	.498	.390699
.467	.359723	.483	.375702	.499	.391699
.468	.360721	.484	.376702	.500	.392699
.469	.361719	.485	.377701		

Note.—The use of the foregoing Table is given in Problem XVII. Rule 3 and the method of constructing it may be seen in Moss's Gauging, page 96

PART III.

LAND-SURVEYING,

AND

MISCELLANEOUS QUESTIONS

IN

Superficial Mensuration.

SECTION I.

LAND-SURVEYING.

Land is commonly measured with a chain, invented by Mr. Gunter, and known by the name of *Gunter's Chain.* It is 4 poles, or 22 yards in length, and divided into 100 equal parts, called links; each link being 7.92 inches.

An acre of land is equal to 10 square chains; that is, 10 chains in length and 1 in breadth; or 220 × 22 = 4840 square yards; or 40 × 4 = 160 square rods, poles, or perches; or 1000 × 100 = 100000 square links.

The measurement of land is generally given in acres, roods, and perches; 4 roods being an acre, and 40 perches a rood.

A statute-pole or perch is 16½ feet long; but in different parts of the kingdom there are, by custom, poles of different lengths; as 15, 18, 21 feet, &c.

The Field-book.

The best method of entering the field-notes, is to begin at the bottom of the page and write upward.

Each page of the field-book must be divided into three columns. In the middle column must be set down the distances on the chain-line, at which any mark, offset, or other observation is made; and in the right and left-hand columns respectively, those marks, offsets, and observations must be entered.

The crossings of fences, rivers, &c. may be denoted by lines drawn across the middle column, or part of the right and left-hand columns, opposite the distances on the chain-line, at which they are crossed; and the corners of fields, and other remarkable turns in the fences, to which offsets are taken, may be defined by lines joining or lying in the same relation to the middle column, as the fences, &c. do to the chain-line.

Thus a tolerably accurate representation of the fences, &c. may be sketched in the field, which will very much assist the surveyor in drawing the plan.

With respect to the characters used to denote stations, the letters of the alphabet will do very well, in small surveys; but in those of a large extent, numeral figures must be used, and the sign + (plus) placed before each figure; thus, + 1, or + 2, which may be read, station first, or cross first, station one, or cross one, &c. Upon the plan they are generally represented by this (⊙) mark.

Note 1. Many surveyors not only begin at the bottom of the field-book, but also at the right-hand-side, and write towards the left, which method I always follow.

2. A straight fence may be denoted in the field-book, by placing S against the line which represents the fence. (See the field-book to Plate III.)

3. In order to assist the memory in planning, a learner sometimes draws a rough sketch of the field or estate he is about to measure, and upon it marks the stations in the same manner as they are put down in taking the survey.

Miscellaneous Instructions.

1. In addition to the chain, the surveyor must provide ten iron arrows, each about 12 or 15 inches in length, and pointed at the bottom, to enter the ground readily; and also bent in a circular form at the top, for the convenience of holding them. To the top of each arrow, a piece of red cloth should be attached, to make it more conspicuous among the long grass, &c. Poles, likewise, generally called "station staves," will be wanted as marks or objects of direction, each about 8 or 10 feet in length, piked with iron at the bottom, and having a red or white flag at the top, that they may be better seen at a distance; and also an *offset staff*, 10 or 12 links long, to which a cross may be fixed, as described in Prob. 4, Part II.

2. In order to measure a line, let your assistant take nine arrows in his left-hand, and one end of the chain and an arrow in his right; then advancing towards the place to which he is directed, at the end of the chain let him put down the arrow which he holds in his right-hand. This the follower must take up with his chain-hand, when he comes to it; the leader, at the same time, putting down another at the other end of the chain. In this manner he must proceed until he has put down his tenth arrow, then advancing a chain further, he must set his foot upon the end of the chain, and call out "change." The surveyor must then walk up to him, if he have no offsets to take; and the arrows being carefully counted, one must be put down at the end of the chain; then proceed as before, until the whole line be measured.

3. Each change ought to be entered in the field-book, or a mistake of 10 chains may happen, when the line is very long.

4. The follower may direct the leader by the motion of his left-hand; moving it to the right or left, as circumstances may require.

5. The arrows should always be put down perpendicularly, and in a right line with the object of direction, otherwise the line will be made too long.

6. If the ditch be in the field which you are about to measure, both it and the hedge usually belong to the adjoining field. In some places 5, and in others 6 links, from the roots of the quickwood, are allowed for the breadth of the ditches between neighbouring estates, but for those ditches adjoining roads, commons, and waste lands, 7 links are commonly allowed.

7. The ditches and fences must always be measured with the fields to which they belong, when the whole quantity of land is required, but in measuring crops of corn, turnips, &c. only so much must be measured as is, or has been occupied by the corn, &c.

8. When your view between two stations is obstructed by an intervening hill, the line must be ranged as follows: With your assistant, go to the place whence you can distinctly see both stations; and turning face to face, at the distance of two or three chains, direct each other to the right or left, until you are both in a right line with the stations, then, either of you putting down a pole, the line will be correctly found.

9. In surveying *hilly* ground, the horizontal measure must always be returned, except for paring, reaping, &c. in which cases the hypothenusal measure must be given; to obtain which, it is frequently necessary to divide a hill into various figures.

10. In order to preserve the horizontal line, in ascending or descending a hill, it will be necessary to elevate the chain. If the inclination of the hill be great, the chain must be elevated at several times.

King's Quadrant is admirably adapted for surveying hilly ground. (See my Land-Surveying, page 172.)

11. When a field is bounded by crooked fences, you must measure a line as near to each as the angles or curves will permit; in doing which, you must take the offset perpendicularly to each corner or angle in the fence. Where the fence is curved, those offsets must be so taken, that a right line drawn from the end of any one to the end of the next, on each side, would neither exclude any part of the land to be measured, nor include any of that which is adjacent. Perpendiculars thus erected will divide the whole offset or piece of land contained between the base-line and the fence, into triangles and trapezoids.

12. Sometimes it is most convenient to measure a line on the outside of the field, and upon it erect perpendiculars to the crooked fence. These are called "insets;" and the area thus included must be deducted from that of the whole figure. (See Ex 2, Prob. I.)

13 When the fences and ditches are to be measured with the field to which they belong, it is generally most practicable to fix the stations within the fences, at a little distance from the corners, and then to measure to the roots of the quickwood, adding or subtracting 5 or 6 links, according to the custom of the place, for the breadth of the ditch. (See Ex 1, Prob. II)

14 When the offsets are small, their places on the base-line may be determined by laying the offset-staff at right angles across the chain, but when large, they must be found by the cross, and measured with the chain.

15 Any scale of equal parts may be used in laying down figures, but an ivory feathered-edged plotting-scale is most convenient, as distances may be pricked off by it, without using the compasses

16. In laying down an offset, apply the scale to the base line, and make a small pencil dot at every place where a perpendicular must be erected, then lay the scale across the base, so that the line which is marked with oo, may coincide with it, the edge of the scale at the same time touching one of the dots From the dot, by the edge of the scale, draw a line (which will be perpendicular to the base, and upon it prick off the offset, or it may be pricked off without drawing a line.

17. The base of each triangle and trapezoid, forming an offset, may be found by subtracting the distances on the chain-line, from each other.

18. In planning or laying down figures relating to surveying, the upper part of the paper or book used should always, if possible, represent the north; then you will have the east on your right-hand, and the west on your left.

19. In taking dimensions, it is an admirable method to measure proof-lines, in order that you may be able to confirm your survey, after it is planned. Some may perhaps deem this tedious and superfluous, but a person had certainly much better be at the pains of discovering his own errors, than expose himself to ridicule, by suffering them to be detected by some other surveyor.

20. The line in which you have the misfortune to lose an arrow, must be re-measured.

PROBLEM I.

To measure a field of three sides.

At each corner of the field place a station-staff; measure along the base till you arrive at the point from which you suppose a perpendicular will rise to the opposite angle; there plant your cross, and turn its index till the mark at each end of the base can be seen through one of the grooves; then apply your eye to the other groove, and if you see the mark at the opposite angle, you are in the right place to measure the perpendicular; if not, move the instrument backward or forward, along the base, till you can see the three marks as above directed. Enter in your field-book the distance from the end of the base to the cross; measure the perpendicular, and then the remainder of the base.

Or, measure the three sides, construct the figure, and determine the length of the perpendicular by the scale; or the area may be found from the three sides, without the perpendicular.

Note 1.—Be especially careful, that in measuring the two parts of the base and the perpendicular, no confusion of arrows take place.

2. Triangles, trapeziums, and trapezoids, may be constructed by Problems 7 and 8, Part I; and their areas found by the Rules given in Problems 4, 5, 7, and 8, Part II.

3. If a triangle be laid down by a scale of 1 chain or 2 chains to an inch, the perpendicular may be ascertained with considerable accuracy.

4. If the examples in this Problem, or any of the following Problems be thought too few, more may be easily supplied by the teacher sketching fields, at pleasure, with his pen, which the learner may measure by a scale.

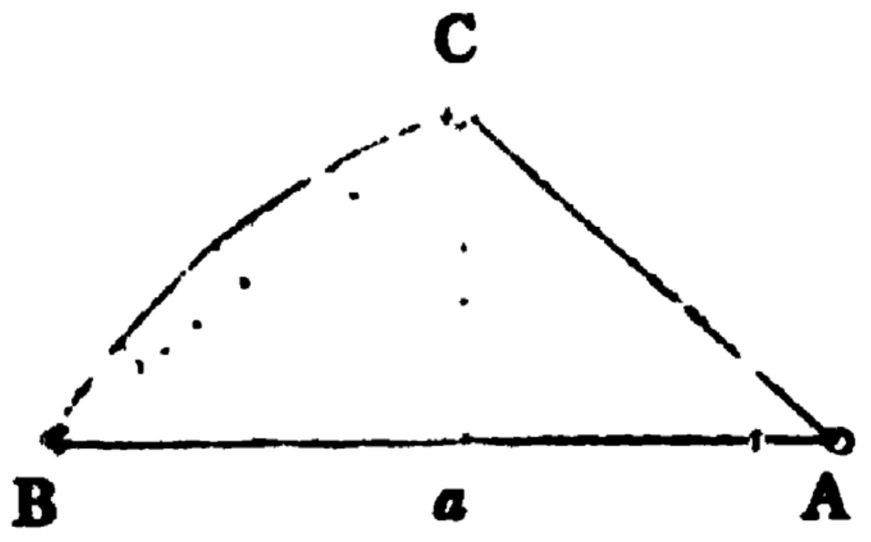
This method will be found very advantageous; as it will give the learner a good idea in what manner he must take his dimensions, and enter his notes, when he commences field-practice.

5. The expression, R. off B, or L. off B, &c. in the field-notes, signifies that you are to turn to the right or left-hand, and measure from B, &c.

EXAMPLES.

1. Lay down a field, and find its area from the following notes.

	BC	
0	846	
64	600	
85	400	
50	200	
0	000	
	R. off B.	
	AB	
	1253	
	1000	
	586	525 C.
Begin at	A, and	go West.
Per. on the left.	*Base line*	*Per. on the right.*



CALCULATION.

Triangle ABC.

1253 base AB.
525 perp. C*a*.

6265
2506
6265

657825 double area.

Offsets taken on the line BC.

50
200
10000

50
85
135
200
27000

85
64
149
200
29800

246
64
984
1476
15744

10000 27000 29800 15744	Double areas of offsets collected.
82544	Sum.

Double Areas.

657825 Triangle ABC.
82544 Offsets.
2)740369
3.70184
4
2.80736
40
32.29440 *Area 3a. 2r. 32p.*

2. Draw a plan of a field, and find its area from the following notes.

	CA
0	1252
37	1000
69	824
45	716
72	610
15	424
55	212
0	000
	R. off C.

	BC	
	683	0
	536	40
	354	64
	229	49
	000	0
	R. off B.	
	AB	
0	973	
48	745	
76	600	
56	495	
25	256	
0	000	
Begin at	A and	go N. East.

ANSWER.

Having constructed the figure, you will find the perpendicular falling from the angle B, upon the side CA, to measure 528 links.

Double Areas.

661056 Triangle ABC.
68543 Offsets taken on the line AB.
95376 Ditto on CA.

824975 Sum.
50154 Insets on BC.

2)774821 Difference.

387410 = *3a. 3r. 19¾p. the area required.*

3. Find the area of a field from the following dimensions.

	CA	
	1435	
	1000	
	L. off C.	
	BC	
	1197	
	1000	
	L. off B.	
	AB	
	1946	
	1000	
From	A, go	East.

CALCULATION.

Here $\frac{1946 + 1197 + 1435}{2} = \frac{4578}{2} = 2289$ = half the sum of the sides; then $2289 - 1946 = 343$, the first remainder; $2289 - 1197 = 1092$, the second remainder; and $2289 - 1435 = 854$, the third remainder; whence $\sqrt{2289 \times 343 \times 1092 \times 854} = \sqrt{732184316136} = 855677$ square links, = 8 *acres, 2 roods, and 9 perches, the area required.*

By Logarithms.

The log. of	2289	= 3.3596458
————	343	= 2.5352941
————	1092	= 3.0382226
————	854	= 2.9314579
Divide the sum of the log. by		2)11.8646204
The quotient is the log. of 855677, the area		5.9323102

4. Lay down a field, and find its area from the following notes.

	DC	proof-line.
	412	
Return	to D.	
	CA	
0	638	
46	465	
32	246	
0	000	
	R. off C.	
	BC	
0	462	
45	300	
32	150	
0	000	
	R. off B.	
	AB	
0	725	
32	565	
40	400	
25	250	D, station for a proof-line.
0	000	
Begin at	A, and	go West.

ANSWER.

Having constructed the figure, you will find the proof-line DC to measure 412 links, as in the field-book; hence, you may conclude there is no error committed in taking or setting down the dimensions. You will also find the perpendicular falling from the angle C, upon the base AB, to measure 402 links; from which, and the offsets, we obtain the area = *1a. 3r. 25p.*

PROBLEM II.

To measure a field of four sides.

Measure a diagonal, and find the perpendiculars falling upon it, from the opposite angles, as already directed. Or, if you measure a diagonal and the four sides, the figure may be constructed, and the perpendiculars determined by the scale.

Note 1. Always make choice of the longer diagonal, because the longer the base-line of a triangle, the more obtuse is its subtending angle; consequently the perpendicular will be shorter, and its place more easily and accurately determined. If you measure the four sides and both the diagonals, one of them will serve as a proof-line, after you have laid down the figure.

2. If neither of the diagonals can be measured, in consequence of obstructions, you must measure tie-lines across the angles of the field, which will enable you to plan it; and hence the diagonal and perpendiculars may be found by the scale.

3. When two perpendiculars cannot be taken upon either of the diagonals, such fields must be divided into two triangles, or into two right-angled triangles and a trapezoid.

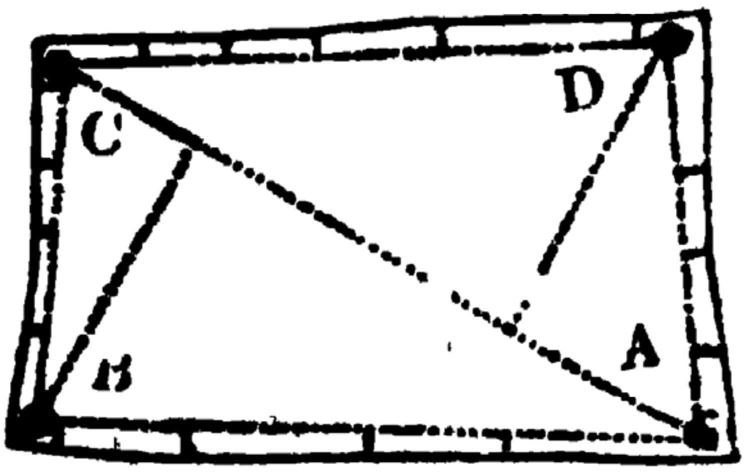
3. Unskilful surveyors affect to reduce trapeziums into rectangles, by adding each two opposite sides together, and taking half their sum respectively for a mean length and breadth, but this method leads to very erroneous results. (See my Land-Surveying, page 124.)

EXAMPLES.

1. Draw a plan of a field, and find its area from the following notes.

	AC	Diag.
	1155	
	1000	
495	915	
	360	520
From	A, go	N. West.
To	the	Fence.
48	660	
53	630	to A.
40	500	
25	380	
50	200	
62	000	
From	D, go	South.

To	the	Fence.
40	1012	
42	950	to D.
54	600	
33	400	
12	260	
30	150	
65	000	
From	C, go	East.
To	the	Fence.
25	615	
35	550	to C.
30	400	
10	240	
22	150	
45	000	
From	B, go	North.
To	the	Fence.
33	1090	
45	1045	to B.
56	800	
40	500	
48	300	
30	000	
Begin at	A, and go	West.



CALCULATION.

Trapezium ABCD.

$\left.\begin{array}{r}520\\495\end{array}\right\}$ perp.

1015 sum.
1155 diag.

5075
5075
1015
1015

1172325 double area.

Offsets taken on the line AB.

30
48

78
300

23400

48
40

88
200

17600

40
56

96
300

28800

56
45

101
245

505
404
202

24745

45
33

78
45

390
312

3510

$\left.\begin{array}{r}23400\\17600\\28800\\24745\\3510\end{array}\right\}$ Double areas collected.

98055 sum.

Offsets taken on the line BC.

$$\begin{array}{r} 45 \\ 22 \\ \hline 67 \\ 150 \\ \hline 3350 \\ 67 \\ \hline 10050 \\ \hline\hline \end{array} \qquad \begin{array}{r} 30 \\ 35 \\ \hline 65 \\ 150 \\ \hline 3250 \\ 65 \\ \hline 9750 \\ \hline \end{array} \qquad \begin{array}{r} \left.\begin{array}{r} 10050 \\ 2880 \\ 6400 \\ 9750 \\ 3900 \end{array}\right\} \text{Double areas collected.} \\ \hline 32980 \text{ sum.} \\ \hline\hline \end{array}$$

$$\begin{array}{r} 22 \\ 10 \\ \hline 32 \\ 90 \\ \hline 2880 \\ \hline\hline \end{array} \qquad \begin{array}{r} 35 \\ 25 \\ \hline 60 \\ 65 \\ \hline 300 \\ 360 \\ \hline 3900 \\ \hline\hline \end{array}$$

$$\begin{array}{r} 10 \\ 30 \\ \hline 40 \\ 160 \\ \hline 2400 \\ 40 \\ \hline 6400 \\ \hline\hline \end{array}$$

Offsets taken on the line CD.

$$\begin{array}{r} 65 \\ 30 \\ \hline 95 \\ 150 \\ \hline 4750 \\ 95 \\ \hline 14250 \\ \hline\hline \end{array} \qquad \begin{array}{r} 30 \\ 12 \\ \hline 42 \\ 110 \\ \hline 420 \\ 42 \\ \hline 4620 \\ \hline\hline \end{array}$$

12
33
45
140
1800
45
6300

54
42
96
350
4800
288
33600

33
54
87
200
17400

42
40
82
62
164
492
5084

14250
4620
6300
17400
33600
5084

} Double areas collected.

81254 sum.

Offsets taken on the line DA.

62
50
112
200
22400

50
25
75
180
6000
75
13500

25
40
65
120
1300
65
7800

40
53
93
130
2790
93
12090

53
48
101
30
3030

22400
13500
7800
12090
3030

} Double areas collected.

58820 sum.

1172325
98055
32980
81254
58820
} Whole double areas collected.

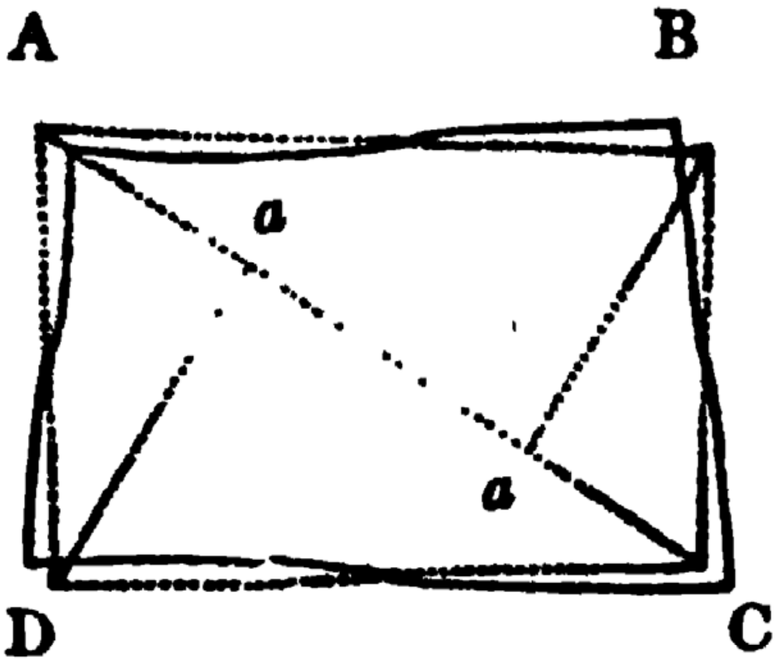
2)1443434 sum.

7.21717
4

.86868
40

34.74720 *Area 7a. 0r. 34¾p.*

To find the area of the foregoing figure by the method generally called "Casting."



Having constructed the figure, and taken out the chain-lines, draw with a pencil, by the straight edge of a clear piece of lantern-horn, the four right lines AB, BC, CD, and DA, in such a manner, that the parts included may be equal to those excluded, as nearly as you can judge by your eye; then the diagonal AC, will be found to measure 1274, and the perpendiculars B*a*, 550, and D*a*, 583 links.

Note. The use of the Parallel Ruler, in straightening crooked fences, is given in the Author's Land-Surveying, in twelve new problems, comprising every case that can possibly occur in practice.

CALCULATION:

550 } perp.
583 }
1133 sum.
1274 diag.
4532
7931
2266
1133
2)1443442
7.21721
4
.86884
40
34.75360 *Ans. 7a. 0r. 34¾p.*

Note.—Although the method of finding the area by casting, is adopted by most practical surveyors, it is certainly less correct than that by offsets, &c., a learner, therefore, ought to practise both, until he can habitually come very near to the truth by the former.

2. Lay down a field, and find its area from the following dimensions.

	AF	
	2718	0
D	1865	953 E.
B	575	1142 C.
	000	0
From	A, go	West.

Area 20a. 3r. 17½p.

3. Required the plan and area of a field from the following notes.

	CB	
	1646	Diag.
	720	385 D.
	L. off C.	
	AC	Base.
	2030	
B 850	620	
From	A, go	East.

K *Area 11a. 3r. 7p.*

4. Draw a plan, and find the area of a field, from the following dimensions.

	BD	
	1460	Diag.
	1000	
Return	to B.	
	AC	
	1480	Diag.
	1000	
	R. off A.	
	DA	
	672	
	R. off D.	
	CD	
0	1244	
47	1000	
70	800	
85	600	
68	400	
30	200	
0	000	
	R. off C.	
	C	
0	720	
85	650	
112	550	
88	450	
	360	Cross the fence.
	300	83
	200	130
	100	100
	000	0
	R. off B.	

	AB	
	1350	
	1000	
Begin at	A, and	go West.

ANSWER.

Having constructed the figure by means of the four sides and the diagonal AC, you will find the diagonal BD to measure 1460 links, as in the field-book; hence, it may be concluded that no error has been made in taking or entering the dimensions. You will also find the perpendicular from the angle A, upon the diagonal BD, to measure 618 links, and the perpendicular from the angle C, upon the same diagonal, 613 links; hence the area is 9*a*. 2*r*. 11*p*.

PROBLEM III.

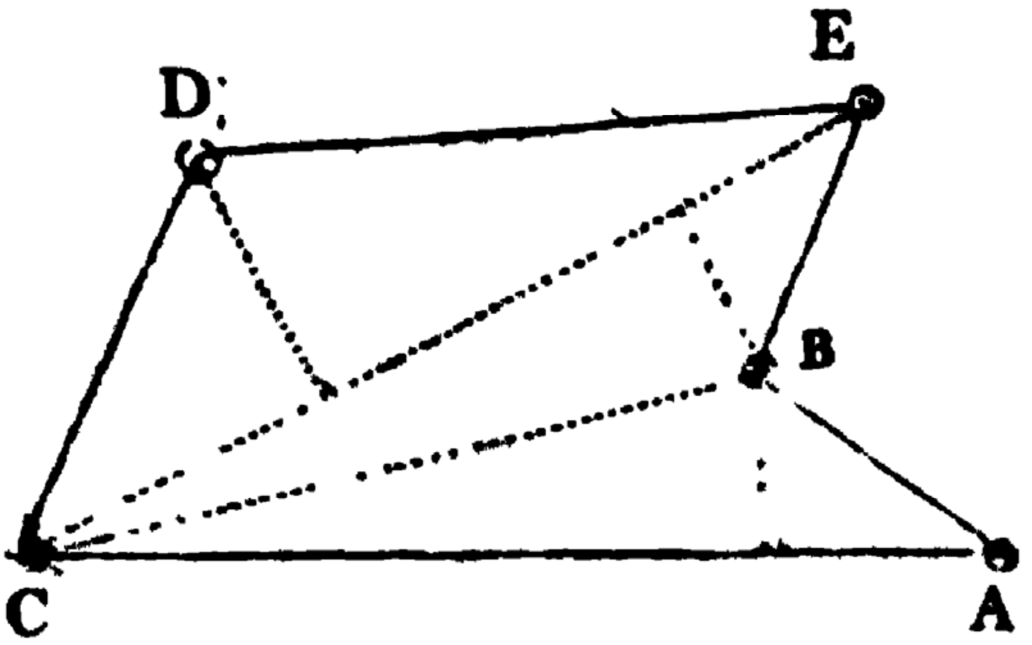
To measure a field of more than four sides.

Any piece of land, consisting of more than four sides, may be surveyed by reducing it into triangles and trapeziums; thus, a field of five sides may be reduced into a triangle and a trapezium; of six sides, into two trapeziums; of seven, into two trapeziums and a triangle; of eight, into three trapeziums, &c.

EXAMPLES.

1. Required the plan and area of a field, from the following dimensions.

	CE	Diag.
	1165	
	912	168 B.
D 308	415	
	R. off C.	
	AC	Base.
	1195	
	293	233 B.
From	A, go	West.



CALCULATION.

Triangle ABC.

1195 base.
233 perp.

3585
3585
2390

278435

Trapezium BCDE.

308 } perp.
168 }

476 sum.
1165 diag.

2380
2856
476
476

554540

Double Areas.

278435
554540

2)832975

4.16487
4

.65948
40

26.37920 *Area* 4*a.* 0*r.* 26*p.*

2. Lay down a field, and find its area from the following notes,

	DB	
	1792	Diag.
E 562	954	
	R. off D.	
	AD	
	2243	Diag.
C 728	1785	
	560	624 B.
From	A, go	N. West.

Area. 20*a*. 0*r*. 31¾*p*.

3. Required the plan and area of a field from the following dimensions.

	CE	
	5126	Diag.
	5000	
	4040	1490 A.
	4000	
	3000	
	2000	
D 1695	1720	
	1000	
	R. off C.	
	AC	
	4300	Diag.
	4000	
	3000	
	2000	
B 1265	1930	
	1000	
From	A, go	West.

Area 108*a*. 3*r*. 12½*p*.

4. Draw a plan of a field, and find its area from the following notes.

BD	
1238	Diag.
1000	
Return to	B.
FC	Diag.
1900	
1000	
Return to	F.
DE	
1500	Diag.
1000	
R. off D.	
FD	
1160	
1000	
R. off F.	
EF	
1260	
1000	
R. off E.	
CE	
1080	
R. off C.	
AC	
2100	Diag.
2000	
1000	
R. off A.	
DA	
900	
R. off D	
CD	
1340	Diag.
1000	
R. off C.	

	BC	
	1000	
	R. off B.	
	AB	
	1560	
	1000	
Begin at	A, and	go West.

ANSWER.

Having laid down the figure, you will find the perpendiculars of the trapezium ABCD = 708 and 390, and of DCEF = 680 and 808 links; *hence the area of the field is 25 acres, 2 roods, and 26½ perches.*

PROBLEM IV.

To measure an irregular, narrow piece of land.

Divide the ground to be surveyed into triangles and trapezoids, by measuring a base-line, in a convenient position, and upon it erecting perpendiculars to the boundaries, on each side.

Note 1. The general method of measuring a few *lands* or *ridges* together, in common fields, &c. is by taking several breadths, dividing their sum by the number for a mean breadth, and multiplying this by the length, for the area, but it is much more accurate to note the place of each breadth upon the base or chain-line, and find the area by the rule for trapezoids.

2. In order to obtain the breadths correctly, you must measure directly across the lands

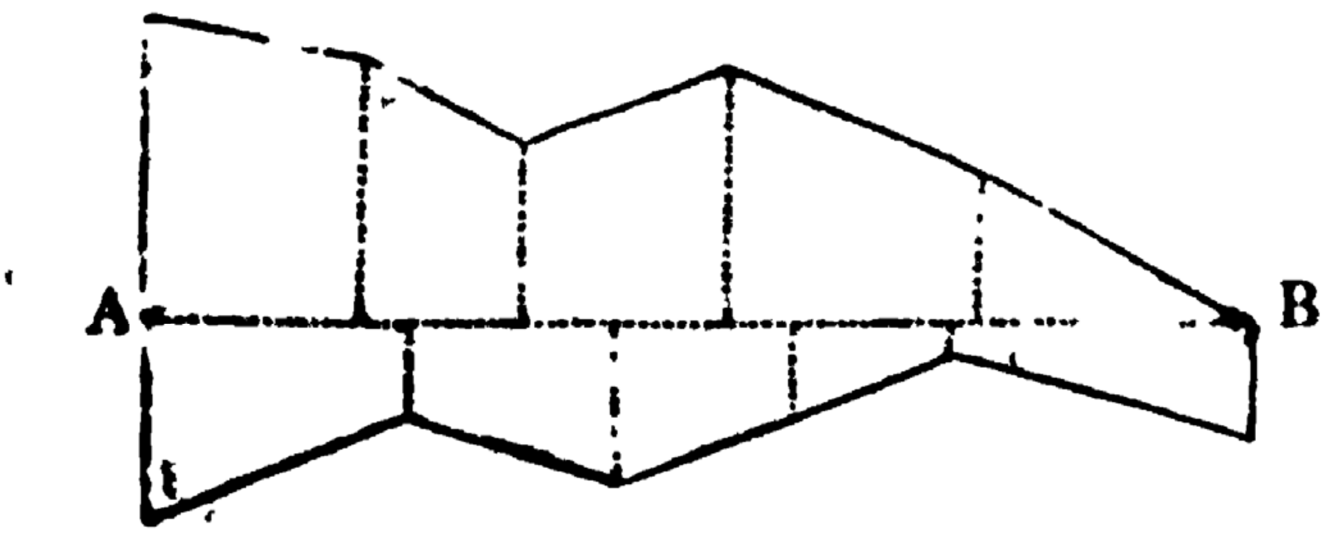
3. If the lands be curved, or longer on one side than on the other, by measuring along the middle, you will obtain the mean length.

4. Paring, reaping, &c. should always be surveyed with a *slack* chain, in order to obtain the measurement of the surface.

EXAMPLES.

1. Draw a plan of an irregular piece of land, and find its area from the following dimensions.

	AB	
0	1814	126
234	1005	
	980	52
	785	125
312	700	
	555	152
215	460	
	335	100
336	260	
360	000	232
From	A, go	East.



ANSWER.

Double Areas.

324337 Offsets on the right.
656476 Ditto on the left.
2)980813 Sum.
490406 = *4a. 3r. 24½p., the area required.*

2. Lay down a field, and find its area from the following notes.

	BC
0	526
65	400
92	300
94	200
73	100
0	000
	R. off B.

	AB	
0	1235	526 to C.
55	1100	435
112	900	328
175	650	250
205	500	224
230	350	245
250	200	280
266	000	365
From	A, go	North.

Area 6*a*. 1*r*. 2*p*.

3. Find the area of five lands, from the following dimensions.

2436	205
2000	210
1700	214
1400	218
1000	220
700	215
400	209
000	205

Area 5*a*. 0*r*. 29*p*.

4. Required the area of ~~six~~ four lands, from the following notes.

3[illegible]65	199
8[illegible]0	
2500	214
2000	
1800	236
1000	
800	201
000	175

Area 7*a*. 0*r*. 8*p*.

Note.—The sum of the breadths in the last example is 1025, which being divided by 5, their number, we obtain 205 for the mean breadth; then 3365 multiplied by 205, is equal to 689825 square links, which are equal to 6 acres, 3 roods, 24 perches, the area, which is too little by 24 perches, proving the inaccuracy of the common method.

If a piece of ground be narrowest about the middle, the common method will give the area too much.

PROBLEM V.

To measure a mere or wood.

By the help of your cross, fix four station-staves on the outside of the mere or wood, so as to form a rectangle; then measure each of its sides, taking insets to the boundaries, which must be treated as before directed.

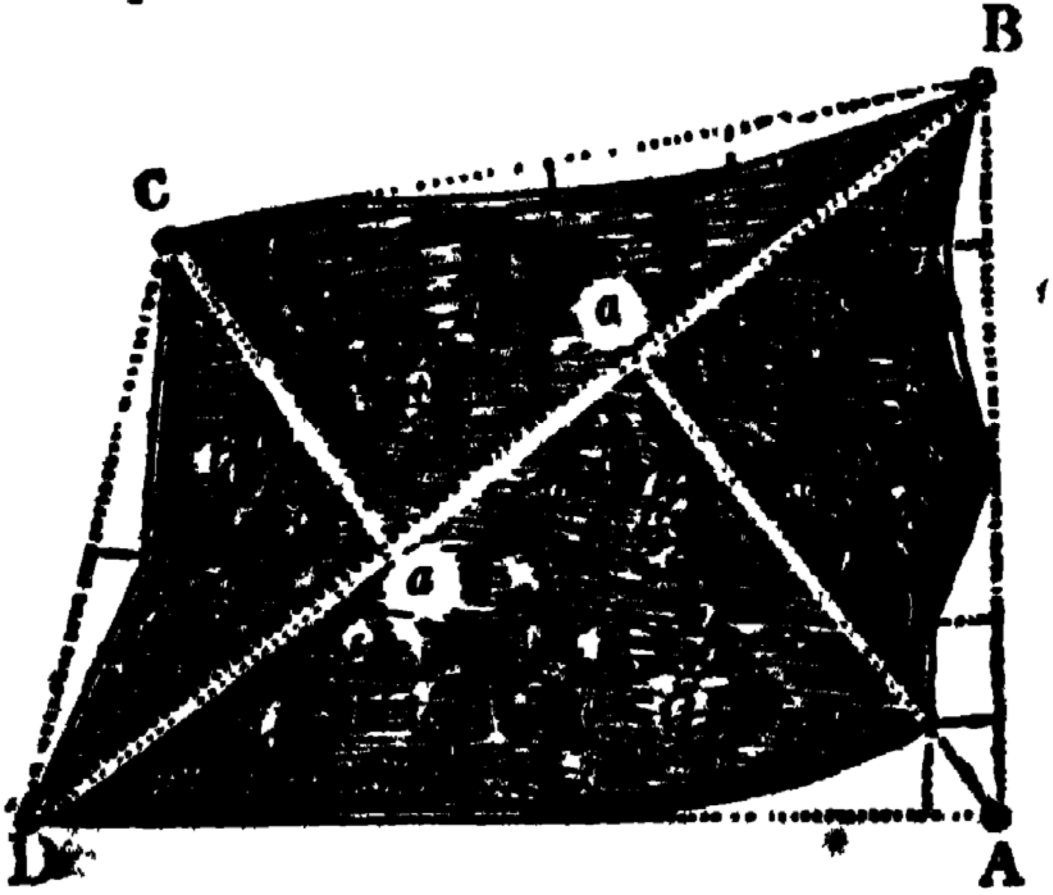
It is, however, a more expeditious method than the former, to measure the four sides of a quadrilateral figure, having one right-angle; then construct the figure, and measure the diagonal and perpendiculars by the scale.

Note 1. If a mere or wood be of a triangular shape, its area may be found by measuring the three sides of a circumscribing triangle, and proceeding as before.

2. By this Problem you may survey fields into which you are not permitted to enter, or which contain obstructions.

EXAMPLE.

Let the following figure represent a wood, the area of which is required.



Set up your cross at station A, and let your assistant fix the marks B and D, so that the angle at A may be a right angle; and measure the line AB, taking insets to the fence, as you proceed. Then fix the mark C, as most convenient; measure the other three lines, and you will find in your field-book the following dimensions.

	DA	
0	1550	
160	1440	
50	1200	
	1000	
0	900	
	L. off D.	
	CD	
0	950	
120	500	
0	000	
	L off C.	
	BC	
	1340	
0	1050	
	1000	
30	700	
60	400	
0	000	
	L off B.	
	AB	
0	1150	
	1000	
50	900	
0	550	
100	300	
110	160	
0	000	
Begin at	A, and	go North.

ANSWER.

Having constructed the figure, you will find the diagonal BD to measure 1930, the perpendicular A*a* 923, and C*a* 605 links; *hence the area of the wood is 12a. 3r. 20p.*

PROBLEM VI.

To measure and plan roads, rivers, canals, &c.

In measuring roads, rivers, or canals, angles or tie-lines must be taken at the different turns, in order to lay down the chain-lines; and offsets must be taken to the boundaries, as you proceed, to enable you to draw the plan.

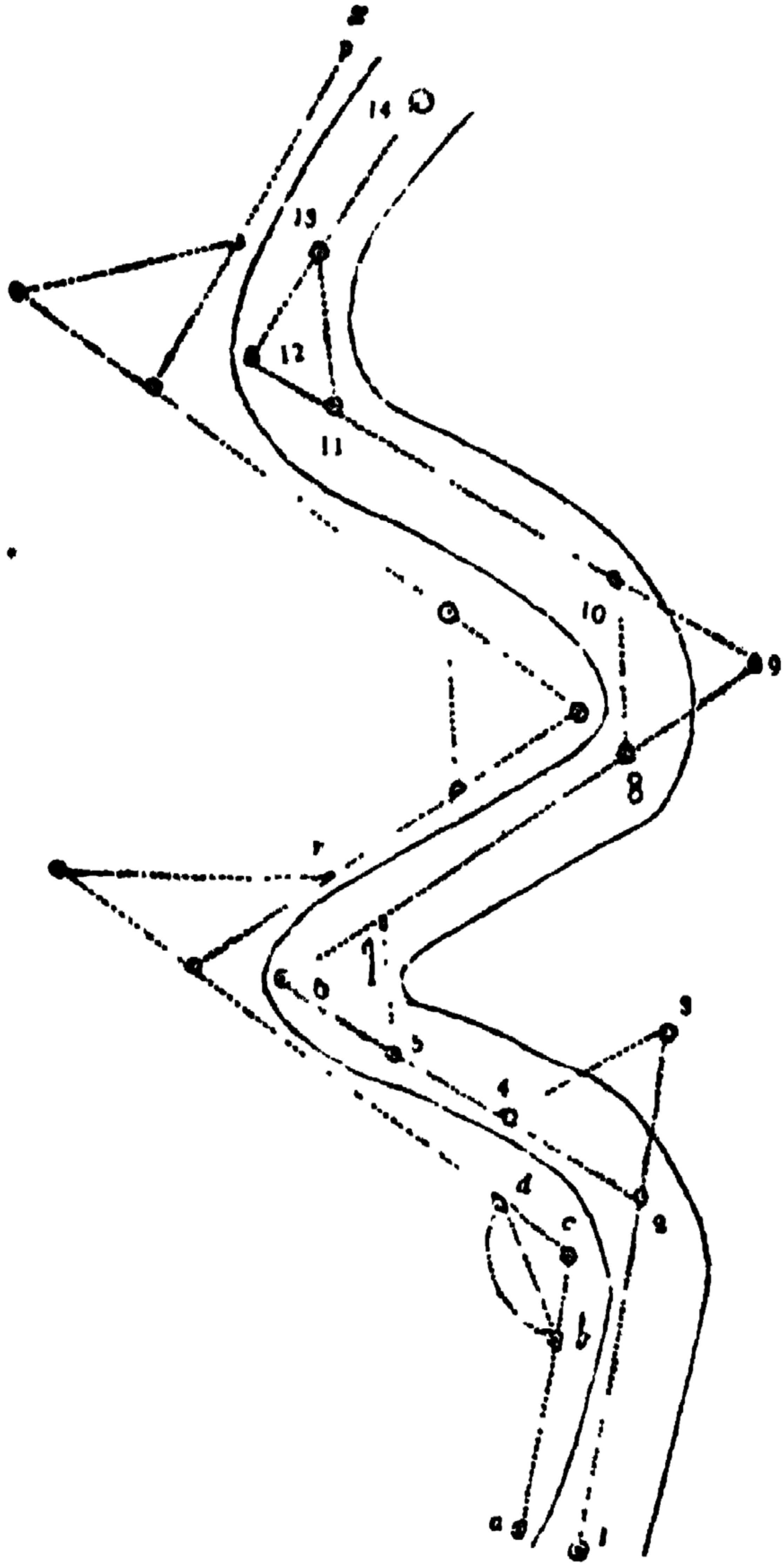
Note 1. The length of a road is generally returned either in miles, furlongs, and poles, or else in miles and yards.

2. A machine called a "Perambulator" is sometimes used to ascertain the lengths of roads. It has a wheel of 8 feet 3 inches, or half a pole in circumference, which being made to pass over the ground, puts in motion the clock-work within; and the distance measured is pointed out by an index on the outside.

This instrument is much more expeditious for measuring the length of a road, than the chain; but it is certainly less correct: for by the wheel passing over stones, sinking into holes, &c. the distance is made to appear more than it is in reality.

EXAMPLES.

1. Let the following figure represent a serpentine road, a plan of which is required.



Begin at + 1, and measure to + 3, taking offsets on both sides, as you proceed. Return to + 2, and measure to + 4, from which run a line to + 3, which will tie the first and second lines. Return to + 4, and continue the line to + 6. From + 6, proceed as before, until you arrive at + 14; and you will have obtained the following dimensions, from which a plan may be drawn.

	to + 14.	
58	350	60
68	200	44
	150	+ 13 is 184 from + 11.
50	100	80
Go from	+ 12,	Line 5.
30	720	to + 12.
70	650	
	600	+ 11.
86	550	33
70	300	50
+ 8 is 200 from + 10	200	
120	135	Cross-fence.
Go from	+ 9,	Line 4.
	700	to + 9.
	600	Cross-fence.
	500	+ 8.
38	480	84
40	300	60
52	180	65
	150	+ 7 is 160 from + 5.
50	100	
Go from	+ 6,	Line 3.
20	512	to + 6.
50	450	
52	380	70
	350	+ 5.
20	300	80
	200	+ 4 is 232 from + 3.
18	100	93
Go from	+ 2,	Line 2.

	600	to + 3.
	480	Cross-fence.
	400	+ 2.
38	350	95
15	300	
28	200	80
55	000	70
Begin	at + 1,	Line 1.

2. Let the foregoing figure represent a river, a plan of which is required.

Begin at *a*, and measure to *c*; taking offsets to the river's edge, as you proceed. From *c* measure to *d*; and there take the tie or chord-line *db*, which will enable you to lay down the first and second lines. Continue the second line to *n*; and from *m*, measure to *r*, at which place take the tie-line *rn*; and thus proceed until you come to the end of your survey at *x*.

If the breadth of the river be every where nearly the same, its breadth taken in different places, by the next Problem, will suffice; but if it be very irregular, dimensions must be taken on both sides, as above.

When the area is required, it must be found from the plan, by dividing the river into several parts; and taking the necessary dimensions by the scale.

Note 1.—Any bog, marsh, mere, or wood, whatever may be its number of sides, may be measured by this Problem.

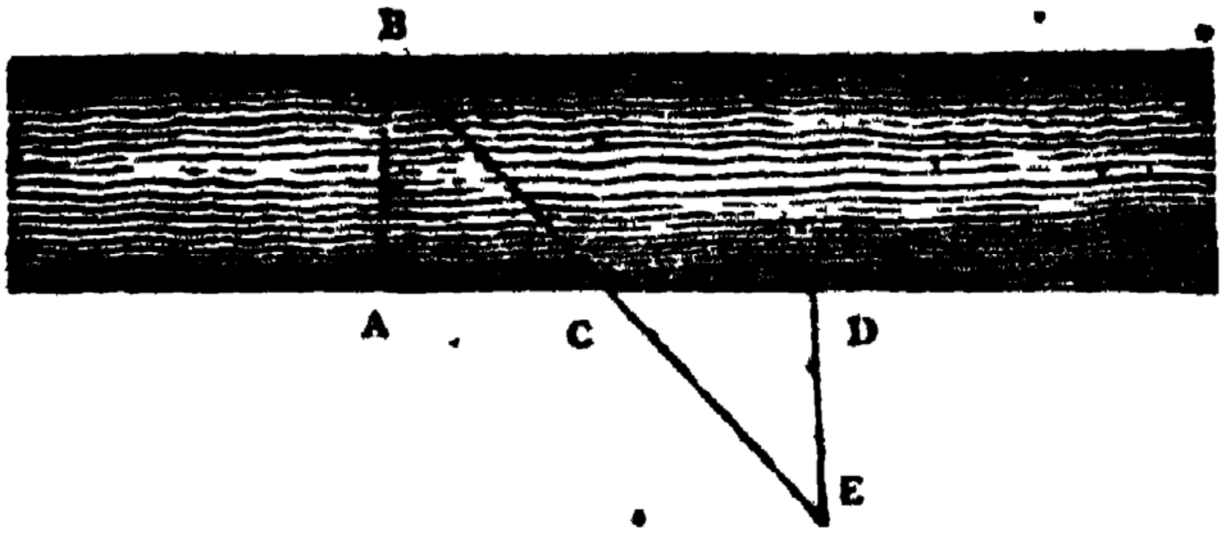
2 In taking an angle with the chain, some surveyors never measure more than 100 links from the angular point, but when the lines, including the angle, are of a considerable length, it is much better to measure the chord-line at a greater distance from the angle, than one chain; because a small inaccuracy in constructing the figure, when the angular distance is short, will throw the lines, when far produced, considerably out of their true position. It sometimes happens, however, in consequence of obstructions, that it is impossible to measure the chord-line at a greater distance from the angular point, than one chain. In such cases, multiply both the angular distance and chord-line by 2, 3, 4, or any larger number, as circumstances may require; and use the products resulting, in laying down the angle. Or, which is the same thing, lay down the angle by a scale 2 or 3 times as large as that by which you intend to draw the plan. By this means the radius or angular distance *bc* equal *cd*, and the chord line *bd*, will be increased in length; and consequently the lines *ac* and *cn*, including the angle, will be more correctly laid down in position.

PROBLEM VII.

To find the breadth of a river.

EXAMPLE.

Let the following figure represent a river, the breadth of which is required.



Fix upon any object B, close by the edge of the river, on the side opposite to which you stand. By the help of your cross, make AD perpendicular to AB, also make AC = CD, and erect the perpendicular DE; and when you have arrived at the point E, in a direct line with CB, the distance DE will be = AB, the breadth of the river; for by Theo. 1, Part I, the angle ACB = DCE, and as AC = CD, and the angles A and D right angles, it is evident that the triangles ABC, CDE are not only similar but equal.

Note 1. The distance between A and the edge of the river must be deducted from DE, when it is not convenient to fix A close by the river's edge.

2. This Problem may also be well applied in measuring the distance of any inaccessible object: for let AC equal 8 CD equal 2, and DE equal 10 chains; then, by similar triangles, as CD : DE :: AC : AB equal to 40 chains.

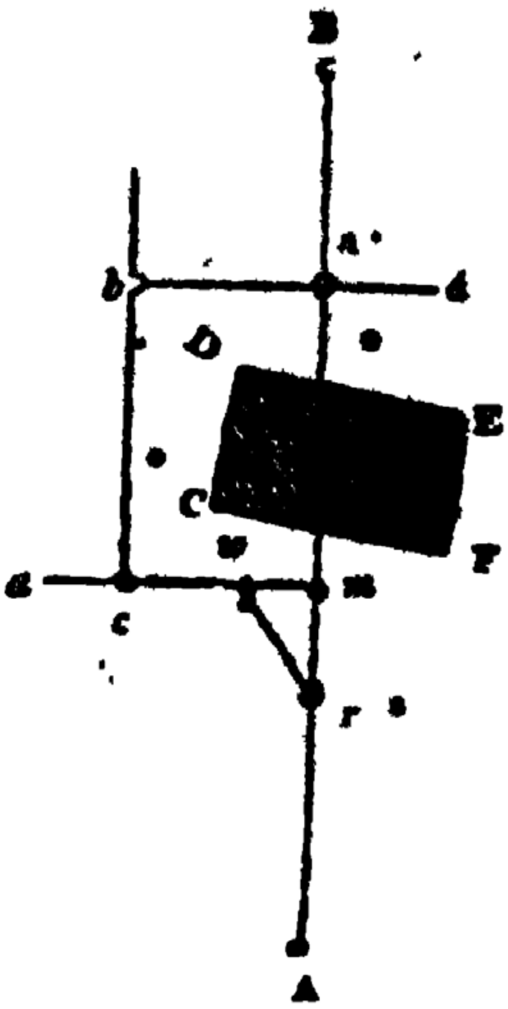
PROBLEM. VIII.

To measure a line upon which there is an impediment.

EXAMPLE.

Let CDEF represent the base of a building, through which it is necessary that a straight line should pass from A.

Measure from A to *m*; at *m*, erect the perpendicular *ma*, which measure until you are clear of the impediment, as at *c*. Erect the perpendicular *ce*, which measure until you are beyond the building, as at *b*. Erect the perpendicular *bd*; and make *bn* equal to *mc*, at which point you will be in a direct line with *m*A. Erect the perpendicular *n*B, which measure; then A*m* added to the sum of *cb* and *n*B, will give the length of the whole line AB.



Note 1. This Problem is very useful when you meet with ponds, bogs, buildings, &c. upon a chain line.

2 When a cross is not at hand, a perpendicular may be erected by the chain, in the following manner Measure 40 links from *m* to *r*, and let one end of the chain be kept fast at *r*, and the eightieth link at *m*; take hold of the fiftieth link, and stretch the chain so that the two parts *mw*, and *rw*, may be equally tight; then will *mw* be perpendicular to *mr*, for *mw*, *mr*, and *rw*, are in proportion to each other as 3, 4, and 5. (See Problem 29, Part [illegible].)

PROBLEM IX.

To part from a rectangular field, any proposed quantity of land, by a line parallel to one of its sides.

RULE.

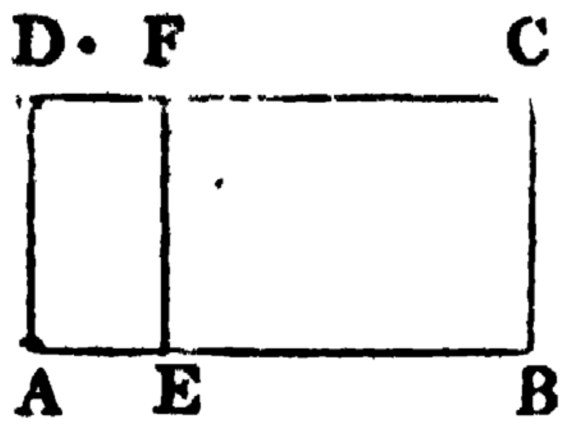
Divide the proposed area, in square links, by the side upon which it is to be parted off, and the quotient will be the length of the other side of the figure required.

Note 1. If there be offsets on the line upon which the proposed quantity is to be parted off, deduct the area of the offsets from it, and proceed with the remainder as above.

2. Acres, roods, and perches may be reduced into square links, by multiplying the whole quantity, in perches, by 625, the number of square links in a perch.

EXAMPLES.

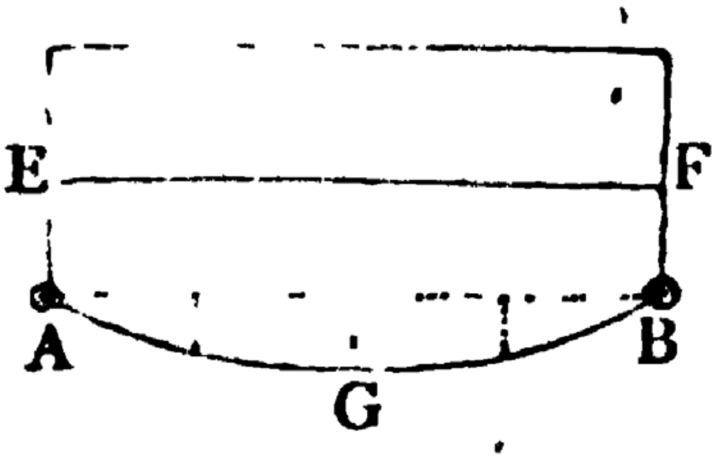
1. From the rectangle ABCD part off 2 acres, 1 rood, and 32 perches, by a line parallel to AD = 700 links.



Here 2*a.* 1*r.* 32*p.* = 392*p.*, and 392 × 625 = 245000 square links; then $\frac{245000}{700} = \frac{2450}{7} = 350$ links = AE = DF.

2. Part off 2*a.* 3*r.* 32*p.* upon the chain-line AB, so that the offsets taken on that line may be included.

	AB	
	1200	0
	1000	
	900	60
	600	80
	300	50
	000	0
From	A, go	East.



2*a.* 3*r.* 32*p.* = 295000 *square links.*
57000 the area of the offsets.
12,00)2380,00 the difference.
198,4 links = AE = BF.

Hence the irregular figure AGBFE, contains 2*a.* 3*r.* 32*p.*

Note. When it is required to part from a trapezium, approaching very nearly to a rectangle, any number of acres, &c. by a line parallel to one of its sides, it may be done by this Problem.

PROBLEM X.

To part from a trapezium or any irregular polygon whatever, any proposed quantity of land, by a line drawn parallel to any of the sides, or by a line drawn from any of the angles, or from any assigned point in one of the sides, to any of the opposite sides.

RULE.

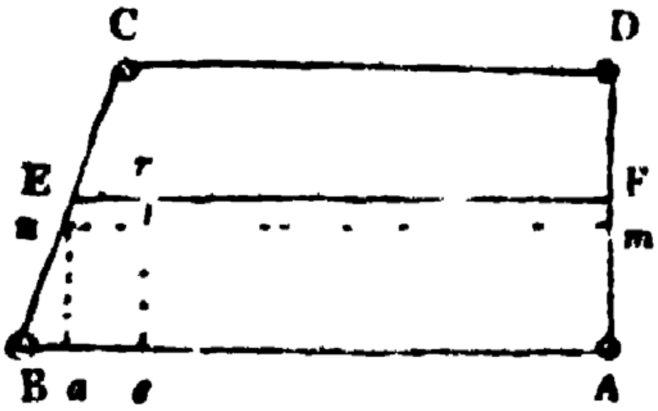
Having surveyed and laid down the whole figure, draw a guess-line in the direction required, parting off, as nearly as can be judged, the proposed quantity; after which, by the scale, measure with the greatest accuracy, the guess-line, and also the quantity thus parted-off. Then, if the guess-line or line of division be drawn from an angle, or from any assigned point in a side, divide the difference between the proposed quantity and the quantity parted off, by half the guess-line; and the quotient will be the perpendicular to be set off, on one side, or the other, of the guess-line, accordingly as the quantity parted off is more or less than the quantity proposed. To the end of this perpendicular, from the point assigned, draw a new line of division; and it will part off the quantity required.

But if the guess-line be drawn parallel to any of the sides, divide the difference before mentioned, by the whole guess-line, and the quotient will be the perpendicular to be set off from each end of the guess-line, on one side, or the other, as above.

EXAMPLES.

1. From a trapezium, whose dimensions are contained in the following notes, part off 2*a*. 2*r*. 24*p*. by a line parallel to the side AB.

	BD	
	1249	Diag.
	1000	
Return	to B.	
	AC	
	1112	Diag.
	1000	
	R. off A.	
	DA	
	550	
	R. off D.	
	CD	
	979	
	R. off C.	
	BC	
	557	
	R. off B.	
	AB	
	1114	
	1000	
Begin	at A,	and go West.



Having laid down the figure, draw the guess-line *mn* parallel to AB; and from *n*, let fall the perpendicular *an*; then suppose *mn* = 1058 links, *an* will be = 230, and A*a* = 1052; therefore B*a* = 1114—1052 = 62 links.

Square links.

Then, 1055×230 = 242650 { the area of the trapezoid *Aanm*.

And 230 × 31 = 7130 { the area of the triangle B*an*.

The sum is = 249780 { the area of the trapezium AB*nm*.

2*a*. 2*r*. 24*p*. = 265000

15220 the difference between the quantity proposed, and the quantity parted off by the guess-line, which being divided by 1058, we obtain 14.4 links, to be set off perpendicularly from *m* and *n* towards D and C. Hence EF is the true line of division; and the trapezium ABEF contains 2*a*. 2*r*. 24*p*.

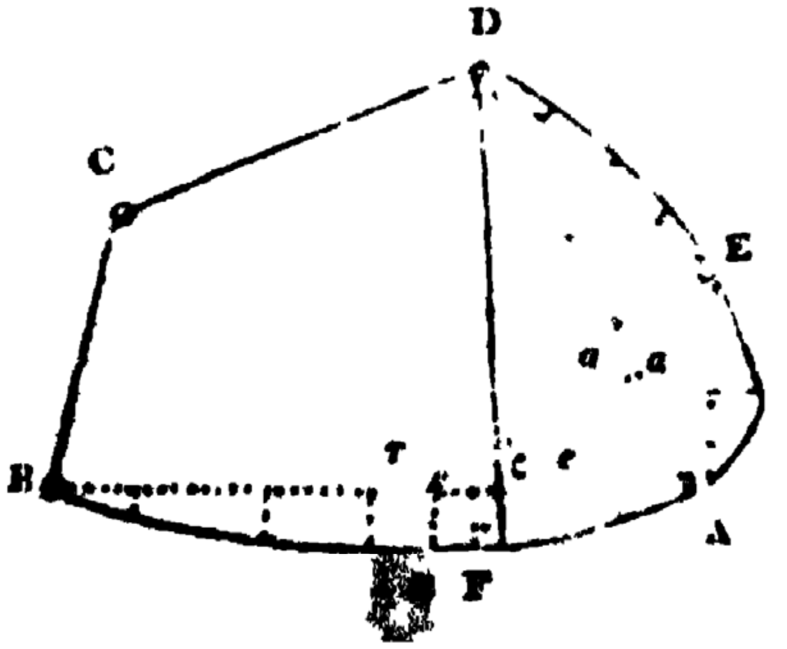
As A is very nearly a right angle, measure, in the field, 230 + 14.4 = 244.4 links, from A to F; and upon any part of the line AB (towards B) as at *e*, erect the perpendicular *er*, which make = 244.4 links; stake out the line E*r*F, and the work will be completed.

Note. In order to prove the operation, find the area of DCEF, then if it be equal to the difference between the area of ABCD and the quantity parted off, the work is undoubtedly right.

2. From any irregular field, whose dimensions are contained in the following notes, part off 2*a*. 3*r*. 20*p*. towards the line AE, by a fence made from the angle D to the side AB.

EB	
1398	Diag.
1000	
R. off E.	
CE	
1240	Diag.
1000	
500	*m*, proof-line, goes to D, and measures 324.
R. off C.	
AC	
1260	Diag.
1000	
R. off A.	

	EA	
0	400	
80	200	
0	000	
	R. off E.	
	DE	
0	600	
25	450	
35	300	
20	150	
0	000	
	R. off D.	
	CD	
	740	
	R. off C.	
	BC	
	550	
	R. off B.	
	AB	
0	1250	
35	1000	
50	800	
60	600	
50	400	
30	200	
0	000	
Begin at	A, and	go West.



Having laid down the figure, draw the guess-line Dn, which suppose $= 766$ links; then the diagonal AD will be $= 824$, the perpendicular $Ea = 278$, and the perpendicular $ar = 372$ links; also re will be $= 228$, and $rn = 52$ links.

Square links.

267800 the area of the trapezium *Ar* DE.

12000	the area of the offsets taken on the line.	DE.
16000		EA.
12348		*Ar.*

The sum is = 308148 the area of A*n*DEA.
2a. 3r. 20p. = 287500

20648 the difference between the quantity proposed, and the quantity parted off by the guess-line, which being divided by 383, (half the guess-line,) gives 54 links, to be set off from *n* towards A. Hence, DF is the true line of division; and the irregular figure AFDE contains *2a. 3r. 20p.*

Now, by the scale, A*c* = 377 links. Measure therefore, in the field, 377 links from A to *c*; stake out the line D*c*F, and the work will be completed.

Note 1. The Rules given in this Problem, for parting off land from irregular fields, are generally adopted by Practical Surveyors, because they may be applied to any irregular figure whatever. Land, however, may sometimes be parted off more directly; for instance, the last example may be performed by finding the area of the irregular figure ADE, and subtracting it from the quantity to be parted off; then, if the difference be divided by half the line AD, the quotient will be the perpendicular of the triangle ADF, the side AB being nearly straight from A to F.

Now, at the distance of this perpendicular, draw a line parallel to AD; and it will intersect AB in F, the point to which the division fence must be made.

2. It is not absolutely necessary to survey and plan the whole figure, in order to part a portion from it, as the guess-line and portion parted off may be measured in the field; but, in my opinion, the former, in general, is a more eligible method than the latter.

3. In order to divide a trapezium or an irregular polygon, among any number of persons, by fences made in a given direction, proceed thus: Part off the first person's share, then from the remainder of the figure, part off the second person's share; and thus continue, till the whole field be divided.

4. Those who desire to see a greater Variety of Examples in surveying single Fields, and to make themselves fully acquainted with the Methods of Laying out, Parting off, and Dividing Land; also of Dividing a Common, &c. of variable Value among any Number of Proprietors, in the Proportion of their respective Interests, may consult my Treatise on Practical Land-Surveying, Sixth Edition, in which I flatter myself they will find these subjects satisfactorily treated.

PROBLEM XI.

To reduce statute-measure to customary, and vice versâ.

It has been before observed, that by custom the perch varies in different parts of England; and with it, consequently, varies the acre in proportion.

In Devonshire and part of Somersetshire, 15; in Cornwall, 18; in Lancashire 21; and in Cheshire and Staffordshire, 24 feet are accounted a perch.

GENERAL RULES.

I. To reduce statute-measure to customary, multiply the number of perches, statute-measure, by the square feet in a square perch, statute-measure; divide the product by the square feet in a square perch, customary measure, and the quotient will be the answer in square perches.

II. To reduce customary measure to statute, multiply the number of perches, customary measure, by the square feet in a square perch, customary measure; divide the product by the square feet in a square perch, statute-measure, and the quotient will be the answer in square perches.

Note 1. It is scarcely necessary to remark that the length of any perch multiplied by itself will give the number of square feet in a square perch of the same measure; hence we have $16.5 \times 16.5 = 272.25$, the statute perch; $15 \times 15 = 225$, the Devonshire and Somersetshire perch; $18 \times 18 = 324$, the Cornwall perch, $21 \times 21 = 441$, the Lancashire perch, and $24 \times 24 = 576$, the Cheshire and Staffordshire perch.

2. When it is intended to find the area of an estate in customary measure only, it is generally thought most convenient to take the dimensions by a chain properly adapted for that purpose. The Devonshire and Somerset chain, is 60 feet the Cornwall chain, 72 feet, the Lancashire chain, 84 feet; and the Cheshire and Staffordshire chain, is 96 feet in length. Each of these chains is divided into 100 equal links, in the same manner as the statute chain; consequently, the customary measure is found by the same Rules as the statute-measure.

3. It may also be observed, that 4840 square yards make one statute acre; 4000 make one Devonshire or Somersetshire acre; 5760 make one Cornwall acre; 7840 make one Lancashire acre; and 10240 square yards make one acre of the customary measure of Cheshire and Staffordshire. *(See the Author's Land-Surveying, Parts II. III. and VI.)*

EXAMPLES.

1. In *25a. 2r. 20p.* statute, how many acres, &c. customary measure, of 15 feet to a perch?

By Rule I. 25A. 2R. 20P.

```
                          25a. 2r. 20p.
                               4
                             ----
                             102
                              40
                            -----
                            4100
                          272.25 = 16.5 × 16.5
                          ------
                           20500
                           8200
                          8200
                        28700
                        8[illegible]
                        ------------  4,0
15 × 15 = 225)1116225.00(496,1
                900          4)124  1
              -----          ---------
                2162         31a. 0r. 1p. Ans.
                2025
                ----
                 1372
                 1350
                 ----
                  225
                  225
                  ---
                  ...
```

2. In 31*a*. 0*r*. 1*p*. customary measure, of 15 feet to a perch, how many acres, &c. statute-measure?

By Rule II. 31A. 0R. 1P.

```
                    4
                  ----
                  124
                   40
                 -----
                 4961
                  225
                 -----
                24805
                9922
               9922
              ------------  4,0
     272.25)1116225.00(410,0
              108900       4)102  20
              ------       ---------
               27225        25a. 2r. 20p. Ans.
               27225
               ------
               .....00
```

3. Reduce 56*a*. 3*r*. 36*p*. statute, to customary measure, of 18 feet to a perch. *Ans*. 47*a*. 3*r*. 20*p*.

4. In 47*a*. 3*r*. 20*p*. customary measure, at 18 feet to a perch, how many acres, &c. statute-measure? *Ans*. 56*a*. 3*r*. 36*p*.

PROBLEM XII.

To survey and plan Estates or Lordships.

Various methods are adopted by different surveyors, in taking the dimensions of Estates or Lordships; I shall, however, describe only four which I conceive to be the most accurate and practical.

METHOD I.

Having made yourself acquainted with the form of the estate, either by actual examination, or by the assistance of a previous plan, select two suitable places, at the greatest convenient distance from each other, as grand stations; and measure a principal base, or what is generally called a "main-line," from one to the other, noting every hedge, brook, or other remarkable object, as you cross or pass it; taking offsets likewise to the bends or corners of the hedges that are near you.

Next, fix upon some other suitable place, towards the outside of the estate, as a third grand station; to which, from each extremity of the diagonal or main-line, or from two convenient points in it, lines must also be run.

These three lines being laid down, will form one large triangle; and in a similar manner, if necessary, on the other side of the diagonal or main-line, a second triangle may be formed.

The survey must then be completed by forming smaller triangles, on the sides of the former; and measuring such lines as will enable you to obtain the fences of each inclosure, and prove the whole of your work.

Note 1. The method of measuring estates by dividing them into triangles, is particularly described in my Surveying; and illustrated with several rough and finished Plans, and an engraved field-book.

It is also exemplified in this Work by Plate I., which is a rough plan of an Estate exhibiting the chain-lines and stations used in taking the survey. The field-notes are not given, as they would have occupied too many pages of copper-plate; it may, however, be observed that the method of entering the notes, sketching the fences, &c. is precisely the same as in the field-book belonging to Plate III.; and the directions of all the lines may be easily ascertained from the following particulars.

The first or main-line leads from + 1 to + 8; the second line from + 8 to + 10; and the third from + 10 to + 1; which three lines form the first large triangle.

The fourth line extends from + 2 to + 15; and the fifth from + 15 to + 8; which two lines and part of the main-line form the second large triangle.

The sixth line leads from + 9 to + 11; the seventh from + 20 to + 6; the eighth from + 7 to + 22; the ninth from + 21 to + 4; the tenth from + 24 to + 13; and the eleventh from + 12 to + 23; which complete the survey of the first triangle.

The twelfth line extends from + 5 to + 17; the thirteenth from + 25 to the main-line, southward of + 3; the fourteenth from + 1 to + 14; the fifteenth from + 14 to + 26; the sixteenth from + 27 to + 16; the seventeenth from + 18 to + 28; and the eighteenth from + 28 to + 19; which finish the whole survey.

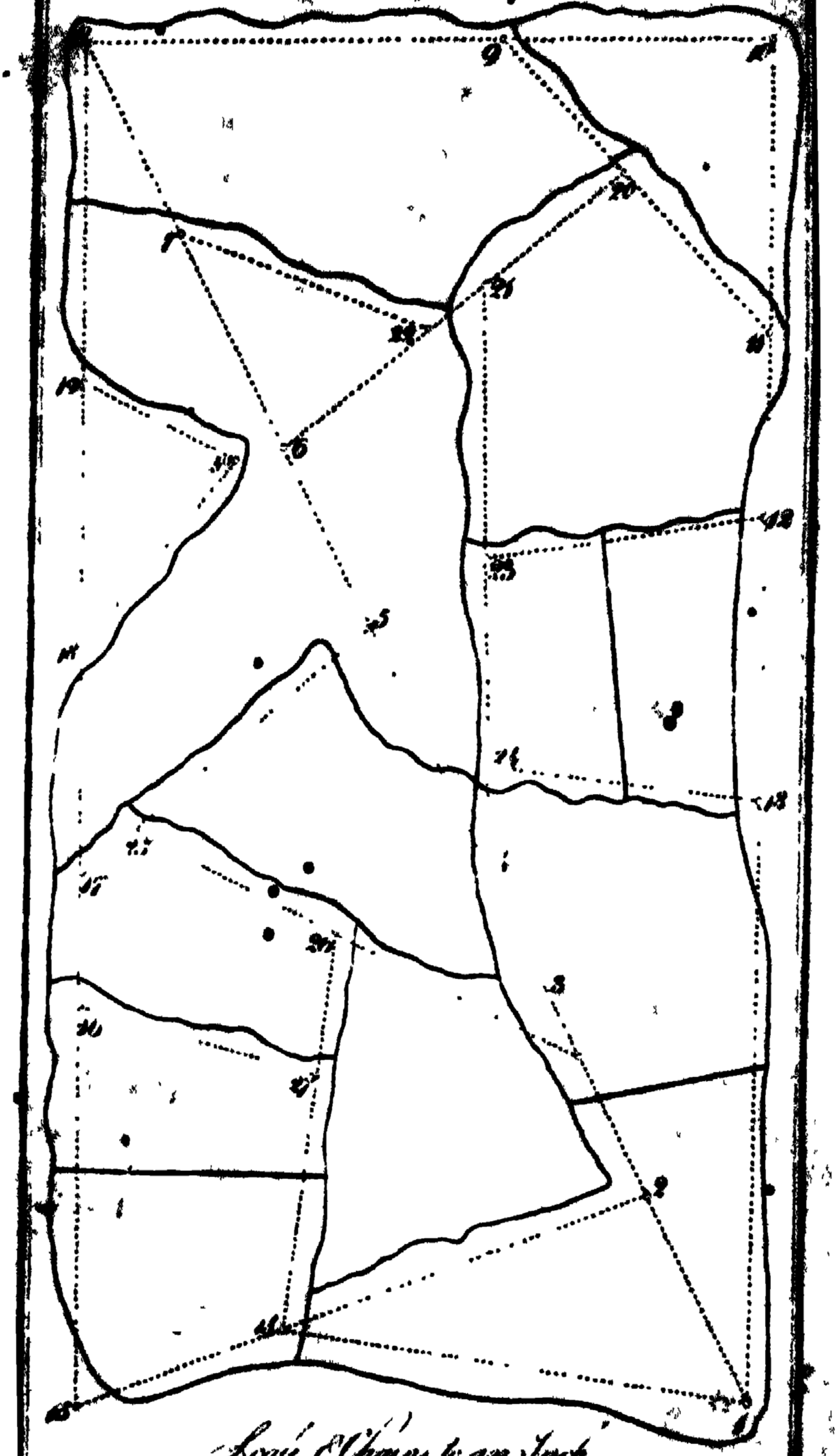
2. The content of the Estate may be found in the following manner: Measure the lines upon the Plan, and take the necessary offsets, by a scale of 8 chains to an inch; and enter the dimensions in a field-book. From the dimensions thus obtained, draw a Plan by a

scale of 2 chains to an inch; then straighten the fences as directed in Problem II.; and measure diagonals, perpendiculars, &c. from which compute the contents of each field.

The diagonals, perpendiculars, and contents may be entered in a Book of Castings, similar to that belonging to Plate III.; and if you should not have a scale of 8 chains to an inch, any other scale will do just the same for practice.

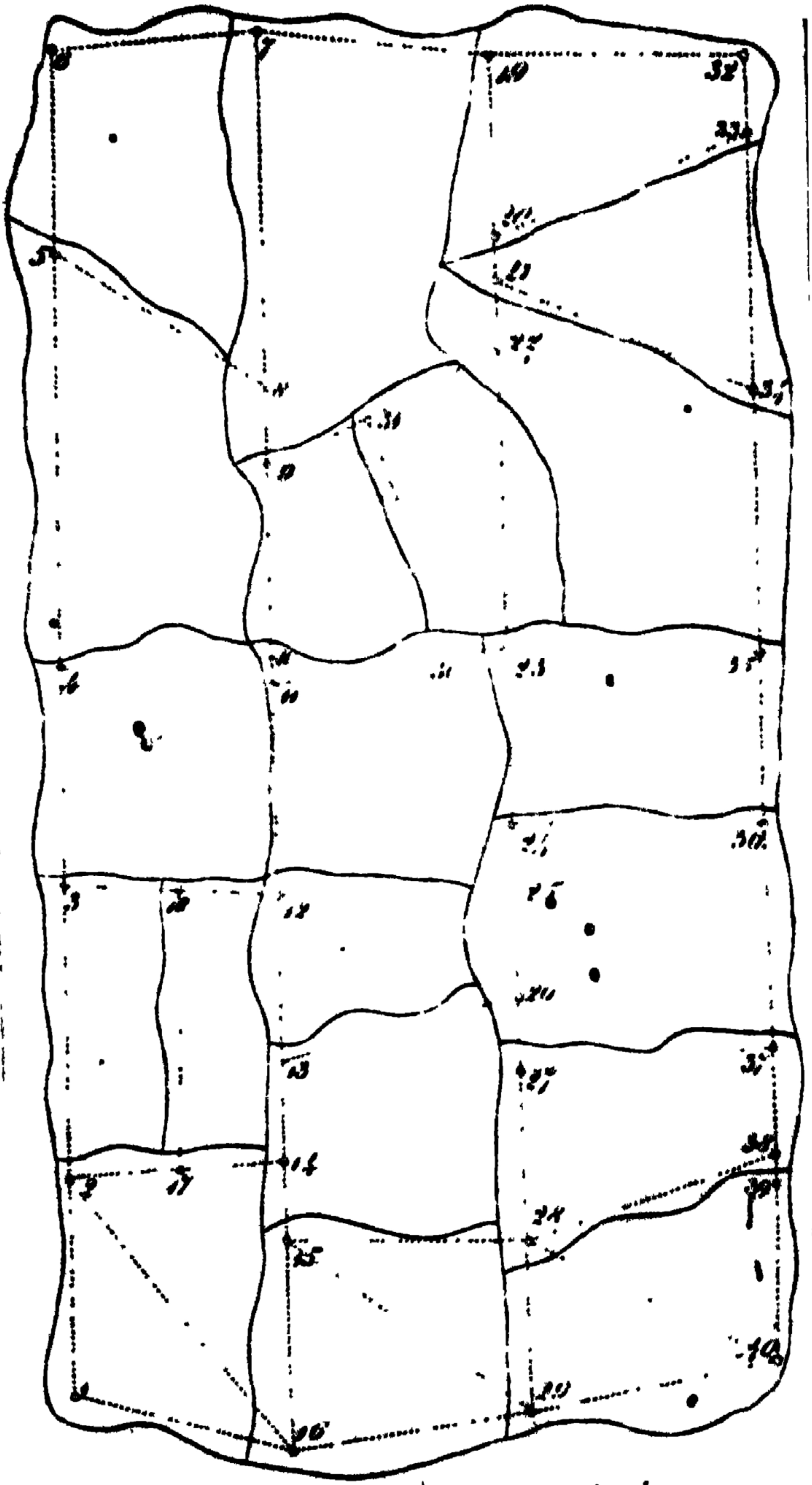
8. Taking the dimensions, &c. as directed in the last note, will be found of infinite service to the learner; as it will tend to make him very expert in entering the field-notes, laying down the lines, and casting the contents, which are no small acquisitions towards becoming a complete Land-Surveyor.

Plan.



Four Chains to an Inch

Plan.



Scale 8 Chains to an Inch.

METHOD II.

Measure a main-line as nearly to one of the out-boundaries of the estate, as the curves in the hedges will permit; noting the crossings of fences, and taking offsets as before directed.

At a convenient distance, measure another main-line parallel or nearly parallel to the first line, so that a number of fences running in that direction may be obtained; and from any two stations in the first line measure lines to any station in the second main-line, forming a triangle; so will a station in the second main-line become determined or fixed.

From the first main-line to the second, or from the second to the first, measure lines in order to obtain all those fences which run in that direction. The remainder of the fences of the inclosures contained between the first and second main-lines being obtained by running lines in the most convenient manner; you will have completed the dimensions of a portion of the estate, which may then be laid down.

Parallel or nearly parallel to the second main-line, and at a proper distance from it, measure a third; and proceed with the internal lines as before, and you will obtain the dimensions of another portion of the estate, which may also be laid down; and thus go forward until you finish the survey.

Note 1. This method is illustrated by Plate II., which displays the chain-lines and stations used in taking the survey.

The first main-line leads from + 1 to + 6; the second line from + 6 to + 7; and the third line, or second main-line from + 7 to + 16. The fourth line extends from + 16 to + 1; the fifth, or tie-line from + 16 to + 2; the sixth from + 2 to + 14; the seventh from + 17 to + 18; the eighth from + 12, through + 18, to + 3; the ninth from + 4 to + 10; and the tenth line leads from + 8 to + 5; thus all the fences between the first and second main-lines are obtained.

The eleventh line, or the third main-line leads from + 19 to + 29; and the directions in which all the rest of the lines were run, may be easily ascertained by inspecting the plan.

2. The content of this Estate may be found in the same manner as directed in Note 2, Method I.

METHOD III.

An estate of four sides may frequently be conveniently surveyed as follows: Measure four lines in such a manner that offsets or insets may be taken to the four out-boundaries of the estate; and tie the first and fourth lines together by a diagonal or tie-line measured from one to the other, at the distance of five, six, or more chains from the angular point, according to the extent of the survey, thus you will be enabled to lay down the first four lines, and also the out-boundaries of the estate.

Next proceed to obtain the internal fences, by measuring lines in the most convenient manner, some of which must be run from the first to the third, or from the second to the fourth line, or in some other more proper direction, so that they may become proofs, and fast-lines, into which other lines may be run with propriety.

In thus proceeding, it is evident that a great deal will always depend upon the dexterity and ingenuity of the surveyor; as no directions can be given that will suit every particular case to be met with in practice.

Note.—This method is illustrated in my Surveying, by Plate VII.; and also in this Work, by Plate III., which is a Rough Plan of an Estate in the Township of Farnley, in the parish of Leeds.

METHOD IV.

The method which I here intend to describe is a compound of all the methods of surveying with the chain; for as there are never two estates to be met with which are exactly alike, sometimes one method claims the preference, and sometimes another; but a skilful surveyor will always adopt that by which he can take his dimensions and proofs with the greatest accuracy by the fewest lines.

If an estate be in the form of an irregular polygon of five six, or more sides, and the fences very crooked, such an estate may generally be most easily surveyed by dividing it into triangles, as in Plate I.; but if many of the fences of the different inclosures run a considerable way in the same direction, and the fields in general be pretty neat trapeziums, it is commonly more eligible to proceed as directed in Method II.

Sometimes an estate varies so much in its shape that all the methods before described may be used with propriety and advantage; and it frequently happens that an ingenious surveyor adopts methods, in particular cases, entirely new to himself; care, however, must always be taken to make one line depend upon another, throughout the whole of the survey, so that when you come to lay it down, you may find no lines whose positions are undetermined.

Note. For the method of measuring Hilly Ground, see my Land-Surveying, Sixth Edition, page 172.

MISCELLANEOUS INSTRUCTIONS.

1. IN ranging a line, you must be extremely careful to keep the poles in a direct line with each other; which you may accurately effect by always having, at least, two behind you, and if your sight be obstructed by hedges, you must cut down the intervening parts.

2. In measuring your main, or any other chain-line, put down stations at every place to which you apprehend it may be ne-

cessary to run lines, in order to complete the survey; and if it happen that you put down more than are wanted, it will be immaterial. At each station you must put down a small stake, called a "station-stake," with the number of the station upon it; and also cut a mark in the ground, which may be easily done by having a small spade upon the bottom of the offset-staff.

3. In measuring your internal lines, it will give you the least trouble to run them from one station to another, if you can make it convenient; if not, you must run them from, and continue them to some chain-line, and measure the distance upon that line, to the nearest station The place where you run upon, or cross a chain-line, may be easily ascertained by setting up poles at two of the nearest stations in that line; and the crossing will be at the place where you are in a direct line with these poles.

4. The angle which the first line makes with the meridian line must be taken with a compass; and in doing this, an allowance must be made for the *variation of the needle*, which is about 24 degrees westerly.

5. In order to plan a large survey, provide a sheet of drawing-paper, of a proper size for use; and trace with a pencil, a meridian or north and south line, in such a position that your first station may be in some convenient point in this line. Then, from your first station, draw your first or main-line, making its proper angle with the meridian, which you may then take out with Indian rubber. Further directions appear to be unnecessary; as any person who is acquainted with the methods of laying down triangles, trapeziums, and trapezoids, will find no difficulty in planning an estate.

6. The most expeditious method of laying down crooked fences, is by means of an offset-scale, which must be used with the plotting-scale in the following manner: Lay one edge of the plotting-scale close by the base-line, and bring the end of the offset-scale in contact with the edge of the plotting-scale, so that the edges of the scales may form a right-angle; then by the edge of the offset-scale, prick *off*, in its proper situation, the first offset, with a pencil finely pointed. Keep the plotting-scale firm, and slide the offset-scale to the place of the next perpendicular, which prick off as before; and thus proceed until all the offsets are finished.

7. In order to find the area of an estate, practical surveyors generally straighten the crooked fences of each field, as directed

in Problem II., and then divide the fields into trapeziums and triangles, and take such dimensions, by the scale, as are necessary to find the area of each field. They then collect all the areas into one sum; afterward find the area of the whole survey, as if it were a single field; and if it be equal or nearly equal to the sum of the separate areas, previously found, they justly infer that their survey is correct.

8. Those who do not approve of finding the area by the method of casting, may make use of the offsets taken in the survey, where convenient, and if more be wanted, they may be measured by the scale, for in measuring a number of small parts by it, some will probably be taken a little too large, and others a little too small, so that, in the end, they will nearly counterbalance each other.

9 Practical surveyors generally lay down their lines by a scale of 4 chains to an inch, when their surveys are very large, and in computing the contents, they measure the bases and diagonals by the same scale, but the perpendiculars by a scale of 2 chains to an inch, consequently, the product of the base and perpendicular of a triangle, will be its area. To treat small surveys, in a similar manner, by a scale of 2 chains and of 1 chain to an inch, must, of course, be very correct.

10. Rivers, large brooks, public roads, and common sewers, should not be included in the area, but only delineated upon the plan; and marshes, bogs, heaths, rocks, &c. should also be represented or specified, and their measurements, separately returned

11. Sometimes the content of each field is entered within the field itself, sometimes the fields are numbered, and their areas set down, one after another, in some convenient part of the plan; and sometimes they are entered in a book of particulars.

12. When you wish to transfer a rough plan to a fresh sheet of paper, or to a skin of parchment or vellum, in order to make a finished plan, proceed thus. Take a sheet of writing-paper of the same size as the rough plan, and rub one side of it with black lead powder; then lay it upon the sheet which you intend for your new plan, with the black side downward; upon both lay the rough plan; and upon the whole place weights or books, to keep them from moving. Next, run your tracer gently over all the boundaries upon the rough plan, so that the black lead under them, may be transferred to the fresh sheet. Separate the papers and trace the lines thus transferred with a fine pen and Indian ink, as common ink ought never to be

used in planning. You must then proceed to enter such names, remarks, or explanations, as may be judged necessary; drawing, by the pen, the representations of hedges, bushes, trees, woods, hills, gates, stiles, bridges, roads, the bases of buildings, &c. in their proper places; running a single dotted line for a foot-path, a double one for a carriage-road, &c. Rivers, brooks, lakes, &c. should be shaded with crooked or waved lines, bold at the edges, and fainter towards the middle. Hills may also be represented by crooked lines, bold about the middle of the hill, but fainter towards the top and bottom; and the bases of buildings must be shaded by straight, diagonal lines. Roads should be shaded with a brownish colour, laid on with a camel-hair pencil; and fields in a variety of forms, with a fine pen and Indian ink. In some convenient part of the Plan, you may write, in conspicuous characters, the title of the Estate, ornamented with a compartment. In another vacancy introduce the Scale by which the plan has been laid down; and also a meridian-line, with the compass or flower-de-luce pointing north.

The whole may then be bordered with black lines at a convenient distance from each other; and the space between them shaded by a hair pencil, with Indian ink.

13. A plan well finished with Indian ink, as directed above, has a very elegant appearance, and is considered by most persons to excel those done in colours; but the work is very tedious, and requires much time to do it well, in consequence of which most surveyors prefer finishing their plans with colours. Some, however, not only embellish them with Indian ink, but also wash the different fields with various shades of colouring.

14 In colouring a plan, meadow and pasture ground should be washed with a transparent green, the pasture rather lighter than the meadow; arable land with a mixture of red and yellow, or of red, brown, and yellow, of various shades, so that too many fields may not appear exactly alike; and some surveyors use both light blue and lake, in colouring plans.

An excellent green, of various shades, may be made by mixing light blue and gamboge.

Rivers, brooks, lakes, &c. should be coloured with Prussian blue mixed with a little Indian ink. The hedges must be done with a strong shade, which should be softened off towards the middle with a lighter one. If the quickwood hedges be not done with a pen and Indian ink, they may be represented by running strong, narrow shades, of various colours, round the boundaries of the different fields.

15. Reeves's and Newman's water-colours are entitled to recommendation. They must be prepared for use in the following manner: Dip one end of the cake in clear water, and rub a little of it upon a clean, earthen plate; then mix it with water, by your hair pencil, until you have brought it to any consistency you please: Indian ink must be prepared for use in the same manner.

16. When parchment is used in planning, it must first be rubbed with clean flannel dipped in the best Paris whiting. This operation clears its surface from grease, and makes the pen slide more freely.

17. The learner will fully comprehend what has been said on the subject of embellishing plans, by examining a well-finished, coloured map.

18. The method of transferring plans by means of a sheet of paper, rubbed with black lead, will do very well for small surveys; but when the survey is large, it is necessary, not only to transfer the original plan, but also to reduce it to a smaller scale, in order that the finished plan may be of a convenient size. This may be effected in different ways; as by squares, by proportional compasses, &c.; but the most expeditious and accurate method, is by an instrument called a Pentagraph.

Those who desire to make themselves acquainted with the method of using this instrument, are referred to my Land-Surveying, Sixth Edition; also to Adam's Geometrical and Graphical Essays, in which valuable work they will likewise find the description and use of the Plane-Table, Theodolite, Spirit-Level, and every other Mathematical Instrument.

Note.—The area of the following Estate, Plate III., was found from a Plan laid down by a scale of 2 chains to an inch. The crooked fences were straightened as directed in Problem II., and the diagonals measured by the scale used in planning; but the perpendiculars by a scale of 1 chain to an inch.

The diagonals, perpendiculars, &c. are contained in the following Book of Castings.

A Book of Dimensions, Castings, &c.

Belonging to Plate III.

No. on Plan.	*Diagonals.*	*First Perpen.*	*Second Perpen.*	*Sum of Perpen.*	*Quantity* IN A. Dec.	*Quantity* IN A.	R.	P.
1	1375	143	227	370	5.08750	5	0	14
2	1542	280	298	578	8.91276	8	3	26
3	1120	264	208	472	5.28640	5	1	5¾
4	1226	217	213	430	5.27180	5	1	3½
5	1225	220	194	414	5.07[illegible]	5	0	11½
6	1552	374	252	626	9.71552	9	2	34½
7	1088	210	168	378	4.11264	4	0	18
8	874	180	—	180	1.57320	1	2	11¾
9	1262	254	76	330	4.16460	5	0	4¾
	1030	84	—	84	.86520			
10	790	136	190	326	2.57540	2	2	12
Whole Quantity ……					52.63652	52	2	21¼

FINIS.

to + 18.

835
740 20
742 26
500 64.
300 48
100 24

From + 7. go 58. Line 11.

to + 8.

662
300 28
100 26

From + 10 go 1. Line 10

to + 10.

1192
28 1100
1000
18 970 +19.
[illegible]
24 800
30 530
18 300
72 30

From + 17. go 21 Line 9

to + 6.

1891
23 1600
50 1200 + 18.
28 1016 5
1000
980 + 17.
50 800
60 400
18 100

From + 15. go A. Line 8.

to + 13.

26 1070
28 900
38 500
16 250
24 70

[illegible] 8

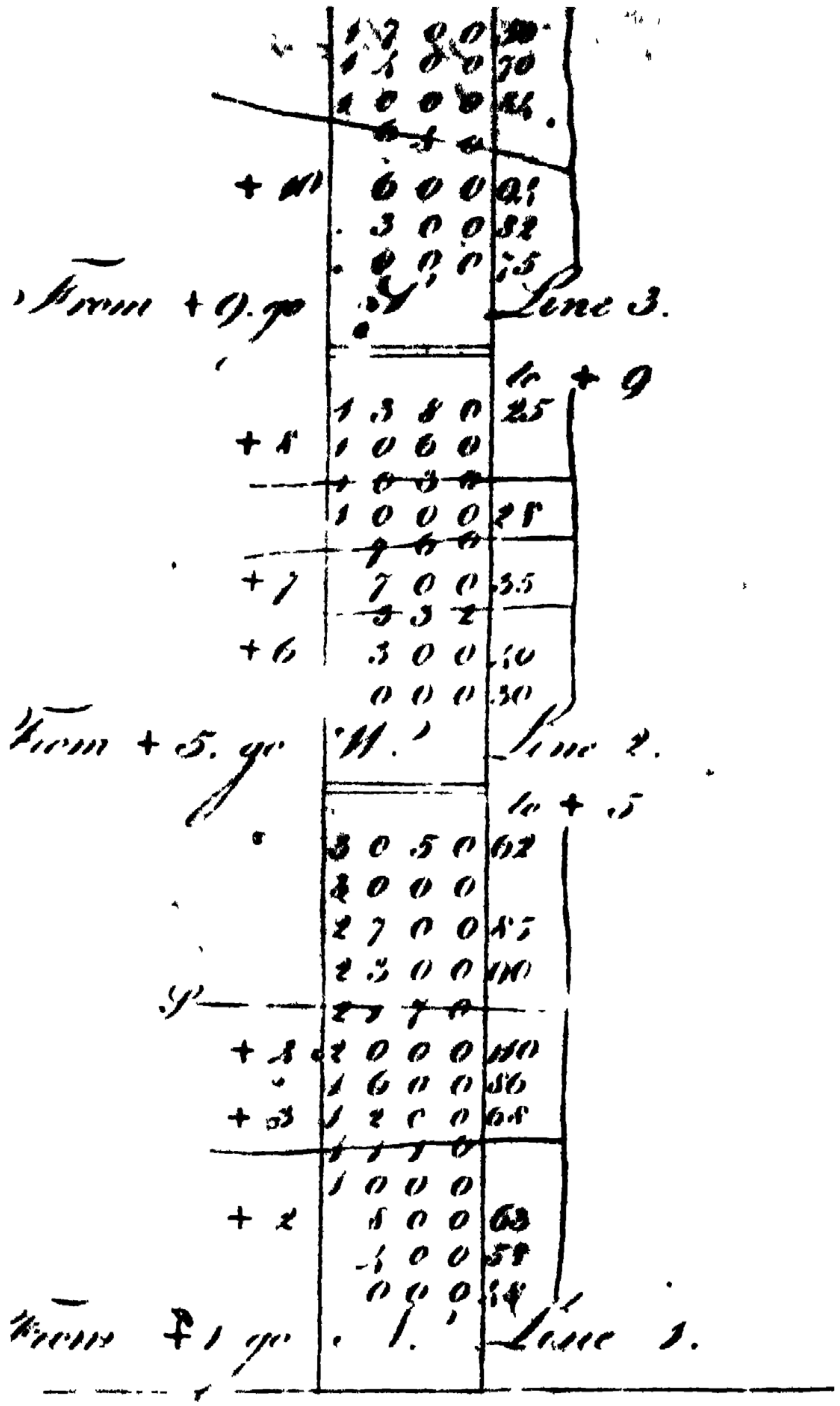
Two Leads 1st off + 11 Line 3.

1007
94 1470
1260 +16
60 1200
1020
1000
925 21
700 26
S 480 14
440 8
445
300 +15
36 300
65 100
From + 3. go M. Line 6

10+2
S 1075
605
From + 14. go VC Line 5 Ding

10 +1
1740 38
S 1450 30
1200 30
1000
+14 900 35
645
+13 620 32
300 30
000 34
From + 12 go E Line 4.

10 + 12.
3040 22
3000
2700 26
2300 36
2250
+ 11 2150
2000 10



1380 20

1180 70

1000 26

040

+10 600 25

300 32

000 75

From +9 go S. Line 3.

to +9

1380 25

+8 1060

1030

1000 28

760

+7 700 35

332

+6 300 30

000 30

From +5 go W. Line 2.

to +5

3050 62

2000

2700 85

2300 110

2270

+4 2000 110

1600 86

+3 1200 68

1110

1000

+2 800 63

400 54

000 34

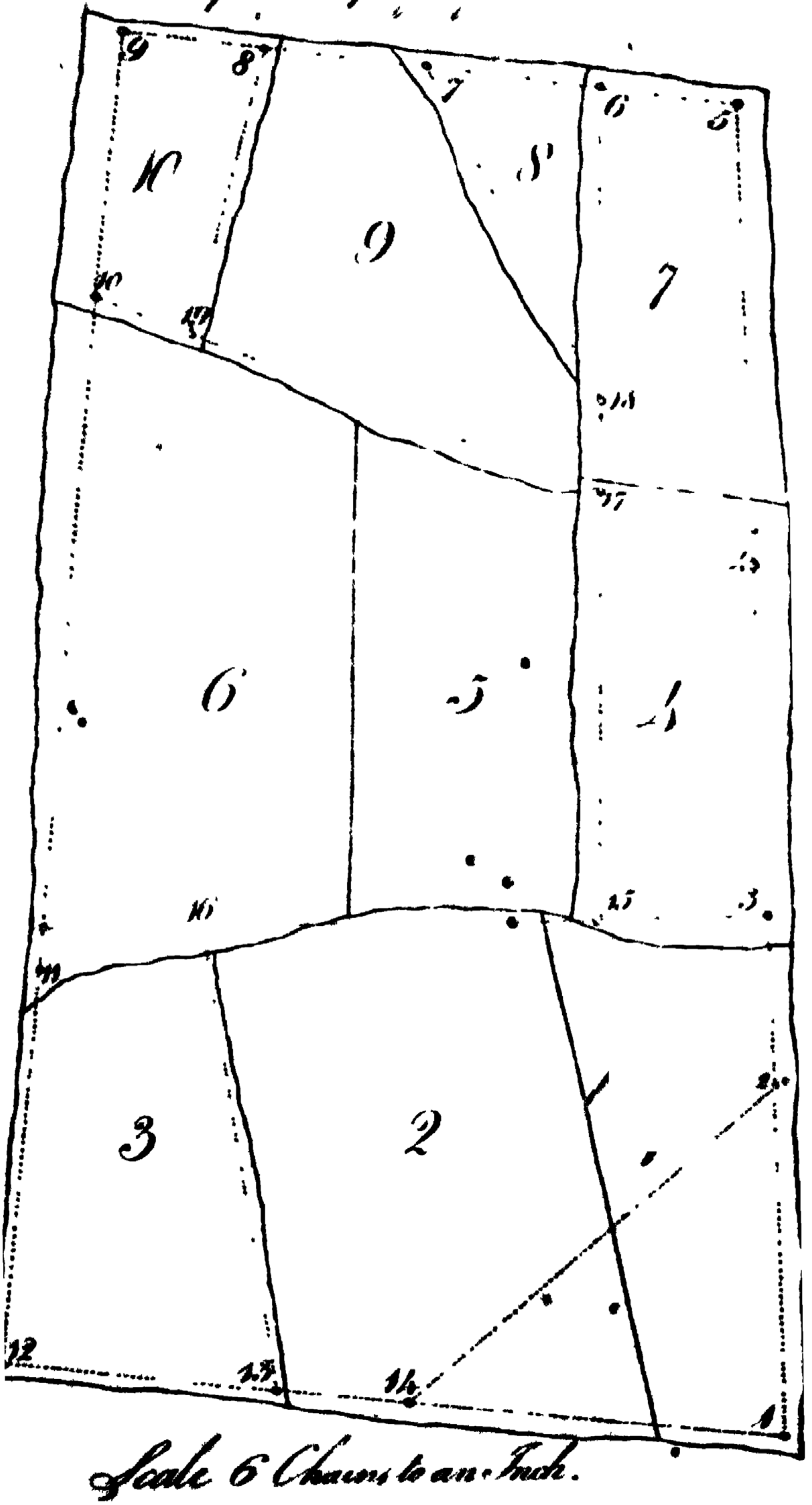
From +1 go N. Line 1.

pril 10th 1825 The Notes of an Estate
ituate in the Township of Farnley
in the Parish of Leeds.

FIELD BOOK.

Plate

Plan from the foregoing Field Book.



Scale 6 Chains to an Inch.

SECTION II.

MISCELLANEOUS QUESTIONS

CONCERNING

Superficial Mensuration.

1. The length of a floor is 35 feet, and its breadth 24 feet; what is the area of three such floors, deducting for two well-holes, one of which measures 10 feet 6 inches by 4 feet 3 inches, and the other 8 feet 9 inches by 3 feet 6 inches? *Ans.* 2444¾ *feet.*

2. From a mahogany plank 16 inches in breadth, 6 square feet are to be sawn off; at what distance from the end must the line be struck? *Ans.* 4½ *feet.*

3. What must be the length of a cord which will strike the circumference of a circular plantation that shall contain just an acre and a half of ground?

Ans. 48.072 *yards.*

4. A gentleman has ordered a rectangular court-yard to be paved, which measures 45 feet 9 inches in front, and 32 feet 6 inches broad. A foot-path 5 feet 6 inches in breadth, leading to the front door of his house, is to be laid with Portland-stone, at 3*s.* 4*d.* per square foot, and the rest with Purbeck-stone, at 2*s.* 3*d.* per square foot; what will be the expense of the whole?

Ans. £176. 19*s.* 1¼*d.*

5. The annual rent of a triangular field is £43. 15*s.*; its base measures 25, and perpendicular 14 chains; what is it let for per acre? *Ans.* £2. 10*s.*

6. The transverse diameter of the ellipse in Grosvenor-square measures 840 links, and the conjugate 612, within the wall; the wall is 14 inches thick; what

quantity of ground does it enclose, and how much does it occupy? *Ans. The wall encloses* 4*a.* 0*r.* 6*p. and occupies* 1760.531 *square feet.*

7. The area of a right-angled triangle is 60, and the hypothenuse 17; required the two legs. *Ans.* 15 *and* 8.

8. Two sides of an obtuse-angled triangle are 5 and 10 chains; what must be the length of the third side, that the triangle may contain just 2 acres of ground? *Ans.* 8.06225 *or* 13.60147 *chains.*

9. Two travellers, A and B, departed from an inn, at the hour of eight in the morning; A proceeded north-west, at the rate of 6 miles an hour, and B north-east, at the rate of 8 miles an hour; how far were they distant from each other, at twelve o'clock of the same day? *Ans.* 40 *miles.*

10. Two boys amusing themselves at a game called snatch-apple, in a room 13 feet high, find that by standing 12 feet from each other, the apple, which is suspended from the ceiling by a string, and in a right line between them, when put in motion, just touches each of their mouths. Required the area of the sector described by the string and apple; the perpendicular height of each boy's mouth, from the ground, being 5 feet. *Ans.* 64.35726 *feet.*

11. What is the area of an isosceles triangle inscribed in a circle whose diameter is 24; the angle included by the equal sides of the triangle being 30 degrees? *Ans.* 134.3538.

12. A maltster has a kiln that is 18 feet square; but he intends to pull it down, and build a new one, that may dry three times as much at once as the old one; what must be its length, if its breadth be 24 feet? *Ans.* 40½ *feet.*

13. The side AB of a triangular field is 40, BC 30, and CA 25 chains; required the sides of a triangle parted off by a division-fence made parallel to AB, and proceeding from a point in CA, at the distance of 9 chains from the angle A. *Ans.* 16, 19.2, *and* 25.6 *chains.*

14. A field in the form of a right-angled triangle is to be divided between two persons, by a fence made from the right angle meeting the hypothenuse perpendicularly, at the distance of 880 links from one end; required the area of each person's share, the length of the division-fence being 660 links.

Ans. 2*a.* 3*r.* 24½*p. and* 1*a.* 2*r.* 2[illegible]*p.*

15. It is required to part from a triangular field whose three sides measure 1200, 1000, and 800 links respectively, 1 acre, 2 roods, and 16 perches, by a line parallel to the longest side.

Ans. The sides of the remaining triangle are 927, 772½, *and* 618 *links respectively.*

16. The perambulator, or surveying wheel, is so contrived, as to turn just twice in the length of a pole, or 16½ feet; what is its diameter? *Ans.* 2.626 *feet.*

17. Required the side of an equilateral triangle whose area is just two acres. *Ans.* 6.79617 *chains.*

18. The sides of a triangle are 20, 30, and 40 respectively; what is the area of its inscribed circle?

Ans. 130.8999.

19. In an isosceles triangle, two circles are inscribed touching each other, and the sides of the triangle; the diameters of the circles are 9 and 25; required the sides of the triangle.

Ans. 44.27083, 44.27083, *and* 41.66666.

20. The base of a field, in the form of a trapezoid, is 30, and the two perpendiculars are 28 and 16 chains respectively; it is required to divide it equally between two persons, by a fence parallel to the perpendiculars.

Ans. The division-fence is 22.8035 *chains, and it divides the base into two parts, whose lengths are* 17.0087, *and* 12.9913 *chains respectively.*

21. A field in the form of an equilateral triangle, contains just half an acre; what must be the length of a tether, fixed at one of its angles, and to a horse's nose, to enable him to graze exactly half of it?

Ans. 48.072 *yards.*

22. The diameter of a circular estate is 25 chains;

what is the length of the chord which divides it into two segments whose areas are to each other as 2 to 1?

Ans. 24.1062 *chains.*

23. In turning a one-horse chaise within a ring of a certain diameter, it was observed that the outer wheel made three turns while the inner wheel made two. The wheels were of equal height; and supposing them fixed at the statutable distance of 5 feet asunder on the axle-tree; what was the circumference of the track described by each wheel?

Ans. The length of the track described by the outer wheel is 94.248 *feet, and that described by the inner wheel* 62.832 *feet.*

24. If the frustrum of a cone whose diameters are 8 and 12 feet respectively, be made to revolve with its slant side upon a horizontal plane, until it returns to its first position; what will be the area of the space passed over by the slant side, the length of which is 10 feet? *Ans.* 1570.8 *feet.*

25. Being desirous of finding the height of a steeple, I placed a looking-glass at the distance of 100 feet from its base, on the horizontal plane, and walking backward 5 feet, I saw the top of the steeple appear in the centre of the glass; required the steeple's height, my eye being 5 feet 6 inches from the ground.

Ans. 110 *feet.*

26. Three men bought a grinding-stone of 50 inches diameter, each paying $\frac{1}{3}$ part of the expense; what part of the diameter must each person grind down for his share? *Ans. The first must grind down* 9.1752, *the second* 11.9573, *and the third* 28.8675 *inches.*

27. Wanting to know the height of the cathedral at York, I measured the length of its shadow, and found it to be 200 feet. At the same time, a staff 5 feet long, cast a shadow of 4 feet; required hence the height of that elegant and magnificent structure. *Ans.* 250 *feet.*

28. At Matlock, near the Peak, in Derbyshire, where there are many surprising curiosities of nature, is a rock by the side of the river Derwent, rising perpen-

dicularly to a wonderful height, which, being inaccessible, I endeavoured to measure in the following manner: At the distance of 340 feet from the bottom of the rock, I fixed a staff, 8 feet in length, perpendicularly to the horizontal plane; and at a convenient distance from this, I fixed another, 3 feet long, so that by looking over both their tops. I could just see the summit of the rock. The distance between the staves I found to be 4 feet 6 inches. From these data I determined the perpendicular altitude of the rock; and you are requested to repeat the process.

Ans. The height of the rock is 128.5925 *yards.*

29. There are two columns in the ruins of Persepolis, left standing upright; one of which is 64 feet above the horizontal plane, and the other 50. Between these, in a right line, stands a small statue, the head of which is 97 feet from the summit of the higher, and 86 feet from the top of the lower column, whose base is just 76 feet from the centre of the figure's base; required hence the distance between the tops of the two columns. *Ans.* 157.03687 *feet.*

30. A gentleman a garden had,
Five score feet long and four score broad;
A walk of equal width half round
He made, that took up half the ground:
Ye skilful in geometry,
Tell us how wide the walk must be.

Ans. 25.96876 *feet.*

Note 1. Persepolis, mentioned in the 29th question, was the ancient capital of the Persian empire. This immense and renowned city was taken by Alexander the Great, about 300 years before the Christian æra, who at the instigation of the depraved courtezan Thais, ordered it to be set on fire. Its magnificent ruins are about 50 miles N. E. of Schiras, the present capital of Persia.

2. The 30th question was first proposed in the Ladies' Diary for the year 1709. The poetry is not very elegant; but the question is more original in its present form than it would be in prose.

PART IV.

MENSURATION OF SOLIDS,

THE

DESCRIPTION AND USE OF THE CARPENTER'S RULE,

TIMBER MEASURE,

And Miscellaneous Questions concerning Solids.

SECTION I.

MENSURATION OF SOLIDS

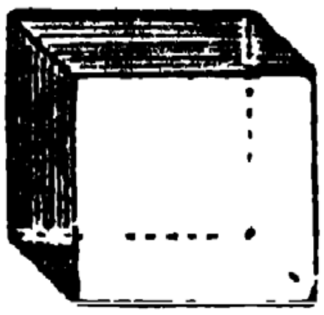
DEFINITIONS.

1. A SOLID is a figure which generally consists of three dimensions; viz. length, breadth, and thickness

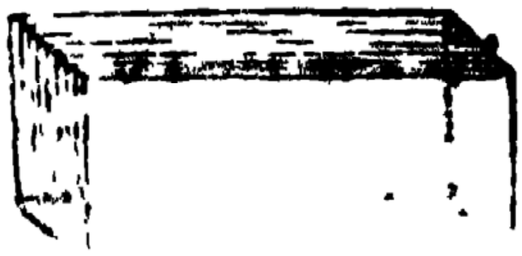
2 The *measurement* of a solid is called its solidity, capacity, or content.

3. The *contents* of solids are estimated by a cube, whose side is one inch, one foot, one yard, &c.; hence the solidity of a body is said to be so many cubic inches, feet, yards, &c. as are contained in that body.

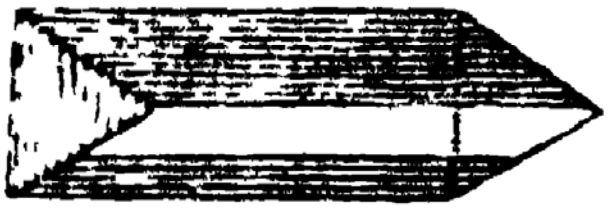
4. A *cube* is a solid having six equal square sides.



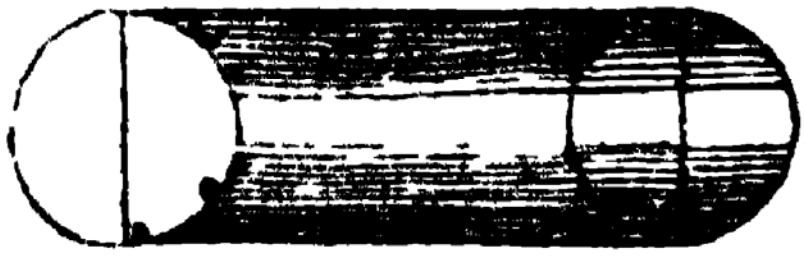
5. A *parallelopipedon* is a solid having six rectangular sides, every opposite two of which are equal and parallel.



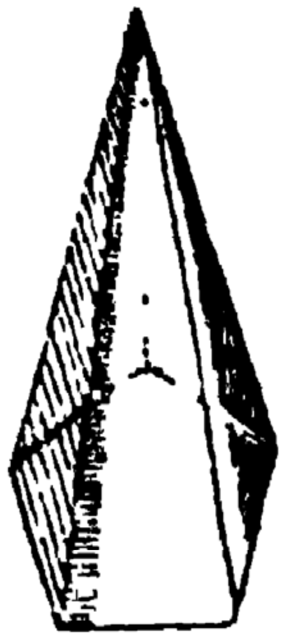
6. A *prism* is a solid whose ends are two equal, parallel, and similar plane figures, and its sides rectangles. It is called a *triangular prism* when its ends are triangles; a *square prism*, when its ends are squares, a *pentagonal prism*, when its ends are pentagons, &c.



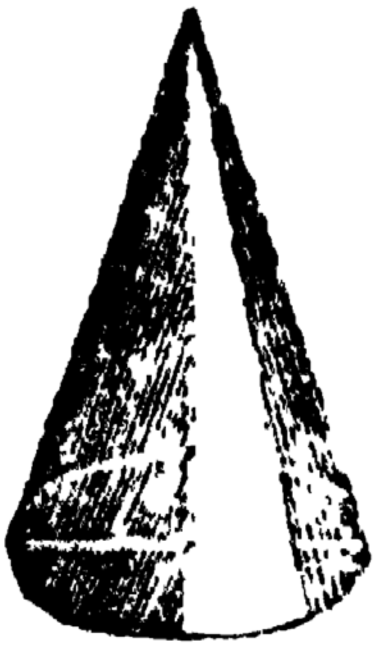
7. A *cylinder* is a solid conceived to be described by the revolution of a right-angled parallelogram about one of its sides, which remains fixed, and is called the *axis* of the cylinder, or it is a solid whose ends are parallel circles, and its sides right lines.



8. A *pyramid* is a solid the base of which is any plane figure whatever, and its sides are triangles, meeting in a point, called the *vertex* of the pyramid.



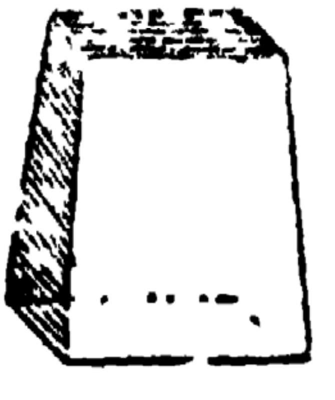
9. A *cone* is a solid conceived to be described by the revolution of a right-angled triangle about one of its legs, which remains fixed, and is called the *axis* of the cone; or it is a pyramid of an infinite number of sides, having a circle for its base.



10. The *frustrum* of a pyramid or cone is that part which remains, when the top is cut off by a plane parallel to the base. The part cut off the top is called a *segment*.

11. A *wedge* is a solid whose base is a rectangle, its two ends plane triangles, and its two opposite sides terminate in an edge.

12. A *prismoid* is a solid whose bases or ends are two right-angled parallelograms, being parallel but not similar to each other, and its sides four plane trapezoids

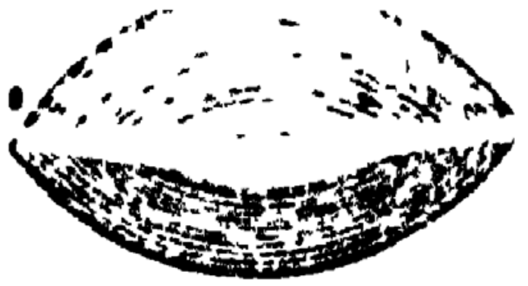


13. A *sphere* or *globe* is a solid conceived to be formed by the rotation of a semi-circle about its diameter, which remains fixed, and is called the *axis* or diameter of the sphere; or it is a solid bounded by one continued convex surface, every part of which is equally distant from a point within, called the centre.

14. The *segment* of a sphere is any part of it cut off by a plane. If the plane pass through the centre, it will divide the sphere into two equal parts called *hemispheres*.

15. The *zone of a sphere* is a part intercepted between two parallel planes; and if these planes be equally distant from the centre, it is called the *middle zone of the sphere*.

16. A *circular spindle* is a solid conceived to be formed by the revolution of a circular segment about its chord, which remains fixed.



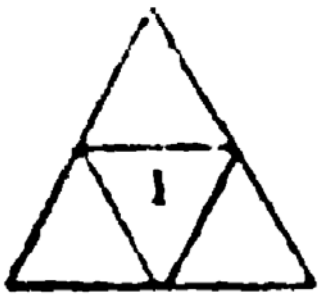
17. A *cylindrical ring* is a round solid, or a cylinder bent into a ring.

18. A *regular body* is a solid contained under a certain number of similar and equal plane figures.

19. The faces of the solid are the plane figures under which it is contained; and the linear sides or edges, are the sides of the plane faces.

20. The greatest number of regular bodies which can be formed is five; viz. 1st, the *tetraedron*, or regular pyramid, which has four triangular faces; 2nd, the *hexaedron*, or cube, which has six square faces; 3rd, the *octaedron*, which has eight triangular faces; 4th, the *dodecaedron*, which has twelve pentagonal faces; and 5th, the *icosaedron*, which has twenty triangular faces.

Note.—If the following figures be made of pasteboard, and the lines cut half through, so that the parts may be turned up and glued together, they will represent the five regular bodies, namely, figure 1 the tetraedron, figure 2 the hexaedron, figure 3 the octaedron, figure 4 the dodecaedron, and figure 5 the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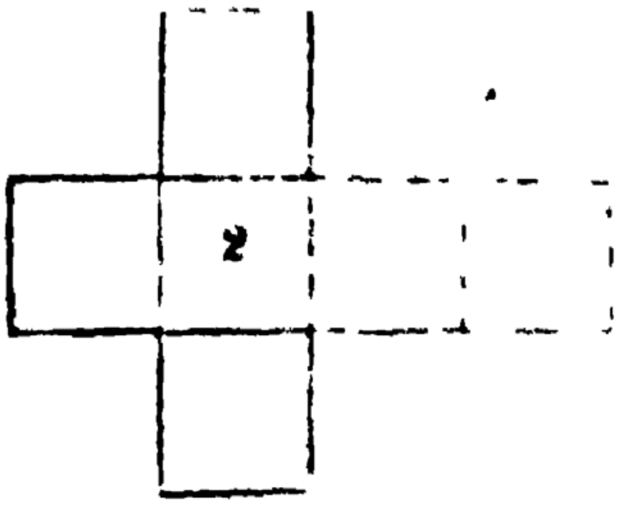
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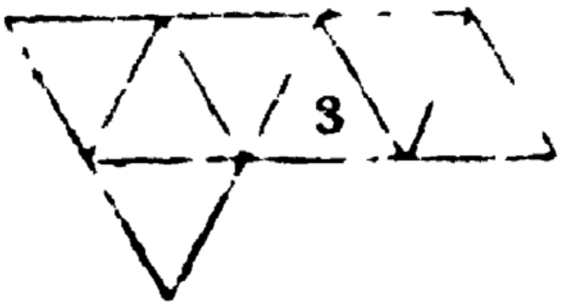
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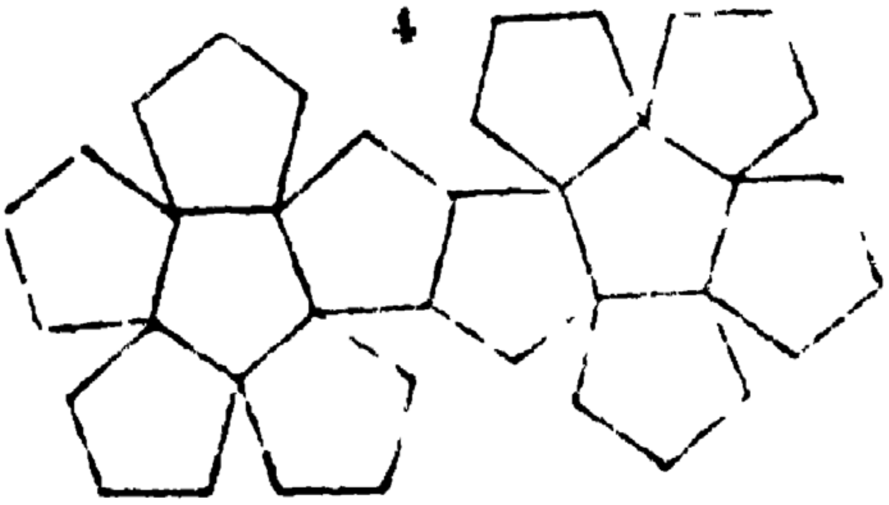
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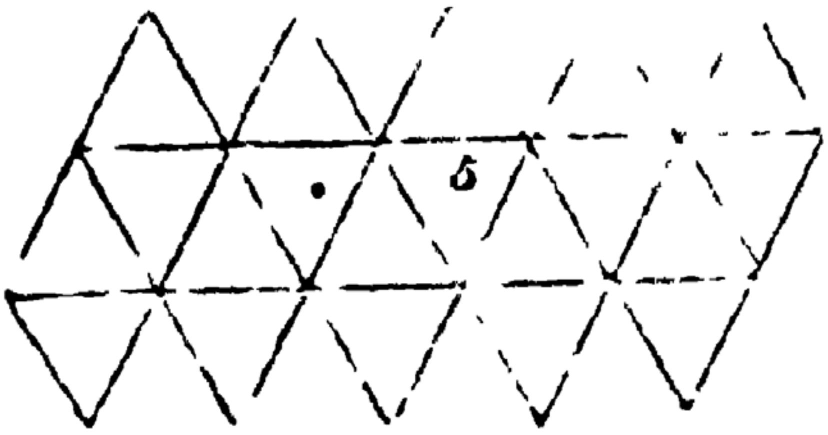

5

A Table of Solid Measure.

Cubic Inches.	*Cubic Foot.*					
1728	= 1	*Cubic Yard.*				
46656	27	= 1	*C. Pole.*			
7762392	4492⅛	166⅜	= 1	*C. Fur.*		
496793088000	287496000	10648000	64000	= 1	*C. Mile.*	
254358061056000	147197952000	5451776000	32768000	512	= 1	

. PROBLEM I.

To find the solidity of a cube.

RULE.

Multiply the side of the cube by itself, and that product again by the side; and the last product will be the solidity required.

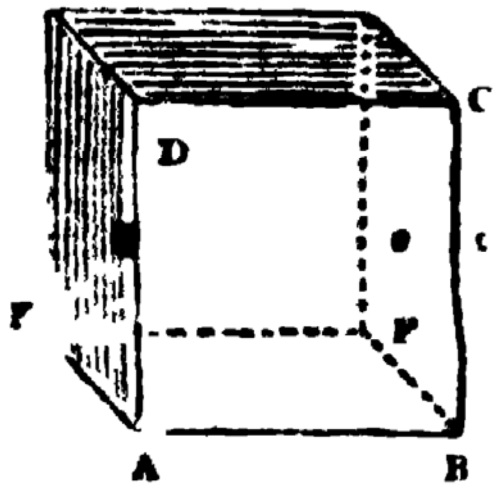
Note 1. The surface of a cube may be found by multiplying the square of its side by six

2 The side of a cube may be found by extracting the cube root of its solidity

3 The best method of giving boys a competent knowledge of the different Problems in *Mensuration of Solids*, is to have the figures formed of wood This will be found very advantageous, as the learner may be taught both the *theory* and the *practice* at the same time

EXAMPLES.

1. The side AB or BC of the cube ABCDEFGH is 5 feet 6 inches, what is its solidity ?



By Decimals.	*By Duodecimals.*			
Feet	*Feet*	*In*		
5.5	5	6		
5.5	5	6		
275	27	6		
275	2	9		
30.25	30	3		
5.5	5	6		
15125	151	3	*pa.*	
15125	15	1	6	
166.375	166	4	6	*Ans.*
12				
4.500				
12				
6.000				

Ans. 166 *ft.* 4 *in.* 6 *pa.*

By Logarithms.

The log of .... 5.5 *is* 0.7403627

Multiply by the index of the power 3

The product is the log. of 166.375, *the ans.* ... 2.2210881

2. What is the solidity of a cube, whose side measures 3 feet 6 inches?

Here 3 feet 6 inches $= 3.5$; *and* $\overline{3.5}|^3 = 3.5 \times 3.5 \times 3.5 = 12.25 \times 3.5 = 42.875$ *feet* $=$ 42 *ft.* 10 *in.* 6 *pa.* (*or* 6'' *seconds*) *the solidity required.*

3. If the side of a cubical piece of timber measures 2 feet 9 inches; what is its solid content?

Ans. 20 *ft.* 9 *in.* $6\frac{3}{4}$ *pa.*

4. What is the solidity of a cubical stone, whose side measures 2 feet 10 inches? *Ans.* 22 *ft.* 8 *in.* 11'' 4'''.

5. What did the inside of a cubical bin, that holds exactly 5 Winchester bushels, cost painting, at $1\frac{1}{2}$*d.* per square foot? *Ans.* 2*s.* $6\frac{1}{4}$*d.*

PROBLEM II.

To find the solidity of a parallelopipedon.

RULE.

Multiply the length by the breadth, and that product by the depth or altitude, and it will give the solidity required.

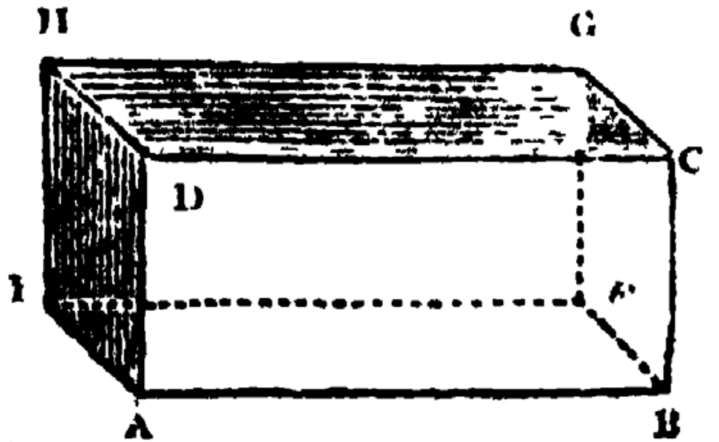
Note 1. In order to [illegible] the surface of a parallelopipedon, multiply the periphery of its end [illegible] length, and the product will be the area of the sides, to which add twice the area of its end, and the sum will be the whole surface.

2. The surfaces of all similar solids are to each other as the squares of their like linear dimensions. This is evident from Theorem 16, Part I.

3. If the content of a parallelopipedon be divided by the product of any two of its dimensions, the quotient will be the other dimension.

EXAMPLES.

1. What i[illegible]he solidity of the parallelopipedon ABCDEFGH, wh[illegible]e length AB is 12 feet 9 inches, breadth BC 8 feet 6 inches, and depth or altitude AE 6 feet 3 inches?



By Decimals.

Feet.
12.75
8.5
6375
10200
108.375
6.25
541875
216750
650250
677.34375
12
4.12500
12
1.50000
12
6.00000 *Ans.* 677 *ft.* 4 *in.* 1″ 6‴

By Duodecimals.

Feet.	In.			
12	9			
8	6			
102	0			
6	4	6″		
108	4	6		
6	3	0		
650	3	0		
27	1	1	6‴	
677	4	1	6	*Ans*

By Logarithms.

The log. of .... 12.75 *is* 1.1055102
——— — .... 8.5 *is* 0.9294189
——— — .... 6.25 *is* 0.7958800
Their sum is the log. of 677.34375, *the ans.* 2.8308091

2. The length of a piece of timber is 24.65 feet, the breadth 2.82 feet, and the depth 1.56 feet; what is its solidity? *Ans.* 108.44028 *feet.*

3. In a quarry near Balbec, is a stone whose length is 70 feet, its breadth 14 feet, and its depth 14 feet 5 inches; what is its solidity? *Ans.* 14128 *ft.* 4 *in.*

4. The length of a cellar is 25 feet, its breadth 15 feet, and its depth 7 feet; what did it cost digging, at 3¾*d.* per solid yard? *Ans.* £1. 10*s.* 4½*d.*

5. A brick-kiln measures 27 feet 6 inches in length, 18 feet 4 inches in breadth, and 10 feet 9 inches in height; how many bricks, each 10 inches long, 5 inches

broad, and 3 inches thick, will it contain at once, admitting them to be close piled? *Ans.* 62436.

6. A parallelopipedon measures 18 feet in length, 12 feet in breadth, and 8 feet 6 inches in depth; what did it cost painting, at 9½*d.* per square yard?

Ans.* £4. 2*s.* 10¼*d.

PROBLEM III.

To find the solidity of a prism.

RULE.

Multiply the area of the base by the perpendicular height of the prism, and the product will be the solidity.

Note 1. If the area of the base in inches, be multiplied by the height in inches, the product must be divided by 1728, in order to obtain the content in cubic feet, but when the area of the base in inches, is multiplied by the height in feet, the product divided by 144, will give the solidity in feet.

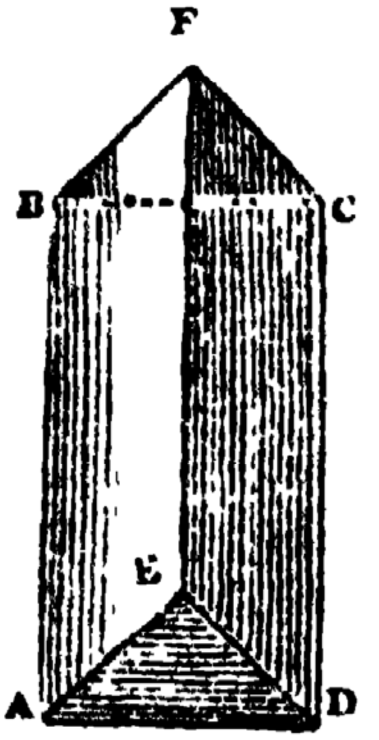
2. The surface of a prism may be found thus: Multiply the area of one side by the number of sides, to the product add twice the area of the base, and the sum will be the surface required. Or, if the circumference of a prism be multiplied by its length, the product will be the surface of all the sides, to which add twice the area of its base, and the sum will be the whole surface.

3. The altitude of a prism may be found by dividing the solidity by the area of the base.

4. The area of the base may be found by Problems 11 and 12, Part II.

EXAMPLES.

1. Required the solidity of the triangular prism ABCDEF, whose length AB is 14 feet 8 inches, and each of the equal sides of the base ADE, 6 feet 4 inches.



Here 14 *ft.* 8 *in.* = 176 *inches, and* 6 *ft.* 4 *in.* = 76 *inches; then by Prob.* 12, *Part II., we have* .4330127 ×

$76^2 = .4330127 \times 5776 = 2501.0813552$, *the area of the base* ADE.

Or, by Prob. 5, Part II., we have $\frac{76 + 76 + 76}{2} = \frac{228}{2} = 114$, *half the sum of the sides; and* $114 - 76 = 38 =$ *each remainder; whence* $\sqrt{114 \times 38 \times 38 \times 38} = \sqrt{6255408} = 2501.08136$, *the area of the base as before; then* $\frac{2501.08136 \times 176}{1728} = \frac{440190.31936}{1728} = 254.73976$ *feet* = 254 *ft.* 8 *in.* 10 *pa. the solidity required.*

2. The perpendicular altitude of a stone pillar is 8 feet 9 inches, and the side of its square base 2 feet 3 inches; what is its solidity?

Ans. 44 *ft.* 3 *in.* 6¾ *pa*

3. The length of a square beam of timber is 32 feet 6 inches, and its circumference or girt 5 feet; what is its solidity? *Ans.* 50 *ft* 9 *in.* 4½ *pa*

4. The side of a pentagonal stone measures 10 inches, and its length 9 feet 4 inches; what is its solidity? *Ans.* 11.15124 *feet.*

5. A hexagonal prism measures 2 feet 6 inches across the centre of its end, from corner to corner, and 25 feet 3 inches in length; what is its solidity?

Ans. 102.50225 *feet*

6. In the walls of Balbec is a stone in the form of a square prism, whose length measures 63 feet, and its breadth and depth each 12 feet; what is its solid content? *Ans.* 9072 *feet.*

7. The gallery of an assembly-room is supported by 16 octagonal prisms of wood, each of which measures 4 feet 8 inches in circumference, and 12 feet in height; required the solidity of the whole, and what they cost painting at 9*d.* per square yard?

Ans. The solidity of the 16 *prisms is* 315.4572368 *feet, and they cost* £3. 14*s.* 8*d. painting.*

Note. Balbec mentioned in the sixth question, is a town of Syria, situated about 37 miles north of Damascus. Its remains of antiquity display, according to the best judges, the boldest plan that ever was attempted.

The inhabitants of Asia consider Solomon as its founder, and the nobleness of the architecture, the beauty of the ornaments, and the stupendous execution of the whole, seem to fix its foundation to a period before the Christian æra, but, in all probability, the Jews knew little of the Grecian style of building and ornamenting in the time of Solomon

PROBLEM IV.

To find the solidity of a cylinder.

RULE.

Multiply the area of the base by the perpendicular altitude of the cylinder, and the product will be the solidity.

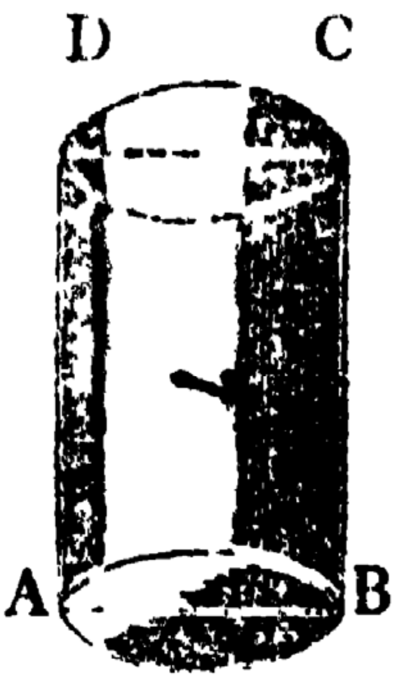
Note 1. If the circumference of a cylinder be multiplied by its height, the product will be the convex surface, to which add twice the area of the base, and the sum will be the whole surface of the cylinder.

2. The altitude of a cylinder may be found by dividing the solidity by the area of the base.

3. The area of the base of a cylinder may be found by Problem IX, Part II.

EXAMPLES.

1. Required the solidity of the cylinder ABCD, the diameter of the base AB of which is 2.76 feet, and the altitude BC 4.58 feet.



Here $2.76^2 \times .7854 = 2.76 \times 2.76 \times .7854 = 7.6176 \times .7854 = 5.98286304$, *the area of the base; and* $5.98286304 \times 4.58 = 27.4015127232$ *feet, the solidity required.*

2. The altitude of a cylindrical stone column is 8 feet 3 inches, and its circumference 5 feet 6 inches; what is its solidity? *Ans.* 19.86018 *feet.*

3. The circumference of a cylindrical piece of timber is 6 feet 8 inches, and its length 24 feet; what is its solid content? *Ans.* 84.88533 *feet*

4. The diameter of a rolling-stone is 18.7 inches, and its length 4 feet 9 inches; required its solidity. *Ans.* 9.05952 *feet.*

5. The diameter of a well is 3 feet 9 inches, and its depth 45 feet, what did it cost sinking at 7s. 3*d* per cubic yard? *Ans.* £6. 13*s* 5¼*d*

6 The greater diameter of a hollow iron roller is 1 foot 9 inches, the thickness of the metal 1¼ inch, and the length of the roller 5 feet; now supposing a cubic foot of cast iron to weigh 464 pounds avoirdupois, what did the roller cost at 19s 9*d* per hundred weight, and how many times will it turn round in rolling 5 acres of land?

Ans. The roller cost 13*l* 1s 6¾*d. and it will turn round* 7923.16926 *times in rolling* 5 *acres of land.*

PROBLEM V.

To find the solidity of a pyramid.

RULE.

Multiply the area of the base by ⅓ of the perpendicular height, and the product will be the solidity.

Note 1 The surface of a pyramid may be found thus Multiply the perimeter of the base by the length of the side, or slant height, and half the product will be the surface of the sides; to which add the area of the base, and the sum will be the whole surface

2. The perpendicular height of a pyramid or cone is a line drawn from the vertex to the middle or centre of the base, and the slant height is the distance between the middle of one side of the base and the vertex

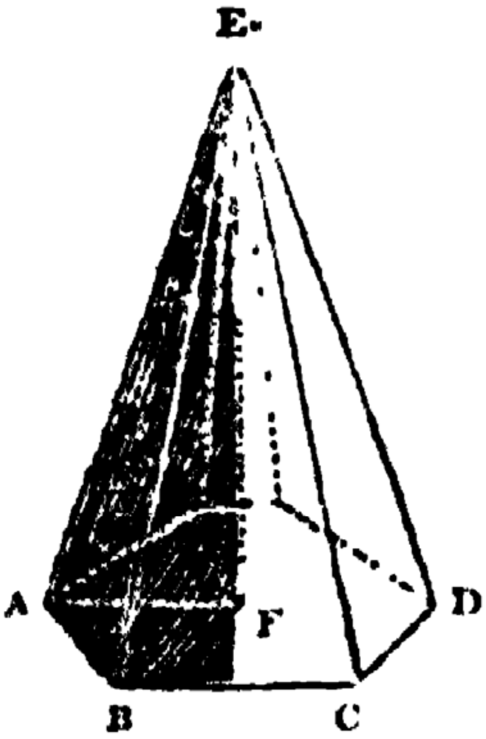
3. In order to find the slant height of a pyramid from the perpendicular height, or the perpendicular height from the slant height, it will be necessary to find the distance between the centre of the base and the middle of one of its sides, which may be done by the following Rule Multiply the number answering to the polygonal base, in the third column of the table of polygons Prob. 10, Part II, by the side of the base, and the product will be the distance required

4 If three times the solidity of a pyramid be divided by the area of the base, the quotient will be the perpendicular altitude

5. The area of the base of a pyramid may be found by Problems 11 and 12, Part II.

EXAMPLES.

1. Required the solidity of the hexagonal pyramid AB CDE, each side of its base being 3 feet 9 inches, and its perpendicular altitude EF 15 feet 9 inches.



By Prob. 12, *Part II., we have* $2.5980762 \times 3.75^2 = 2.5980762 \times 14.0625 = 36.5354465625$, *the area of the base*, *and* $36.5354465625 \times \frac{15.75}{3} = 36.5354465625 \times 5.25 = 191.811094453125$ *feet, the solidity required.*

2. The three sides of a triangular pyramid are 6, 7, and 8 feet, and its perpendicular altitude 18 feet; what is its solidity? *Ans.* 121.998975 *feet.*

3. The altitude of a square pyramid is 15 feet 9 inches, and the side of its base 2 feet 6 inches; required its solidity. *Ans.* 32 *ft.* 9 *in.* 9 *pa.*

4. The spire of a church is a regular hexagonal pyramid, whose perpendicular height measures 48 feet, the side of its base 10 feet, the perpendicular height of the cavity or hollow part 42 feet, and its side at the base 8 feet; how many cubic feet of stone are contained in the spire? *Ans.* 1829.0456448 *feet.*

5. The slant height of a regular, pentagonal, stone pyramid, measures 18 feet 6 inches, and the side of its base 3 feet, what was its whole expence; 6*s.* 9*d.* being paid for each cubic foot, and 6½*d.* per square foot, for polishing the sides? *Ans.* £35. 15*s.* 7¾*d.*

PROBLEM VI.

To find the solidity of the frustum of a pyramid.

GENERAL RULE.

To the areas of the two ends of the frustum add the square root of their product; and this sum being multiplied by ⅓ of the perpendicular height, will give the solidity.

Note 1.—This rule will also give the solidity of the frustum of a right cone, and likewise that of the frustum of an elliptical cone, generally called a cylindroid. (*See Prob. 4, Part VIII Gauging.*)

2. To find the perpendicular altitude of the frustum of a pyramid. To the areas of the two ends add the square root of their product, for a divisor; then, divide three times the solidity or content, by this divisor, and the quotient will be the altitude required.—This rule will hold good, whatever may be the form of the ends of the frustum.

RULE II.

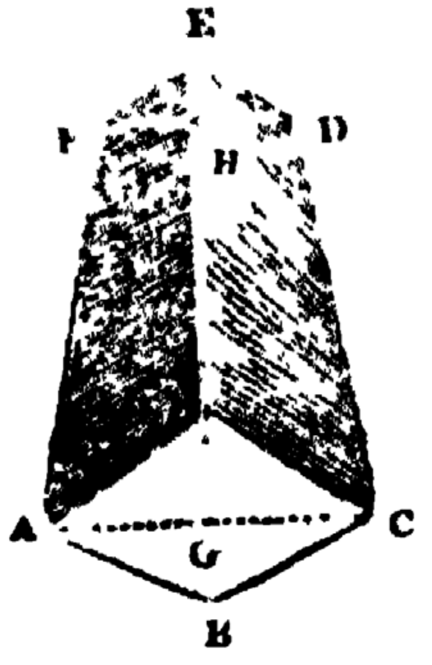
If the ends be regular polygons.

Add together the square of a side of each end, and the product of those sides; multiply the sum by $\frac{1}{3}$ of the height, and this product by the tabular area belonging to the polygon, (Prob. 12, Part II.) and the last product will be the solidity.

Note The surface of the frustum of a pyramid may be found thus: Multiply half the sum of the perimeters of the two ends, by the slant height and the product will be the surface of the sides, to which add the areas of the ends, and the sum will be the whole product of the frustum.

EXAMPLES.

1. What is the solidity of the frustum ABCDEF, of a square pyramid; the side AB of the greater end being 5 feet, the side DE of the less end 2 feet, and the perpendicular height GH 12 feet?



By the General Rule.

Here $5 \times 5 = 25$, *the area of the greater end; and* $2 \times 2 = 4$, *the area of the less; also,* $\sqrt{25 \times 4} = \sqrt{100} = 10$, *the square root of their product; then* $\overline{25 + 4 + 10} \times 4 = 39 \times 4 = 156$ *feet, the solidity required.*

By Rule II.

Here $5^2 + 2^2 + \overline{5 \times 2} = 25 + 4 + 10 = 39$; *then* $39 \times 4 \times 1 = 156$ *feet, the solidity as before.*

2. Each side of the greater end of a piece of squared timber is 28 inches, each side of the less end 14 inches, and its length 18 feet 9 inches; how many solid feet does it contain? *Ans.* 59.548611 *feet.*

3. The length of a piece of timber is 17 feet 3 inches, and its ends are similar rectangles; the length of the greater being 36, and its breadth 20 inches; the length of the less 18, and its breadth 10 inches, how many solid feet are contained in the frustum?

Ans. 50.3125 *feet.*

4. What is the solidity of an octagonal stone pillar; each side of the greater end being 12 inches, each side of the less end 8 inches, and the perpendicular altitude 9 feet? *Ans.* 30.58003 *feet.*

5. The roof of a portico is supported by four regular, hexagonal, marble pillars, whose perpendicular altitude is 15 feet, the side of the greater end 8 inches, and the side of the less end 5 inches; what did they cost at 4*l.* 10*s.* per solid foot? *Ans.* £209. 9*s.* 4$\frac{3}{4}$*d.*

6 The perpendicular altitude of a square chimney, in Leeds, is 120 feet 9 inches, the side of its base 10 feet 9 inches, and the side of its top 5 feet 9 inches. The cavity or hollow part is a square prism, whose side is 2 feet 6 inches; how many solid feet are contained in the chimney, and what is the area of its four sides?

Ans. The solidity of the chimney is 7715.421875 *cubic feet, and the area of the four sides is* 3985.603941 *square feet.*

PROBLEM VII.

To find the solidity of a cone.

RULE.

Multiply the area of the base by the perpendicular height, and $\frac{1}{3}$ of the product will be the solidity.

Note 1. The altitude of an oblique cone, or one whose axis does not make a right angle with the plane of a base, may be found by demitting a perpendicular from the vertex of the cone, upon the plane.

2. If the circumference of the base of a cone be multiplied by half its slant height, the product will be the convex surface; to which add the area of the base, and the sum will be the whole surface of the cone.

3. The solidity of every cone is equal to one-third of the solidity of its circumscribing cylinder.

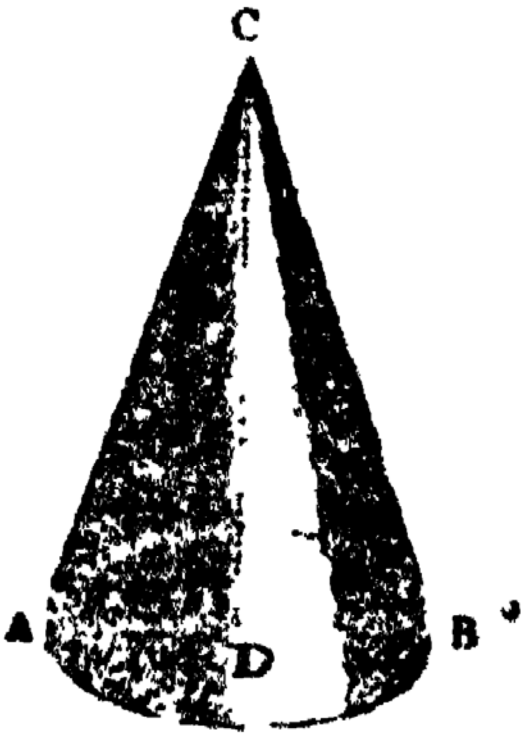
4. All prisms, cylinders, pyramids, and cones, of the same altitude, are to each other as their bases; consequently, prisms, cylinders, pyramids, and cones of equal bases, are to each other as their altitudes.

5. The perpendicular altitude of a cone may be found by dividing three times the solidity by the area of the base.

6. The area of the base of a cone may be found by Problem 15, Part II.

EXAMPLES.

1. Required the solidity of the cone ABC; the diameter AB of the base being 12 feet, and the perpendicular altitude DC 18 feet 6 inches.



Here $.7854 \times 12^2 = .7854 \times 144 = 113.0976$, *the area of the base; and* $\frac{113.0976 \times 18.5}{3} = \frac{2092.3056}{3}$ $= 697.4352$ *feet, the solidity required.*

2. The diameter of the base of a conical piece of timber is 5 feet 6 inches, and the length of its slant side 25 feet 9 inches; what is its solidity?

Ans. 202.75934 *feet.*

3. The circumference of the base of a conical stone is 12 feet 6 inches, and its perpendicular altitude 8 feet 9 inches; what is its solidity?

Ans. 36.266927 *feet.*

4. What will be the expence of tooling or dressing a conical spire, at 6½*d.* per square foot; the circumference of the base being 30, and the slant height 45 feet?

Ans. £18. 5*s.* 7½*d.*

PROBLEM VIII.

To find the solidity of the frustum of a cone.

RULE I.

To the product of the diameters of the two ends, add the sum of their squares; then this sum being multiplied by the height, and again by .2618, (one-third of .7854,) will give the solidity.

RULE II.

Divide the difference of the cubes of the diameters of the two ends, by the difference of the diameters; then this quotient being multiplied by .7854, and the product by $\frac{1}{3}$ of the height, will give the solidity.

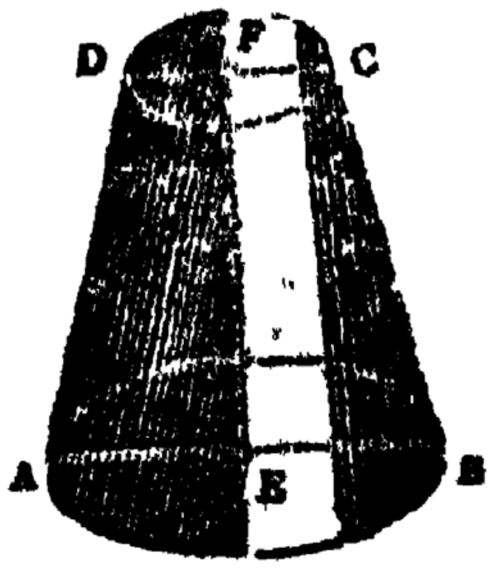
Note 1. The solidity of the frustum of a cone may also be found by the *general rule*, given in Problem 6

2. The surface of the frustum of a cone may be found thus: Multiply half the sum of the perimeters of the two ends, by the slant height, and the product will be the convex surface; to which add the areas of the ends, and the sum will be the whole surface of the frustum.

3. To find the perpendicular altitude of the frustum of a cone: To the product of the diameters of the two ends add the sum of their squares, and multiply this result by .2618 for a divisor, then, divide the solidity of the frustum by this divisor, and the quotient will be the perpendicular altitude.

EXAMPLES.

1. What is the solidity of the frustum ABCD of a cone; the diameter AB of the greater end being 7 feet, the diameter CD of the less end 5 feet, and the perpendicular altitude EF 8 feet 3 inches?



By Rule I.

Here $\overline{7\times5} + 7^2 + 5^2 = 35 + 49 + 25 = 109$; *and* $109 \times 8.25 \times .2618 = 899.25 \times .2618 = 235.42365$ *feet, the solidity required.*

By Rule II.

Here $\frac{7^3 - 5^3}{7-5} \times .7854 \times \frac{8.25}{3} = \frac{343 - 125}{2} \times .7854 \times 2.75 = \frac{218}{2} \times .7854 \times 2.75 = 109 \times .7854 \times 2.75 = 85.6086 \times 2.75 = 235.42365$ *feet, the solidity as before.*

2. The greater diameter of a piece of timber is 2 feet 6 inches, the less 1 foot 3 inches, and the length 25 feet 9 inches; what is its solidity?

Ans. 73.73351 *feet.*

3. The circumference of a piece of round timber is 65 inches at the greater end, 35 inches at the less end, and its length 33 feet 9 inches; what is its solid content? *Ans.* 48.027773 *feet.*

4. The pediment in the front of the court-house, in Leeds, is supported by four stone columns, whose perpendicular altitude is 22 feet 9 inches, the diameter at the bottom 2 feet 9 inches, and at the top 2 feet 3 inches; required their solidity?

Ans. 448.185287 *feet.*

5. The slant height of the *hovel* or chimney of a pot-kiln is 45 feet, the circumference at the bottom 113 feet 9 inches, and at the top 31 feet 8 inches; required its convex surface. *Ans.* 3262.5 *feet.*

6. The top diameter of a lime-kiln is 15 feet 10 inches, the bottom diameter 8 feet 8 inches, and the perpendicular depth 21 feet 4 inches; how many bushels of lime will it burn at once; allowing five pecks, or 2688 cubic inches to a bushel?

Ans. 1156.80698 *bushels.*

7. The shaft of Pompey's pillar is a single stone of granite whose perpendicular altitude is 90 feet, the diameter at the base 9 feet, and at the top 7 feet 6 inches; required its solidity *Ans.* 4824.8195 *feet.*

Note.—Pompey's Pillar is situated near [illegible]dria, in Egypt. It is a fine regular column, of the Corinthian order; and its whole height, including the capital and pedestal, is 114 feet. Upon it there has been a statue, which must have been of a gigantic stature, to have appeared of the ordinary size of a man, to a spectator at the bottom.

The remains of this statue, consisting of a foot and an ancle, were only discovered about 30 years ago, by some *jolly sons of Neptune*, that ascended the pillar, and drank a bowl of punch upon the top of it, amidst the shouts and acclamations of the natives, who were astonished at the *address* and *boldness* of the *British tars*.

This object they accomplished by means of a rope which they contrived to draw over the top of the pillar, by the assistance of a paper-kite. One of the company then ascended; and a kind of shroud being constructed, seven persons more went up; and the whole descended again, without the least accident, except the falling of a portion of the capital, which they carried off in triumph.

Learned men and travellers have made many fruitless attempts to discover in honour of whom this majestic monument was erected. The best informed have concluded that it could not be in honour of Pompey, since neither Strabo nor *Diodorus* Siculus has mentioned it. The Arabian historian Abulfeda in his description of Egypt, calls it "The Pillar of Severus;" hence Mr. Savary conjectures that it was erected to the honour of the emperor Severus by the inhabitants of Alexandria, to whom, as history informs us, he granted several favours.

Severus died at York, A. D. 211, and Pompey's Pillar did not receive that name until the fifteenth century.

PROBLEM IX.

To find the solidity of a cuneus or wedge.

RULE.

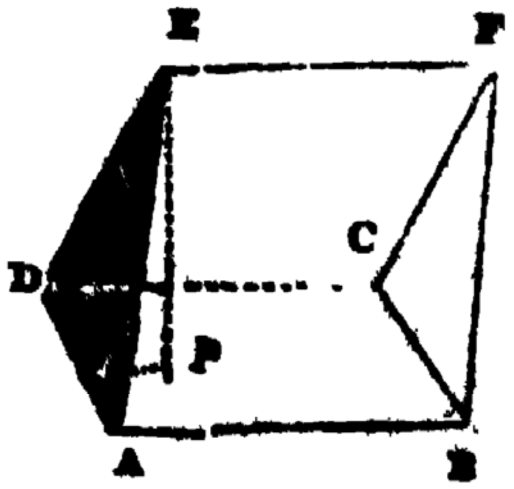
To twice the length of the base, add the length of the edge, multiply the sum by the breadth of the base, and this product by the perpendicular height of the wedge; and $\frac{1}{6}$ of the last product will be the solidity.

Note 1. The surface of a wedge may be obtained by finding the area of each of its sides or parts separately, and adding them into one sum.

2. When the length of the edge is equal to the length of the base, the solidity of the wedge is evidently equal to half the solidity of a prism of the same base and altitude; but if the edge be longer or shorter than the base, the wedge is manifestly greater, or less than half a prism.

EXAMPLES.

1. Required the solidity of the wedge ABCDEF, the length AB of its base being 26 inches, the breadth BC 18 inches, the length of the edge EF 15 inches, and the perpendicular height PE 28 inches.



Here $\overline{26\times2}+15=52+15=67$; *and* $\frac{67\times18\times28}{6}$ $=67\times3\times28=5628$ *cubic inches, the solidity required.*

2. Required the solidity of a wedge, the length and breadth of the base being 82 and 35 inches, the length of the edge 115 inches, and the perpendicular height 8 feet. *Ans.* 90.41666 *feet.*

3. What is the solidity of a wedge, the length and breadth of whose base is 5 feet 9 inches and 3 feet 6 inches, the length of the edge 4 feet 3 inches, and the perpendicular height 9 feet? *Ans.* 82.6875 *feet.*

PROBLEM X.

To find the solidity of a prismoid.

RULE.

To the sum of the areas of the two ends add four times the area of a section parallel to and equally distant from both ends; multiply this sum by the perpendicular height, and $\frac{1}{6}$ of the product will be the solidity.

Note 1. The length of the middle section is equal to half the sum of the lengths of the two ends, and its breadth is equal to half the sum of their breadths.

2 The surface of a prismoid may be obtained by adding the areas of its four sides and two ends into one sum

3. As the ends of a prismoid are rectangles, their areas may be found by Problem 2, Part II.

4 To find the perpendicular height of a prismoid. To the sum of the areas of the two ends add four times the area of the middle section, for a divisor; then, divide six times the content by this divisor, and the quotient will be the perpendicular altitude This Rule will hold good for cylindroids, whatever may be the form of their ends; and also for all frustums and solids whose parallel sections are similar figures

REMARK.

The last Rule, and the similar Rules given in the preceding Problems, will be found useful when the content or capacity of a solid, and all the horizontal dimensions are given, to find the perpendicular altitude. This is often the case in giving orders for the making of cisterns, couches, tuns, vats, tubs, and other vessels of various descriptions. *(See Gauging, Part VIII.)*

SCHOLIUM.

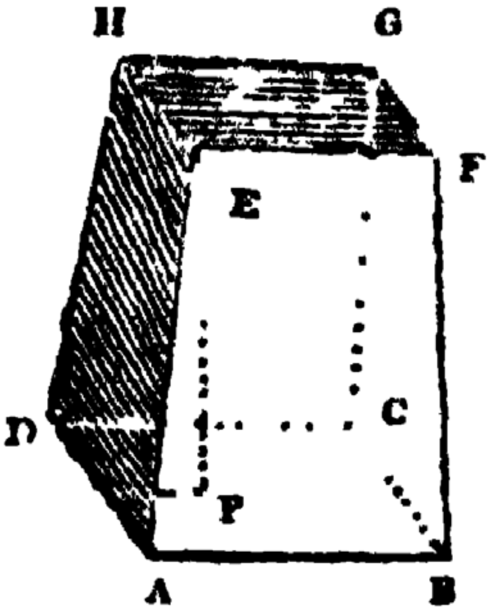
The Rule given in this Problem for a prismoid, is very clearly demonstrated in Simpson's Fluxions, page 178, first edition; and also in a similar manner at page 302, of Holliday's Fluxions.

There is likewise a very elegant demonstration given in Proposition III., Section I., Part IV., of Dr. Hutton's Mensuration, third edition, in which it is shewn, to be true for all frustums whatever, and for all solids whose parallel sections are similar figures; and Mr. Fletcher, in the second edition of his Universal

Measurer, Part III., page 54, and Mr. Moss, in his Gauging, page 175, third edition, say that it is *nearly* true for any other solid, whatever may be its form.

EXAMPLES.

1. What is the solidity of the prismoid ABCDEFGH, the length AB of the greater end being 18, and its breadth BC 8 inches; the length EF of the less end 12, and its breadth FG 6 inches; and the perpendicular height PE 18 inches?



Here $18 \times 8 + 12 \times 6 = 144 + 72 = 216$, *the sum of the areas of the two ends.*

Also, $\frac{18 + 12}{2} = \frac{30}{2} = 15$, *the length of the middle section; and* $\frac{8 + 6}{2} = \frac{14}{2} = 7$, *the breadth of the middle section; then* $15 \times 7 \times 4 = 105 \times 4 = 420$, *four times the area of the middle section; whence* $\overline{216 + 420} \times \frac{18}{6}$ $= 636 \times 3 = 1908$ *cubic inches, the solidity required.*

2. How many solid feet of timber are contained in a beam whose ends are rectangles; the length and breadth of the greater being 32 and 20 inches; the length and breadth of the less 16 and 8 inches; and the perpendicular length 25 feet? *Ans.* 61⅓ *feet.*

3. The perpendicular altitude of a stone pillar is 8 feet 6 inches; the length and breadth of the greater end are 26 and 16 inches; and the length and breadth of the less end 18 and 10 inches; required the solidity of the stone. *Ans.* 17.118055 *feet.*

4. The length and breadth of a fish-pond at the top, are 132 and 64 yards; the length and breadth at the bottom 116 and 48 yards; and the perpendicular depth 12 feet 9 inches; what did it cost digging, at 6¾*d.* per cubic yard? *Ans.* £832. 11*s.* 6*d.*

5. What is the capacity of a coal-waggon, the inside dimensions of which are as follow: at the top, the length is 80½, and breadth 56 inches; at the bottom, the length is 41½, and the breadth 32 inches; and the perpendicular depth 47 inches? *Ans.* 129814 *cubic inches.*

Note Coals are frequently carried from pits in prismoidal waggons, having four cast iron wheels, which run upon a rail-way.

They are prevented from leaving the rail-way by a particular construction of the wheels; the inner edge of the rim of each wheel being made to project Coals are conveyed, in waggons of the above description, from the pits in the vicinity of Newcastle, to the river Tyne; from those near Sunderland, to the river Wear, and from Middleton Colliery, to the town of Leeds

In the last question are given the dimensions of a Leeds coal waggon, which differs but little from those of other places

Instead of rails, the waggon-road between Middleton and Leeds, is made with bars of cast-iron, and the waggons are drawn by a machine which moves along the road before them by the force of steam. This machine was invented by Mr. Blenkinsop, of Middleton, and is called "Blenkinsop's Patent Steam Carriage" It is considered to be one of the most curious pieces of mechanism, that ever appeared in England, or perhaps in the world

It moves at the rate of about three miles and a half in an hour, and is capable of drawing 30 waggons at once, upon level ground, which are computed to weigh 105 tons

PROBLEM XI.

To find the solidity of a sphere or globe.

RULE.

Multiply the cube of the diameter by .5236, or the cube of the circumference by .016887, and the product will be the solidity.

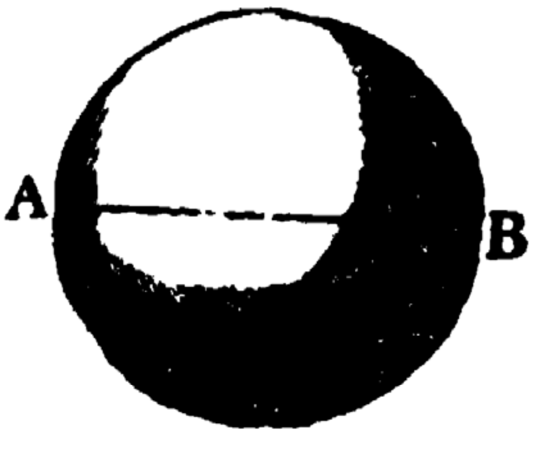
Note 1. The convex surface of a sphere may be found by multiplying the circumference by the diameter. Or, multiply the square of the diameter by 3.1416, and the product will be the convex surface.

2. The solidity of a sphere is equal to two-thirds of the solidity of its circumscribing cylinder.

3. The surface of a sphere is equal to 4 times the area of a circle of the same diameter as the sphere, or to the area of a circle whose diameter is double that of the sphere, or to the convex surface of the circumscribing cylinder

EXAMPLES.

1. What is the solidity of a globe whose diameter AB is 25 inches?



Here $25^3 \times .5236 = 25 \times 25 \times 25 \times .5236 = 625 \times 25 \times .5236 = 15625 \times .5236 = 8181.25$ *cubic inches, the solidity required.*

2. What is the solidity of a spherical stone whose circumference measures 5 feet 6 inches?

Ans. 2.80957 *feet.*

3. If the circumference of a cannon-ball be 18.6 inches; what is its solidity? *Ans* 108.66541 *inches.*

4. The diameter of the moon is 2180 miles; what is her solidity in cubic miles?

Ans. 5424617475.2 *miles.*

5. The diameter of the earth is 7957¾ miles; what is its solidity in cubic miles, and its convex surface in square miles?

Ans. The solidity is 263858149120 *cubic miles, and the convex surface is* 198944286 *square miles.*

.PROBLEM XII.

To find the solidity of a segment of a sphere.

RULE I.

To three times the square of the radius of the segment's base, add the square of its height; and this sum multiplied by the height, and the product again by .5236, will give the solidity.

RULE II.

From three times the diameter of the sphere subtract twice the height of the segment; multiply the remainder by the square of the height, and that product by .52361 for the solidity.

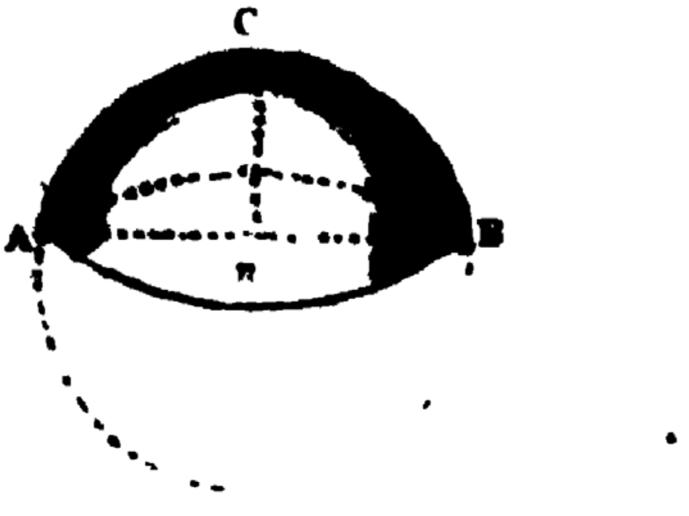
Note 1. The surface of the segment of a sphere may be found thus. Multiply the circumference of the whole sphere by the height of the segment, and the product will be the convex surface; to which add the area of the base, and the sum will be the whole surface.

2 The area of a spherical triangle, or the spherical surface included by the arcs of three great circles of the sphere, intersecting each other, may be found by the following rule:

As two right angles, or 180 degrees,
Are to the area of a great circle of the sphere,
So is the excess of the three angles above two right angles,
To the area of the triangle.

EXAMPLES.

1. The radius An of the base of the segment ABC is 9 inches, and the height Cn 7 inches; what is the solidity of the segment?



By Rule I., we have $9^2 \times 3 + 7^2 = 81 \times 3 + 49 = 243 + 49 = 292$; *and* $292 \times 7 \times .5236 = 2044 \times .5236 = 1070.2384$ *inches, the solidity required.*

2. If the diameter of a sphere be 3 feet 6 inches; what is the solidity of a segment whose height is 1 foot 3 inches? *Ans.* 6.545 *feet.*

3. What is the solidity of a spherical segment whose height is 24.8 inches, and the radius of its base 30.6 inches? *Ans.* 25.73099 *feet.*

4. Required the solidity and convex surface of each of the frigid zones, whose height is 330.0075 miles, and the diameter of the earth 7957¾ miles.

Ans. The solidity is 1323679753.4398 *cubic miles, and the convex surface is* 8250209.7425 *square miles.*

PROBLEM XIII.

To find the solidity of the frustum or zone of a sphere

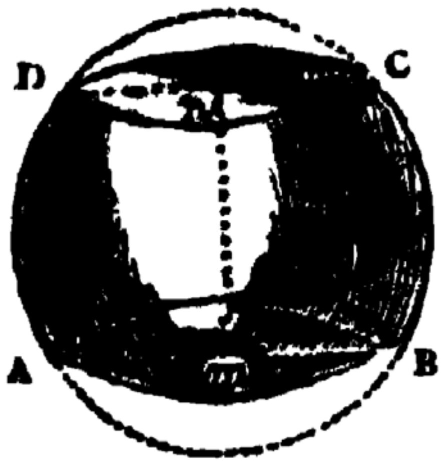
RULE.

To the sum of the squares of the radii of the two ends, add $\frac{1}{3}$ of the square of their distance, or the height of the zone; and this sum being multiplied by the said height, and the product again by 1.5708, will give the solidity.

Note The convex surface of a zone may be found by multiplying the circumference of the whole sphere by the height of the zone.

EXAMPLES.

1. Required the solidity of the zone ABCD; the greater diameter AB being 28, the less diameter CD 20, and the height *mn* 15 inches.



Here $14^2 + 10^2 + \frac{15^2}{3} = 196 + 100 + \frac{225}{3} = 296 + 75 = 371$; *and* $371 \times 15 \times 1.5708 = 5565 \times 1.5708 = 8741.502$ *inches, the solidity required.*

2. What is the solidity of a zone whose greater diameter is 9 feet 3 inches, less diameter 6 feet 9 inches, and height 5 feet 6 inches? *Ans.* 370 32425 *feet.*

3. Required the solidity and convex surface of the torrid zone, the top and bottom diameters of which are each 7297.735 miles, its height 3173.14565 miles, and the circumference of the earth 25000 miles?

Ans. The solidity is 149455316338.46698 *cubic miles, and the convex surface is* 79328641.25 *square miles.*

PROBLEM XIV.

To find the solidity of a circular spindle.

RULE.

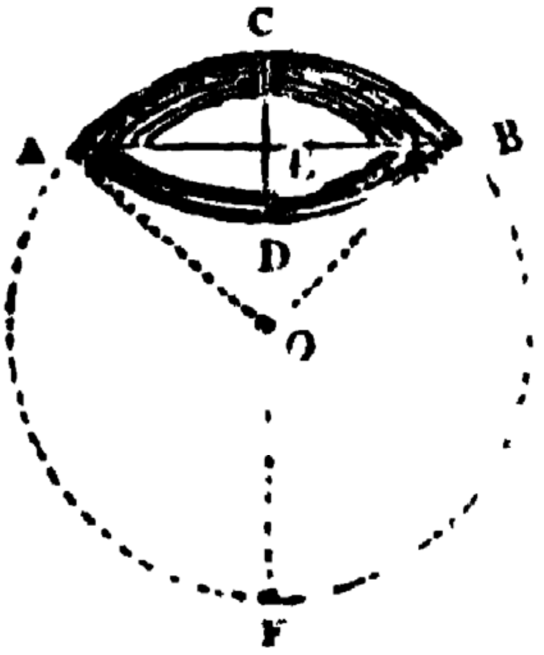
Find the area of the revolving segment ACBEA, which multiply by half the central distance OE. Subtract the product from ⅓ of the cube of AE, half the length of the spindle; and multiply the remainder by 12.5664, and the product will be the solidity.

Note 1. The surface of a circular spindle may be found by the following rule. Multiply the length AB of the spindle by the radius OC of the revolving arc ACB. Multiply also the length of the said arc by the central distance OE, or distance between the centre of the spindle and the centre of the revolving arc. Subtract the latter product from the former, and the remainder being multiplied by 6.2832, will give the surface required.

9. The convex surface of any segment or zone of a circular spindle, cut off perpendicularly to the chord of the revolving arc, may be found in the same manner, using the length of the solid, and the part of the arc which describes it, instead of the length of the whole spindle and whole arc.

EXAMPLES.

1. Required the solidity of the circular spindle AB CD, whose length AB is 30, and middle diameter CD 16 inches.



By Rule 2, *Prob.* 14, *Part II., we have* $\frac{15^2}{8} + 8 = \frac{225}{8} + 8 = 28.125 + 8 = 36.125$, *the diameter* CF ; *and* $8 \times \frac{2}{3} \div (36.125 - \frac{8 \times 41)}{50} = 5.333\dot{3} \div \overline{36.125 - 6.56} = 5.3333 \div 29.565 = .18039$; *then* $\overline{1 + .18039} \times 30 = 1.18039 \times 30 = 35.4117$, *the length of the revolving arc* ACB ; *and by Rule I. Prob.* 16, *we have* $35.4117 \times 18.0625 = 639.6238$, *half of which is* 319.8119, *the area of the sector* AOBC.

Again, $CO - CE = 18.0625 - 8 = 10.0625 = OE$, *and* $OE \times AE = 10.0625 \times 15 = 150.9375$, *the area of the triangle* AOB; *then* $319.8119 - 150.9375 = 168.8744$, *the area of the revolving segment* ACB.

Now, by the Rule, we have $(\frac{15^3}{3} - 168.8744 \times \frac{10.0625}{2}) \times 12.5664 = (\frac{3375}{3} - \overline{168.8744 \times 5.03125}) \times 12.5664$

= (1125 − 849.6493) × 12.5664 = 275.3507 × 12.5664 = 3460.16703648 *inches, the solidity required.*

2. What is the solidity and superficial content of a circular spindle, whose length is 96, and middle diameter 72 inches?

Ans. { *Solidity* 138.51593 *feet.*
{ *Superficies* 130.82761 *feet*

PROBLEM XV.

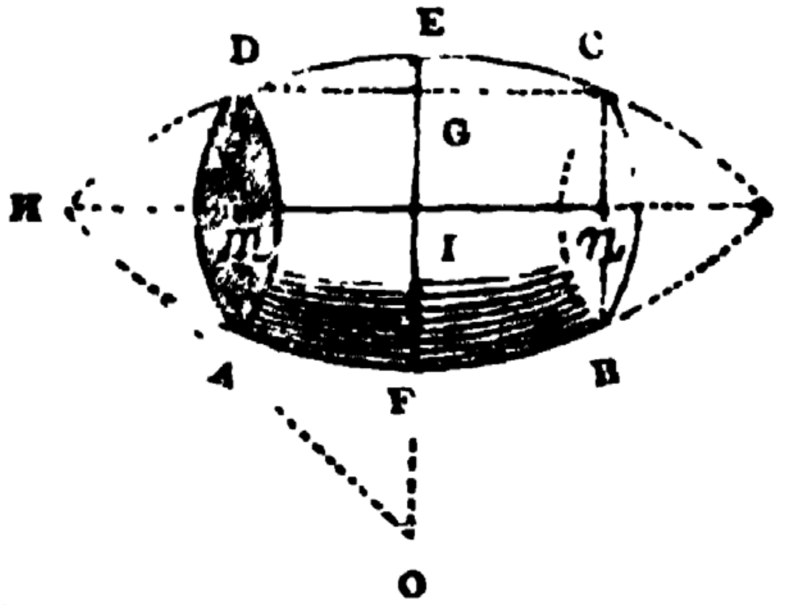
To find the solidity of the middle frustum or zone of a circular spindle.

RULE.

From the square of half the length of the whole spindle, take ⅓ of the square of half the length of the frustum; and multiply the remainder by the said half length of the frustum. Multiply the revolving area which generates the frustum, by the central distance, and subtract this product from the former, then the remainder being multiplied by 6.2832, will give the solidity required.

EXAMPLES.

1. What is the solidity of the frustum ABCD, whose length *mn* is 40, its greater diameter EF 32, and its least diameter AD or BC 24 inches?



Draw DG *parallel to mn, then we have* DG = ½ *mn* = 20. *Also,* ½ EF − ½ AD = 16 − 12 = 4 = EG, *and*

$DG^2 + EG^2 = 20^2 + 4^2 = 400 + 16 = 416$, *the square of the chord* DE; *then by Theo.* 12, *Part I.*, $DE^2 \div EG = \frac{416}{4} = 104$, *the diameter of the generating circle, half of which is* 52, *the radius* OE *or* OH; *consequently* $OE - EI = 52 - 16 = 36$, *the central distance* OI.

Again, $OH^2 - OI^2 = 52^2 - 36^2 = 2704 - 1296 = 1408$, *the square of* HI, *half the length of the whole spindle.*

By Rule 3, *Prob.* 17, *Part II., we have* $\frac{4}{104} = 03846$, *the tabular height, and the corresponding Area Seg. is* .00994; *then* $00994 \times 104^2 = .00994 \times 10816 = 107.51104$, *the area of the segment* DEC. *Also,* $mn \times Dm = 40 \times 12 = 480$, *the area of the rectangle* $mDCn$; *and* $107.51104 + 480 = 587\ 51104$, *the generating area* $mDCn$.

Now, by the Rule, we have $(1408 - \frac{20^2}{3}) \times 20 = (1408 - \frac{400}{3}) \times 20 = \overline{1408 - 133.33333} \times 20 = 1274\ 66667 \times 20 = 25493\ 33334$, *the first product, and* $587.51104 \times 36 = 21150.39744$, *the second product, which taken from the first, leaves* 4342 9359; *then* $4342.9359 \times 6.2832 = 27287.53484688$ *inches, the solidity required.*

2. What is the solidity of the middle frustum of a circular spindle, whose length is 50, greatest diameter 40, and least diameter 30 inches? *Ans.* 30.84257 *feet.*

PROBLEM XVI.

To find the solidity of a cylindrical ring.

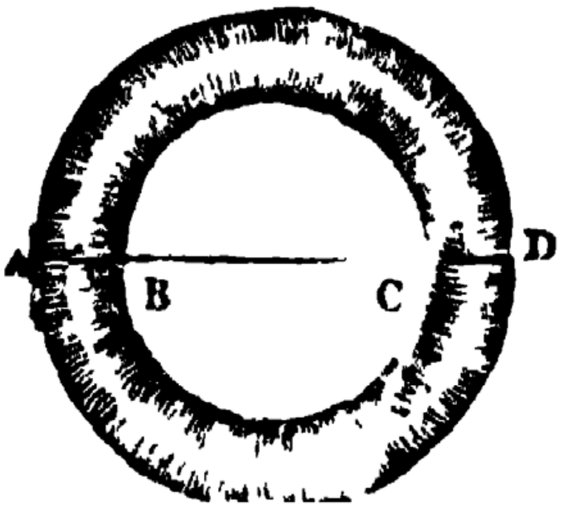
RULE.

To the thickness of the ring, add the inner diameter; and this sum being multiplied by the square of the thickness, and the product again by 2.4674, will give the solidity.

Note. The surface of a cylindrical ring may be found by the following rule To the thickness of the ring, and the inner diameter, and this sum being multiplied by the thickness, and the product again by 9.8696, will give the surface required.

EXAMPLES.

1. What is the solidity of a cylindrical ring, whose thickness AB or CD is 6, and the inner diameter BC 20 inches?



Here $\overline{20 + 6} \times 6^2 \times 2.4674 = 26 \times 36 \times 2.4674 = 936 \times 2.4674 = 2309.4864$ *inches, the solidity required.*

2. What is the solidity of an anchor-ring, the inner diameter of which is 24.6 inches, and its thickness 6.4 inches? *Ans.* 3133.005824 *inches.*

3. Required the solid and superficial contents of a cylindrical ring, whose thickness is 9, and inner diameter 32 inches.

Ans. $\begin{cases} \textit{Solidity } 8194.2354 \textit{ inches.} \\ \textit{Superficies } 3641.8824 \textit{ inches.} \end{cases}$

PROBLEM XVII.

To find the solidity or superficies of any regular body.

RULES.

1. Multiply the tabular solidity, in the following table, by the cube of the linear edge of the body, and the product will be the solidity.

2. Multiply the tabular area, by the square of the linear edge, and the product will be the superficies.

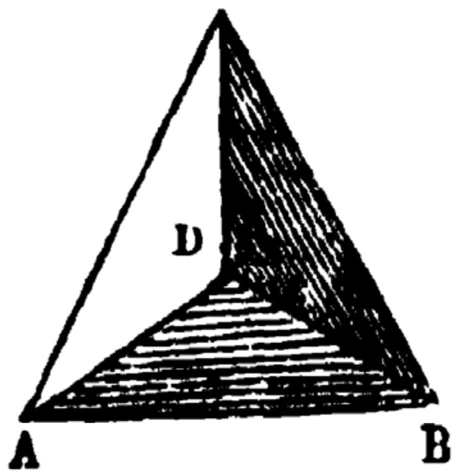
Solidities and Surfaces of Regular Bodies.

No. of Sides.	*Names.*	*Solidities.*	*Surfaces.*
4	Tetraedron	.1178511	1.7320508
6	Hexaedron	1.0000000	6.0000000
8	Octaedron	.4714045	3.4641016
12	Dodecaedron	7.6631189	20.6457288
20	Icosaedron	2.1816949	8.6602540

Note. The above table shews the solidity and superficies of the five Regular Bodies, when the linear edge of each is 1, or unity.

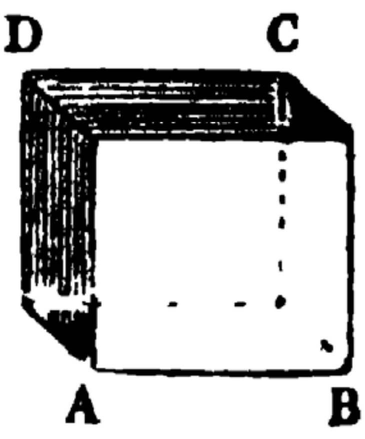
EXAMPLES.

1. What is the solidity and superficies of the tetraedron ABCD, whose linear edge is 8 inches?



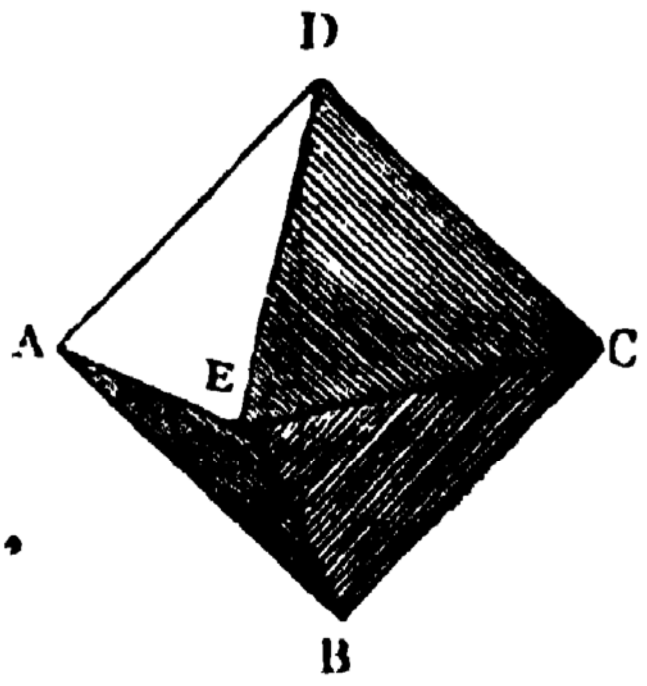
Here $.1178511 \times 8^3 = .1178511 \times 512 = 60.3397632$ *inches, the solidity; and* $1.7320508 \times 8^2 = 1.7320508 \times 64 = 110.8512512$ *inches, the superficies.*

2. What is the solidity of the hexaedron ABCD, whose side AB is 6 feet?



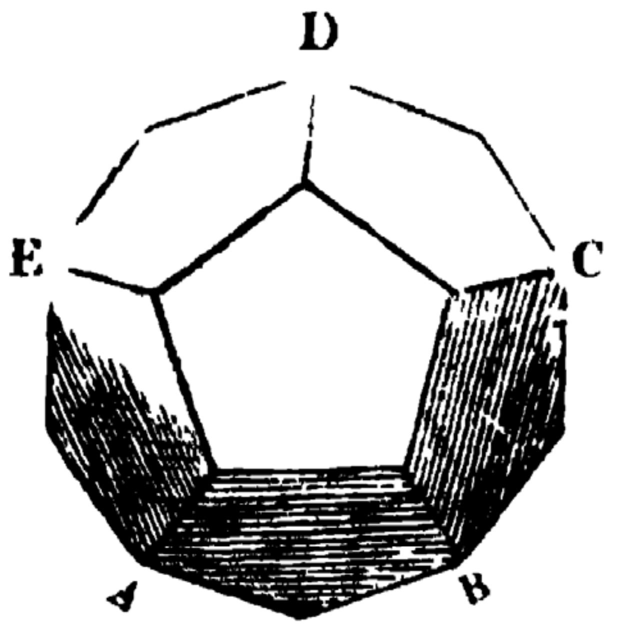
Ans. 216 *feet.*

3 Required the solidity of the octaedron ABCDE, whose linear side is 22 inches.



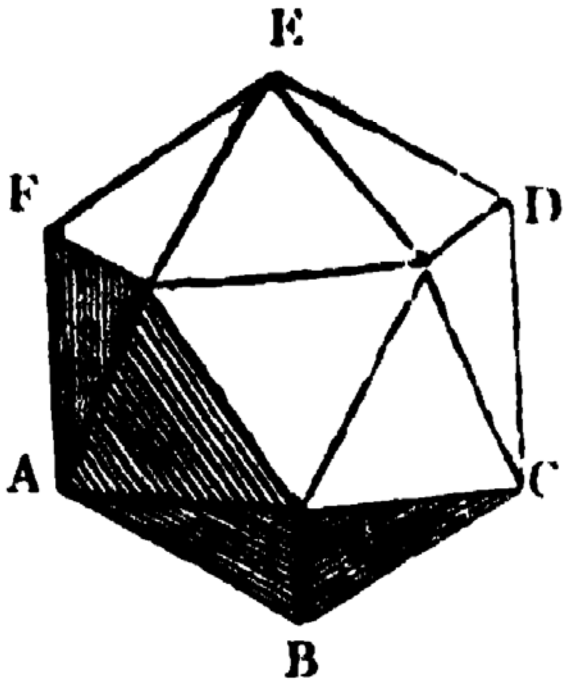
Ans 5019.515116 *inches.*

4. The linear side of the dodecaedron ABCDE is 5.68 feet; what is its solidity?



Ans. 1404.26984 *feet.*

5. What is the solidity and superficies of the icosaedron ABCDEF, whose linear edge is 15?



Ans. { *Solidity* 7363 22028.
{ *Superficies*, 1948 55715.

Note. The five Regular Bodies are sometimes called Platonic Bodies.

It appears from an ancient Greek Epigram, quoted by Scarburgh in his "English Euclid," (Oxford, 1705,) that "the five Platonic Bodies, which the wise Pythagoras found out, were indeed discovered by him, but Plato elucidated and taught them in the clearest manner, and Euclid took them in as the foundation of his own imperishable renown."

Pythagoras was a native of the island of Samos, and was born about 5[illegible] years before Christ, Plato was born at Athens, B.C. 429, and Euclid flourished at Alexandria, about 300 years before the Christian æra.

PROBLEM XVIII.

To find the solidity of an irregular solid.

RULES.

1. Divide the irregular solid into different figures, and the sum of their solidities, found by the preceding Problems, will be the solidity required.

2. If the figure be a compound solid, whose two ends are equal plane figures; the solidity may be found by multiplying the area of one end by the length.

3. To find the solidity of a piece of wood or stone, that is craggy or uneven, put it into a tub or cistern, and pour in as much water as will just cover it; then take it out, and find the content of that part of the vessel through which the water has descended, and it will be the solidity required.

4. If a solid be large and very irregular, so that it cannot be measured by any of the above Rules, the general method is to take lengths, in two or three different places; and their sum divided by their number, is considered as a mean length.

A mean breadth, and a mean depth are found by similar processes.

Sometimes the length, breadth, and depth, taken in the middle, are considered as mean dimensions.

Note. The unhewn blocks, in the freestone quarries in the vicinity of Leeds, are generally measured by the method described in the last Rule. The dimensions, however, are not, in general, taken to the extremities of the stones, in order to make an allowance for the waste in hewing.

EXAMPLES.

1. The lower part of a stone is a parallelopipedon, the breadth of whose end is 7, and its depth 5 feet. The upper part is a triangular prism, the perpendicular of whose end is 4 feet; required the solidity of the stone, its length being 18 feet.

By Rule I.

Here $7 \times 5 \times 18 = 35 \times 18 = 630$, *the solidity of the lower part; and* $7 \times 2 \times 18 = 14 \times 18 = 252$, *the solidity of the upper part; then* $630 + 252 = 882$ *feet, the solidity of the whole stone.*

By Rule II.

Here $7 \times 5 + 7 \times 2 = 35 + 14 = 49$, *the area of the end; and* $49 \times 18 = 882$ *feet, the solidity as before.*

2. Being desirous of finding the solidity of an irregular piece of wood, I immersed it in a cubical vessel of water; and when it was taken out, the water descended 6 inches; required the solidity of the wood, the side of the vessel being 30 inches.

By Rule III.

Here $30 \times 30 \times 6 = 900 \times 6 = 5400$ *inches, the solidity required.*

3. The lengths of an irregular block of marble, taken in different places, are 8 feet 6 inches, 8 feet 10 inches, and 9 feet 2 inches, the breadths 4 feet 7 inches, 4 feet 2 inches, and 4 feet 9 inches, the depths 3 feet 2 inches, 3 feet 5 inches, and 3 feet 11 inches; required its solidity.

By Rule IV.

LENGTHS.		BREADTHS.		DEPTHS.	
Feet	*In.*	*Feet.*	*In.*	*Feet*	*In*
8	6	4	7	3	2
8	10	4	2	3	5
9	2	4	4	3	11
3)26	6	3)13	6	3)10	6
8	10 mean.	4	6 mean.	3	6 mean.

Feet	*In*		
8	10		mean length
4	6		mean breadth.
35	4		
4	5		
39	9		
3	6		mean depth
119	3		
19	10	6″	
139	1	6″	*Ans.*

4. Wanting to know the solidity of an irregular block of marble, I immersed it in a cylindrical tub of water, whose diameter was 34.8 inches; and on taking it out, I found the fall of the water to be 12.6 inches, what was the solidity of the marble?

Ans. 11984 50028 *inches.*

5. Required the solidity of an irregular block of Yorkshire-stone, whose dimensions are as follow, viz. lengths taken in different places, 11 feet 3 inches, and 11 feet 9 inches, breadths, 5 feet 5 inches, 5 feet 9 inches, and 6 feet 4 inches; depths, 4 feet 5 inches, 4 feet 8 inches, and 5 feet 2 inches.

Ans. 318 *ft* 7 *in.* 9 *pa.*

PROBLEM XIX.

To find the magnitude or solidity of a body from its weight

RULE.

As the tabular specific gravity of the body,
Is to its weight in avoirdupois ounces,

So is one cubic foot, or 1728 cubic inches,
To its content in feet, or inches, respectively.

A TABLE

Of the specific gravities of bodies.

Fine Gold	19640	Chalk	1793
Standard Gold	18888	Sand	1520
Quick Silver	14000	Lignum-vitæ	1333
Lead	11325	Ebony	1331
Fine Silver	11091	Coal	1250
Standard Silver	10535	Pitch	1150
Copper	9000	Mahogany	1063
Gun Metal	8781	Box-wood	1030
Cast Brass	8000	Sea-water	1026 or 1030
Steel	7850	Common water	1000
Iron	7645	Oak	925
Cast Iron	7425	Gunpowder, shaken	922
Tin	7320	Logwood	913
Load-stone	4930	Beech	852
Granite	3000	Ash	800
Marble	2700	Yew	797
Glass	2642	Apple-tree	793
Flint	2570	Plum-tree	785
Bristol-stone	2510	Maple	755
Portland-stone	2496	Cherry-tree	715
Mill-stone	2484	Pear-tree	661
Yorkshire-stone	2442	Cedar of Lebanon	613
Clay	2160	Elm	600
Grind-stone	2143	Willow	585
Burford-stone	2049	Fir	550
Brick	2000	Poplar	383
Light earth	1984	Cork	240
Ivory	1825	Atmospheric air	1.2

Note 1.—The specific gravities of bodies are their relative weights contained under the same given magnitude, as a cubic foot, a cubic inch, &c.

2. As a cubic foot of water weighs just 1000 ounces avoirdupois, the numbers in the foregoing Table, express not only the specific gravities of the several bodies, but also the weight of a cubic foot of each, in avoirdupois ounces.

3. The several sorts of wood, mentioned in the preceding Table, are supposed to be dry.

4. In some tables, sea-water is 1026, and in others 1030.

EXAMPLES.

1. What is the solidity of a marble chimney-piece, whose weight is 210 *lb.* 15 *oz.* avoirdupois?

oz. *lb.* *oz.* *ft.* *ft.*
As 2700 : 210 15 :: 1 : $1\frac{1}{4}$ *Ans.*
16
1275
210
3375
1
2700)3375(1.25
2700
6750
5400
13500
13500

2. What is the solidity of an irregular Yorkshire-stone, whose weight is 228 *lb.* 15 *oz.* avoirdupois?

Ans. $1\frac{1}{2}$ *foot*

3. A piece of carved mahogany weighs 49 *lb* 14 *oz* avoirdupois, what is its solidity?

Ans. 1297.21919 *inches*

PROBLEM XX.

To find the weight of a body from its magnitude or solidity

RULE.

As one cubic foot, or 1728 cubic inches,
Is to the content of the body,
So is its tabular specific gravity,
To the weight of the body.

EXAMPLES.

1. The solidity of a grindstone is 3 feet; what is its weight?

ft. ft. oz. lb. oz.
As 1 : 3 :: 2143 : 401 13 *Ans.*
3
1)6429
16)6429(401
64
29
16
13

2. The solidity of a beam of dry oak is 25 feet; what is its weight? *Ans.* 1445 *lb.* 5 *oz.*

3. Required the weight of a block of marble, whose length is 63 feet, and its breadth and thickness each 12 feet, these being the dimensions of one of the stones in the walls of Balbec.

Ans 683·4375 *tons, which is nearly equal to the burthen of an East India ship.*

SECTION II.

Description and Use of the Carpenter's Rule.

DESCRIPTION.

THIS instrument is commonly called *Cogeshall's Sliding Rule,* and is much used in measuring timber, and artificers' work; not only in taking the dimensions, but also in casting up the contents

It consists of two pieces of box, each one foot in length, and connected together by a folding joint.

One side or face of the rule is divided into inches and half-quarters, or eighths; and on the same face there are also several plane scales, divided into twelfth parts, which are designed for planning such dimensions as are taken in feet and inches.

On one part of the other face are four lines marked A, B, C, D; the two middle ones, B and C, being upon a slider.

Three of these lines, viz. A, B, C, are exactly alike, and are called double lines, because they proceed from 1 to 10, twice over. The fourth line D, is a single one, proceeding from 4 to 40, and is called the *girt line*.

The use of the double lines A and B, is for working proportions, and finding the areas of plane figures, and the use of the girt-line D, and the other double line C, for finding the contents of solids.

When 1 at the beginning of any line is accounted 1, then the 1 in the middle will be 10, and the 10 at the end 100; and when 1 at the end is accounted 10, then the 1 in the middle is 100, and the 10 at the end 1000, &c.; and all the smaller divisions are altered in value accordingly.

Upon the girt-line are also marked W G at 17.15, and A G at 18.95, the wine and ale guage points, to make the instrument serve the purpose of a Gauging Rule.

On the other part of this face there is commonly either a table of the value of a load, or 50 cubic feet of timber, at all prices, from 6*d.* to 24*d.*, per foot; or else several plane scales divided into twelfth parts, and marked 1, $\frac{3}{4}$, $\frac{1}{2}$, and $\frac{1}{4}$, signifying that the inch, $\frac{3}{4}$ inch, &c. are each divided into 12 equal parts.

The edge of the rule is generally divided decimally, or into tenths; namely, each foot into 10 equal parts, and each of those into ten other equal parts. By this scale dimensions may be taken in feet, tenths, and hundredths of a foot, which is a very commodious method of taking dimensions, when the contents are to be cast up decimally.

The Use of the Sliding Rule.

PROBLEM I.

Multiplication by the sliding rule.

RULE.

Set 1 upon A, to the multiplier upon B; then against the multiplicand on A, will be found the product upon B.

Note.—When the third term runs beyond the end of the line, seek it on the other radius or part of the line, and increase the product 10 times, or 100 times, &c. as the case requires.

EXAMPLES.

1. What is the product of 24 multiplied by 12?
As 1 *on* A : 12 *on* B .: 24 *on* A : 288 *on* B, *the Ans.*

2. Required the product of 36 multiplied by 18.
Ans 648.

3. Multiply 12.8 by 6.5. *Ans.* 83.2

4. What is the product of 68 multiplied by 35?
Ans. 2380.

PROBLEM II.

Division by the sliding rule.

RULE.

Set the divisor on A, to 1 upon B; then against the dividend on A, is the quotient on B.

Note.—When the dividend runs beyond the end of the line, diminish it by 10 or 100 times, in order to make it fall upon A, and increase the quotient in the same proportion.

EXAMPLES.

1. What is the quotient of 432 divided by 12?
As 12 *on* A : 1 *on* B :: 432 *on* A : 36 *on* B, *the Ans.*

2. What is the quotient of 9752 divided by 46?
Ans. 212.

3. If a board be 8 inches broad; what length must be sawn off, to make a foot square?
Ans. 18 *inches.*

PROBLEM III.

To square any number.

RULE.

Set 1 upon D, to 1 upon C; then against the number upon D, will be found the square upon C.

EXAMPLES.

1 What is the square of 25?

As 1 *on* D : 1 *on* C :: 25 *on* D : 625 *on* C, *the Ans*

2 Required the square of 36 *Ans* 1296

3. What is the square of 58? *Ans* 3364

PROBLEM IV.

To extract the square root of any number

RULE.

Set 1 upon C, to 1 upon D; then against the number on C, is the root on D

Note.—In order to value this rightly, you must suppose the 1 on C to be one of these squares, 1, 100, 10000, &c which is nearest to the given number; and then the root corresponding will be the value of the 1 upon D

EXAMPLES.

1 What is the square root of 144?

As 1 *on* C : 1 *on* D : 144 *on* C : 12 *on* D, *the Ans.*

2. Required the square root of 900. *Ans* 30.

3. What is the square root of 9586? *Ans.* 97.9

PROBLEM V.

To find a mean proportional between two numbers

RULE.

Set one of the numbers on C, to the same on D; then against the other number on C, will be the mean on D.

EXAMPLES.

1. What is the mean proportional between 9 and 16?

As 9 *on* C : 9 *on* D :: 16 *on* C : 12 *on* D, *the Ans.*

2. Required a mean proportional between 15 and 27. *Ans.* 20.1.

3. The segments of the hypothenuse of a right-angled triangle made by a perpendicular from the right-angle, are 18 and 32; what is the perpendicular? *Ans.* 24.

PROBLEM VI.

To find a fourth proportional to three numbers, or to perform the Rule of Three.

RULE.

Set the first term on A, to the second on B; then against the third term on A, stands the fourth on B.

Note.—The finding a third proportional is exactly the same, the second number being twice repeated.

Thus, suppose a third proportional was required to 80 and 60.

As 80 on A : 60 on B :: 60 on A : 45 on B, the third proportional sought.

EXAMPLES.

1. What is the fourth proportional to the three numbers, 12, 24, 36?

As 12 *on* A . 24 *on* B .: 36 *on* A : 72 *on* B, *the Ans.*

2. If 1 foot of timber cost 3*s*; what will 180 feet cost? *Ans.* 540*s.*

3. If 50 feet of timber cost 7*l.*; what will 2500 feet cost? *Ans.* £350.

PROBLEM VII.

To find the areas of plane figures by the sliding rule.

RULES.

1. *To find the area of a rectangle or rhomboides.*

As 1 upon A, is to the perpendicular breadth upon B, so is the length upon A, to the area upon B.

2. *To find the area of a triangle.*

As 2 upon A, is to the perpendicular upon B; so is the base upon A, to the area upon B.

3. *To find the area of a trapezium.*

As 2 upon A, is to the sum of the two perpendiculars upon B; so is the diagonal upon A, to the area upon B.

4. *To find the area of a regular polygon.*

As 2 upon A, is to the perpendicular upon B; so is the sum of the sides upon A, to the area upon B.

5. *To find the diameter and circumference of a circle, the one from the other.*

As 7 upon A, is to 22 upon B; so is the diameter upon A, to the circumference upon B, and *vice versâ*.

6. *To find the area of a circle.*

As 4 upon A is to the diameter upon B, so is the circumference upon A, to the area upon B.

Or, as 1 upon D, is to .7854 upon C; so is the diameter upon D, to the area upon C.

Note—In order to exemplify the foregoing Rules, the learner may work the questions in the respective Problems of Part II

PROBLEM VIII.

To find the contents of solids by the sliding rule.

RULES.

1. *To find the solidity of a cube*

As 1 upon D, is to the side upon C; so is the side upon D, to the solidity upon C.

2. *To find the solidity of a parallelopipedon.*

As 1 upon A, is to the breadth of the base or end upon B; so is the length of the base upon A, to the area of the base upon B, and, as 1 upon A, is to the length of the solid upon B; so is the area of the base upon A, to the solidity upon B.

3. *To find the solidity of a prism, or a cylinder.*

Find the area of the base by the last Problem, with which proceed as in the last Rule.

4. *To find the solidity of a cone, or a pyramid.*

Find the area of the base by the last Problem; then, as 3 upon A, is to the length of the solid upon B; so is the area of the base upon A, to the solidity upon B.

Note.—Examples for practice may be found in Section I.

SECTION III.

TIMBER MEASURE.

PROBLEM I.

To find the superficial content of a board or plank.

RULE.

Multiply the length by the breadth, and the product will be the superficial content.

Note If the board taper, add the breadth of the two ends together, and half their sum will be a mean breadth.

By the Sliding Rule.

As 12 upon B, is to the breadth in inches upon A; so is the length in feet upon B, to the content upon A, in feet and fractional parts.

EXAMPLES.

1. If the length of a plank be 12 feet 6 inches, and its breadth 1 foot 3 inches; what is its superficial content?

By Decimals.	*By Cross Multiplication.*		
Feet.	*Feet.*	*In*	
12.5	12	6	
1.25	1	3	
625	12	6	
250	3	1	6'
125	15	7	6' *Ans.*
15.625 *Ans.*			

By the Sliding Rule.

As 12 *on* B . 15 *on* A :: 12.5 *on* B : 15.6 *on* A.

2. Required the content of a board whose length is 25 feet 8 inches, and breadth 1 foot 7 inches.

Ans. 40 *ft.* 7 *in.* 8 *pa.*

3. What is the content of a board whose length is 18 feet 10 inches, and breadth at the broader end 2 feet 3 inches, and at the narrower 1 foot 7 inches?

Ans. 36 *ft.* 1 *in.* 2 *pa*

4. If the length of a mahogany plank be 35 feet 9 inches, and its breadth 3 feet 6 inches; what is the value of three such planks at 2*s.* 9*d.* per square foot?

Ans. £51. 12*s.* 3¼*d.*

Note. Mahogany is a production of the warmest parts of America. It is also found plentifully in the islands of Cuba, Jamaica, and Hispaniola.

This tree grows tall and straight, rising often sixty, and sometimes a hundred feet from the ground to the arms, and is about 4 feet in diameter. The foliage is a beautiful deep green, and the appearance of the whole tree is very elegant.

The mahogany brought from Jamaica, is most valued, in consequence of its firmness, durability, and beauty of colour.

This wood has been used in medicine with the same effect as Peruvian bark.

PROBLEM II.

To find the solidity of squared or four-sided timber.

RULE.

Multiply the mean breadth by the mean thickness, and this product by the length, and it will give the solidity, *according to the customary method.*

By the Sliding Rule.

As 1 upon A, is to the mean breadth upon B; so is the mean thickness upon A, to the mean area upon B, or the area of the middle section; and, as 1 upon A, is to the length upon B; so is the mean area upon A, to the solidity upon B.

Note 1. If the tree be equally broad and thick throughout, the breadth and thickness taken in any part, will be the mean breadth and thickness, but if it taper regularly from one end to the other, the breadth and thickness must be taken in the middle. Or, take the dimensions of the two ends, and half their sum respectively will be the mean breadth and thickness.

2. Some measurers multiply the square of one-fourth of the circumference, or quarter girt, taken in the middle, by the length, for the solidity; but when the ends are not equal squares, this method always gives *too much*; and the more the breadth and thickness differ, the greater will be the error.

3. The Rule given in this Problem, is generally used in *practice*, and when the tree is equally broad and thick throughout, it gives the true content; but in tapering timber, it always gives *too little*, consequently, the method mentioned in the last note is sometimes nearer to the truth.

When the true content of a tapering tree is wanted, it must be found by the *General Rule*, for the frustum of a pyramid, given in Problem 6, Section 1, or by the Rule for a prismoid, given in Problem 10.

EXAMPLES.

1. If the length of a piece of timber be 9.8 feet, its breadth 2.6 feet, and its thickness 1.5 foot; what is its solidity?

Feet.
2.6
1.5
130
26
3.90
9.8
3120
3510
38.220 *Ans.*

By the Sliding Rule.

As 1 *on* A : 2.6 *on* B :: 1.5 *on* A : 3.9 *on* B, *the mean area.*

As 1 *on* A : 9.8 *on* B :: 3.9 *on* A : 38.22 *on* B, *the solidity.*

2. The mean breadth of a piece of timber is 2.86 feet, its mean thickness 1.93 foot, and its length 18.64 feet; what is its solidity? *Ans.* 102.889072 *feet.*

3. Each side of the greater end of a piece of squared timber is 28 inches, each side of the less end 14 inches, and its length 18 feet 9 inches; how many solid feet does it contain? *Ans.* 57.421875 *feet.*

Note. The true content, measured as the frustum of a pyramid, is 59.5485 feet. See Example 2, Problem VI., Section I.

4. The length of a piece of timber is 17 feet 3 inches; at the greater end, the breadth is 36 inches, and the thickness 20 inches; and at the less end, the breadth is 18 inches, and the thickness 10 inches; what is the solidity? *Ans.* 48.515625 *feet.*

Note. The true content, measured as the frustum of a pyramid, is 50.3125 feet. See Example 3, Problem VI., Section I.

5. A piece of timber is 32 inches broad, and 20 inches deep, at the greater end; 16 inches broad, and 8 inches deep, at the less end; and its length is 25 feet; what is its solidity? *Ans.* $58\frac{1}{3}$ *feet.*

Note. The true content, measured as a prismoid, is $61\frac{1}{9}$ feet See Example 2, Problem X., Section I.

PROBLEM III.

To find the solidity of round or unsquared timber.

RULE I.

Multiply the square of $\frac{1}{4}$ of the circumference, or quarter girt, by the length, and the product will be the content, *according to the common practice.*

By the Sliding Rule.

As the length upon C, is to 12 upon D; so is $\frac{1}{4}$ of the girt in inches upon D, to the content upon C.

RULE II.

Multiply the square of $\frac{1}{5}$ of the girt by twice the length, and the product will be the solidity *nearly.*

By the Sliding Rule.

As twice the length upon C, is to 12 upon D; so is $\frac{1}{5}$ of the girt upon D, to the content upon C.

Note 1. The girt must be taken in the middle of the tree, if it taper regularly, if not, take several girts, and their sum divided by their number, will give something near a mean girt. If the tree be very irregular, divide it into two or more parts, and find the content of each separately.

2. The girt of a tree is generally taken with a string, which being folded into four equal parts, and applied to a carpenter's rule, gives the quarter girt, but it is more expeditious to take the girt with a *tape* divided into equal parts of 4 inches each, and numbered at 4 inches from the end with 1, at 8 inches with 2, at 12 inches, with 3, &c. by which means the quarter girt can be obtained by merely girting the tree.

If the second Rule be used, the girt may be taken with a tape divided into equal parts of 5 inches each.

Each division of 4 or 5 inches, may also be subdivided into halves and quarters, and numbered successively with $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$; hence the quarter girt may be obtained with accuracy. The other side of the tape may be divided into feet and inches.

3. In measuring a tree which has its bark on, an allowance is generally made, by deducting so much from the girt as is judged sufficient to reduce the tree to such a circumference as it would have without its bark. This deduction may be most easily made from the quarter girt, and should be proportioned according to the thickness of the bark.

For ash, beech, elm, young oak, &c. $\frac{1}{2}$ or $\frac{3}{4}$ of an inch for every foot of the quarter girt, will generally be sufficient: but for old oak, whose bark is much thicker, it will be found necessary to allow an inch, an inch and a half, and sometimes two inches, for every foot of the quarter girt.

4. That part of the boughs, or of the trunk of a tree, which is less than 24 inches in circumference, or 6 inches quarter girt, is generally cut off, and sold at an inferior price, not being considered timber.

5. The first Rule in this Problem gives the solidity about $\frac{1}{4}$ part less than the true quantity, or nearly what the quantity would be if the tree were squared; so that it seems intended to make an allowance for the squaring of the tree.

6. The second Rule gives nearly the *true* content, whether the tree be a cylinder, or the frustum of a cone; and as it is full as easy in *practice* as the first, it ought always to be used when accuracy is required; if the Rules for a cylinder, the frustum of a cone, &c. be thought too prolix. (See Examples 3 and 4.)

7. If a piece of round tapering timber be cut through exactly in the middle, the two parts will measure to the most possible, and to more than the whole by either of the Rules.

8. To find where a round tapering tree must be cut, so that the part next the greater end may measure the most possible. From the greatest girt take 3 times the least; then, as the difference of these girts, is to the remainder, so is one-third of the whole length, to the length to be cut off from the less end. Or, cut the tree where the girt is one-third of the greatest girt.

When the greatest girt does not exceed 3 times the least, this Rule is impracticable.

9. Forty feet of round timber, measured by the quarter girt method, are called a load; but when the content of timber is correctly found, fifty feet are accounted a load.

Fifty feet of dry oak weigh rather more than a ton and a quarter.

10. The content of a piece of timber, according to the quarter girt method, may be readily found from the following Table, thus: Multiply the area corresponding to the quarter girt, in inches, by the length of the timber, in feet, and the product will be the solidity in feet and decimal parts.

A TABLE *for measuring* TIMBER.

Quarter GIRT.	*AREA.*	*Quarter* GIRT.	*AREA.*	*Quarter* GIRT.	*AREA.*
Inches.	*Feet.*	*Inches.*	*Feet.*	*Inches.*	*Feet.*
6	.250	12	1.000	18	2.250
6¼	.272	12¼	1.042	18½	2.376
6½	.294	12½	1.085	19	2 506
6¾	.317	12¾	1.129	19½	2.640
7	.340	13	1.174	20	2.777
7¼	.364	13¼	1.219	20½	2.917
7½	.390	13½	1.265	21	3.062
7¾	.417	13¾	1.313	21½	3.209
8	.444	14	1.361	22	3.362
8¼	.472	14¼	1.410	22½	3.516
8½	.501	14½	1.460	23	3.673
8¾	.531	14¾	1.511	23½	3 835
9	.562	15	1.562	24	4.000
9¼	.594	15¼	1.615	24½	4.168
9½	.626	15½	1.668	25	4.340
9¾	.659	15¾	1.722	25½	4.516
10	.694	16	1.777	26	4.694
10¼	.730	16¼	1.833	26½	4.876
10½	.766	16½	1.890	27	5 062
10¾	.803	16¾	1.948	27½	5 252
11	.840	17	2.006	28	5.444
11¼	.878	17¼	2.066	28½	5.640
11½	.918	17½	2.126	29	5.840
11¾	.959	17¾	2.187	29½	6.044
				30	6 250

EXAMPLES.

1. What is the solidity of a tree whose girt in the middle is 100 inches, and length 18 feet?

By Rule I.

In. *In.*
25 = $\frac{1}{4}$ *of* 100.
25
125
50
625
18
5000
625
144)11250(78.125 *ft. Ans.*
1008
1170
1152
180
144
360
288
720
720

Ft. In.
2 1
2 1
4 2
2 1'
4 4 1'
18
78 1 6' *Ans.*

By the Table.
4.340
18
34720
4340
78.120 *Ans.*

By the Sliding Rule.

As 18 *upon* C : 12 *upon* D :: 25 *upon* D : 78 *upon* C

By Rule II.

In. *In.*
20 = $\frac{1}{5}$ *of* 100.
20
400
36
144)14400(100 *ft. Ans.*
144
00

Ft. In.
1 8
1 8
1 1 4'
1 8
2 9 4'
36
100 0 0 *Ans.*

By the Sliding Rule.

As 36 *upon* C : 12 *upon* D :: 20 *upon* D : 100 *upon* C.

2. The length of a tree is 32 feet 6 inches, and its girt in the middle, after allowing for the bark, is 60 inches; what is its content?

Ans. { *By the first Rule,* 50.78125 *feet.*
By the second Rule, 65 *feet.*

3. The circumference of a cylindrical piece of timber is 6 feet 8 inches, and its length 24 feet; what is its solidity?

Ans. { *By the first Rule,* 66 *ft.* 8 *in.*
By the second Rule, 85 *ft.* 4 *in.*

Note.—The *true* content measured as a cylinder, is 84 feet 10 inches See Example 3, Problem IV., Section I

4. The circumference of a piece of round, tapering timber is 65 inches at the greater end, 35 inches at the less end, and its length 33 feet 9 inches; what is its solidity?

Ans. { *By the first Rule,* 36.621 *feet*
By the second Rule, 46.875 *feet.*

Note.—The *true* content measured as the frustum of a cone, is 48 feet See Example 3, Problem VIII., Section I

5. What is the content of an oak tree whose dimensions are as follow: the length of the trunk is 45 feet, and its quarter girt, in the middle, 30 inches; the length of a large bough 16 feet, and its quarter girt 14 inches; and the length of another bough 12 feet, and its quarter girt 10 inches? *Ans.* 311.36111 *feet.*

PROBLEM IV.

To measure and value standing Timber.

To find the content.

RULE.

Obtain the dimensions by some of the methods described in the following pages; and find the solidity by the Rules given in the last Problem. The first Rule is generally adopted, and the content found by the Sliding Rule.

MISCELLANEOUS INSTRUCTIONS

FOR

FINDING THE DIMENSIONS

OF

STANDING TIMBER,

AND FOR

Setting out Woods and valuing them previously to a Fall

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1. VARIOUS methods are used in order to obtain the dimensions of standing trees; but their girt and altitudes may be found most correctly by means of a ladder, and a long staff divided into feet and inches, and numbered from the top to the bottom, on one side, and from the bottom to the top on the opposite side, for the convenience of taking dimensions.

The ladder will enable you to take the girt at or near the middle of the tree, and with the staff you may measure the boughs and the upper part of the trunk. The lower part of the trunk may generally be measured by a tape.

2. Divide the arc of the quadrant of a circle of any convenient radius, into 90 equal parts, or degrees; and figure them at every 10 degrees thus; 10, 20, 30, &c. to 90. Upon that radius which is contiguous to 90 degrees, place two small brass sights, and from the centre or angular point suspend a plummet. Fix the quadrant to a square staff, of a convenient length for use, by means of a nail passing through both; and upon the end of this nail screw a small nut, so that the quadrant may be made fast to the staff at pleasure.

The neck of the nail should be round, so that the quadrant may be readily turned upon it; but that part which is contained within the staff, should be square, to prevent the nail from turning round with the quadrant.

Then, in order to find the height of a tree, screw the quadrant fast to the staff, so that the plummet may hang exactly at 45 degrees, when the staff is perpendicular to the horizon;

move the staff backward or forward, always keep it perpendicular, until you can see the top of the tree through both the sights; measure the distance between the bottom of the staff and the bottom of the tree, to which add the height of your eye; and the sum will be the height of the tree, supposing the ground to be horizontal.

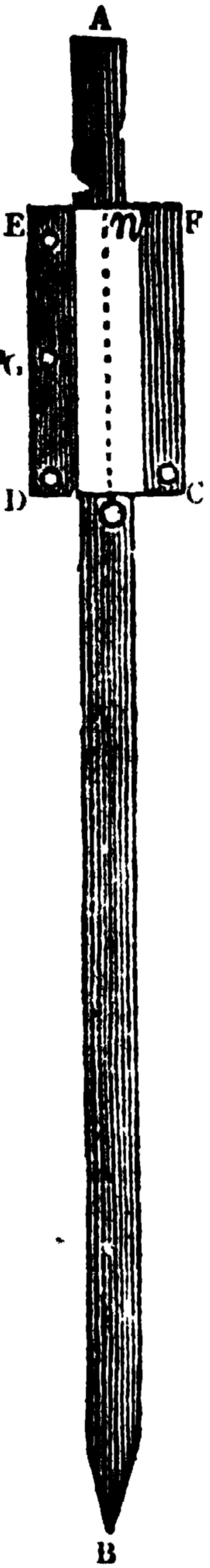
If, by reason of impediments, you cannot retire so far from the tree as directed above, make the quadrant fast, so that the plummet may hang at 63½ degrees, when you view the top of the tree through both the sights; then twice your distance from the tree added to the height of your eye, will give the height of the tree.

If this also be impracticable, take an angle of altitude, and measure the distance to the bottom of the tree; then, by a scale of equal parts, draw a line equal to the measured distance; and at one end of this line erect the perpendicular, and at the other end, by Problem 23, Part I., make an angle equal to the angle of altitude. Measure the perpendicular of the right-angled triangle thus formed, by the same scale from which the base was taken, to which add the height of your eye; and you will obtain the height of the tree

3. Divide a square staff, AB, of about 7 or 8 feet in length, into feet and inches, for the convenience of measuring the distance between the place of observation and the tree, or taking any other dimensions.

Upon one side of this staff, at a commodious distance from the bottom, fix a rectangular board, CDEF, whose length DE is exactly equal to twice its breadth CD, which breadth may be about 4 or 5 inches. At C and D fix sights, or small iron pins; and also at G and E; making DG and GE each equal to CD.

Then, when the top of a tree is seen through the sights at C and G, the tree's height is equal to your distance from its bottom added to the height of your eye; but if seen through the sights at C and E, its height is equal to twice your distance from its bottom, adding the same height as before.



In making an observation with this instrument, it ought to be fixed perpendicularly to the horizon, which may be done by means of a plummet suspended from *n*.

In taking the altitude of a tree growing upon an inclined plane, you must endeavour to make your observations from a place upon a level with the bottom of the tree. If this cannot be done, direct the horizontal sights at C and D, towards the lower part of the tree, and let your assistant make a mark upon it; then find the height of the tree above this mark, as before, to which add the distance of the mark from the ground, which must, in this case, be considered the height of the eye; and the sum will be the height of the tree.

The simplicity of this instrument seems to give it a preference to the quadrant; *practice* will, however, soon enable a person to judge pretty correctly of the heights of trees in general, independently of any instrument.

4. In order to obtain the girt of a tree, at or near the middle, proceed thus: If the tree taper pretty regularly, take the girt about two feet from the bottom, to which add 24 inches, the supposed girt at the top, and half the sum will be nearly the girt at the middle; as the height of the trunk ought only to be taken to that part which will measure 24 inches in circumference. Or, take the quarter girt about 2 feet from the bottom, to which add 6 inches; and half the sum will be the quarter girt at the middle nearly.

If the upper part of the trunk be nearly as thick as the bottom, which is sometimes the case; take the girt as far from the ground as you can reach, from which girt make a small deduction, at discretion, in order to reduce it as nearly as you can to the girt at the middle.

5. In measuring timber in a wood, you will frequently meet with 8, 10, 12, or more trees, nearly of the same dimensions, so that by measuring one of a medium size, you may, by it, estimate the contents of the rest, without measuring them, and thus be enabled to proceed with considerable expedition.

6. When a *fall* of wood takes place, it is often necessary to cut down many young trees, some of which will not contain more than a foot of wood, in order to make room for others to come to a greater maturity. If the quarter girt of these be less than 4 inches, their contents cannot be found by the Sliding Rule, consequently they must be computed by the Pen; *practice* will, however, soon enable a person to estimate the contents

of small trees with considerable accuracy, independently of calculation.

Young trees are called *pole-wood*, and valued at a less price than larger trees, or such as are called timber.

7. In valuing a wood, the branches, the bark of oak trees, and the under-wood must be taken into consideration, as well as the timber and pole-wood.

The strongest of the under-wood, and the boughs or branches which are not timber, are called *cord-wood*, and valued in Yorkshire and other northern counties, by the statute chord of 128 cubic feet; viz. 4 feet in breadth, 4 feet in height, and 8 feet in length.

Cord-wood is sometimes piled up and sold in parcels of those dimensions, and sometimes it is made into piles much larger, and the number of chords they contain found by admeasurement.

In Sussex, and several other southern counties, a pile of wood containing 126 cubic feet, viz. 3 feet broad, 3 feet high, and 14 feet long, is accounted a chord of wood.

The strongest of the cord-wood is made into props for the roofs of coal-mines, posts, rails, &c.; and the other into charcoal, which is used in manufacturing gunpowder, polishing copper or brass, making powerful fires for melting metals, &c. The price of cordwood is at present from nine to fifteen shillings per cord.

Bark is valued by the ton; and the price fluctuates in proportion as we have supplies from foreign countries. The price is, at present, from £.12 to £.15, according to the quality of the article.

Bark is a very strong astringent, and is chiefly used in tanning the hides of cattle into leather.

Under-wood is valued by the acre, if the number of acres contained in the wood be known; if not, it must be estimated by the great, at the discretion of the valuer. The price of under-wood is regulated by the quantity, quality, and demand there is for it. In some woods it is worth five pounds per acre, and in others it is scarcely worth five shillings.

Part of the under-wood is used for wicker work, such as panniers, coal-baskets, &c.; and some of it is only fit for hedging materials.

8. When a *fall* of wood is about to take place, the Valuer or Woodman must first mark all the trees which are intended to stand, by causing a ring of white or red paint to be made quite round the trunk of each tree. This is called "setting-out."

The trees that are permitted to stand should be at proper distances from each other; of different ages; and such as are likely to make the greatest improvement. Those which are straight and elegant, and have few branches, are generally preferred; this part of the business, however, depends very much upon the skill and experience of the woodman.

When the wood has all been *set-out*, those trees which are not marked must be valued; and in doing this, attention must be paid to the sizes, qualities, and species of the trees. Large trees, if sound, are always worth more per foot, than small ones; and if the crooked oak trees, and large crooked boughs, be particularly adapted for ship *timbers*, they are worth more per foot than straight trees.

In measuring and valuing, some persons number every tree, by a *scribe-iron*, or with paint; others only count them, and mark each tree with a cross, to shew that it has been measured; and others number the large trees, and count the small ones, setting down the contents of 10, 15, or 20 of these in one sum.

9. No regular rules can be given by which the quantity of bark, and the cord-wood a tree contains, may be computed, as the bark of trees differs very much in thickness, and the cord-wood depends entirely upon the quantity of top. Sometimes there is a great deal more bark upon the top of a tree, than upon the bole.

If the bole of a tree measures 30 feet or 360 inches in length, 60 inches in circumference in the middle, and the bark be half an inch* thick; we have 360 multiplied by 60 multiplied by ½, equal to 10800 cubic inches, the solidity of the bark, which being divided by 2150, the cubic inches in a Winchester bushel, we obtain 5¼ bushels, the quantity of bark upon the trunk, to which must be added the quantity estimated to be on the top, practitioners, however, are never at the trouble of calculating the quantity of bark upon the trunk, but make an estimate of what they suppose to be upon the whole tree. Besides, when bark is chopped, it will measure to a great deal more than in a solid state, as found by the above calculation.

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* In most places in the West Riding of Yorkshire, 6 pecks Winchester measure, or 3225 cubic inches are accounted a bushel

of bark, and 9 such bushels a quarter; and it has been found by experiment, that about 8 quarters of the bark of pretty large trees, or about twelve of that of small trees, or *pole-wood*, will weigh a ton; consequently, we may reckon about 10 quarters, upon a medium, to make a ton.

10. When an Estate, containing a wood, or a great number of trees in the hedge-rows, is intended to be sold, all the trees should be valued previously to the sale; as there have been instances where Estates have been sold, without considering the woods; and the purchaser has immediately disposed of the timber for nearly the price of the Estate.

11. When a rough calculation of a large wood is wanted, or time will not permit you to be very accurate, let an acre be measured off in that part of the wood which appears to be of a mean value, and measure and estimate the timber, &c. which it contains; then this estimation being multiplied by the number of acres in the wood, will give the value of the wood nearly. Or measure and estimate an acre of the best and an acre of the worst, and multiply half the sum of the two estimations by the number of acres in the wood, for the whole value.

12 In measuring and valuing standing timber, various methods are adopted, by different persons, in setting down the dimensions, &c. Some enter the oaks, elms, &c. separately in different parts of the book, and others enter them promiscuously, having a column in which they specify the name of each tree. Some valuers estimate the quantity of cord-wood in the top of each tree, or in the tops of two or three trees taken together; and others value the top of each tree.

The two following examples will serve to illustrate what has been said on the subject of measuring and valuing standing timber. The content of each tree was found by the Sliding Rule, and the learner ought to repeat the process, in order to make himself expert at casting by the Rule.

EXAMPLES.

1. Find the contents of six oak trees, and the value of the timber contained in each tree, from the following dimensions and values.

Number of Trees	*Bush. of Bark*	*Cords of Wood.*	*Lengths in Feet*	*Quarter Girts in Inches*	*Contents in Feet*	*Value per Foot*		*Value of each Tree*		
						s.	*d.*	*£.*	*s.*	*d.*
1	1¼	0	12	8½	6	2	0	0	12	0
2	3½	0	16¾	12¼	17½	2	6	2	3	9
3	32	½	39¼	18	89	4	3	18	18	3
4	15	¼	23	15¾	39¾	3	0	5	19	3
5	2½	0	15	9½	9½	2	3	1	1	4½
6	80	¾	44	28½	248	6	0	74	8	0
Total	134¼	1½	—	—	—	-	-	103	2	7½

2. Required the contents of ten trees, and the value of the timber in each tree, from the following dimensions and values.

No. of Trees	*Names of the Trees.*	*Value of the Tops.* £.	s.	d.	*Lengths in Feet.*	*Quarter Girts in Inches.*	*Contents in Feet.*	*Value per Foot.* s.	d.	*Value of each Tree* £.	s.	d.
1	Elm	0	5	6	42½	12¼	44¼	3	0	6	12	9
2	Ash	0	5	0	38¼	11¾	36½	3	0	5	9	6
3	Pine......	0	3	0	36	14½	52½	3	6	9	3	9
4	Fir	0	1	6	28	10	19¼	1	9	1	13	8¼
5	Beech ...	0	4	6	39¾	16½	75	2	6	9	7	6
6	Sycamore	0	3	6	26¼	9¾	17¼	2	6	2	3	1½
7	Poplar ...	0	2	6	48½	12¾	54¾	3	0	8	4	3
8	Yew	1	2	6	9½	38¼	96½	4	9	22	18	4½
9	Fir	0	2	0	32½	13¼	39½	2	0	3	19	0
10	Larch ...	0	2	6	30¾	15½	51¼	3	6	8	19	4½
	Total	2	12	6	—	—	—	-	-	78	11	3¾

Note. Having described the methods of measuring and valuing standing timber, and in the last Example, mentioned a variety of trees, it is presumed that this section cannot be better concluded than by a short description of the natures, properties, and uses of these trees.

Besides, it is absolutely necessary in order to become a valuer of timber, to be made acquainted with a few of the leading properties of trees, and their comparative usefulness.

A DESCRIPTION

OF THE

NATURE AND PROPERTIES

Of some of the most useful

TIMBER TREES.

THE OAK.

THE OAK stands at the head of all British timber-trees, as well for its utility, as for its majestic appearance. It is slow of growth, but if permitted to stand, it arrives at a size equal, if not superior, to that of any other tree of the forest; and by the vast arms which it throws out on every side, it forms a mass which fills the eye of the spectator, and impresses him with gigantic ideas of its masculine strength. The oak delights most in a rich, strong soil, in which it strikes its root to a great depth. It loves hills, better than boggy ground, and thrives best in large plantations.

The uses to which oak is applied, are numerous. It is a firm wood, and will endure all weathers, climates, and seasons, hence it is used for posts, rails, window-frames, casks, water-pails, carts, waggons, wheel-spokes, &c. In machinery, no other wood is equal to it, where a great stress is to be borne, as in mill-work, &c ; and in water-works, it is inferior to none. It is also used for household furniture, such as tables, bed-steads, chests of drawers, &c. ; but it has acquired its chief fame, in this country, by its use in ship-building, being much superior to foreign oak ; and has, no doubt, contributed very materially to the *naval* glory of OLD ENGLAND.

Some oak trees have arrived at an enormous size. In Dr. Hunter's edition of Evelyn's Sylva, is given a figure of the old oak of Cowthorpe, in Yorkshire, which measures 48 feet in circumference, at a yard from the ground. About a mile and a half from Shrewsbury, there is an oak whose girt is 44 feet at the bottom, 27 at the distance of 8 feet from the ground; and it is 41½ feet in height. In Hainault Forest, near Barking, in Essex, there is an oak which measures 36 feet in circumference. The tree has been known through *many centuries* by the name of FAIRLOP. Mr. Gilpin, in his "Remarks on Forest Scenery and other Woodland Views," says that the tradition of the country traces this tree half-way up the Christian æra.

Some of our best poets have noticed the usefulness and remarkable longevity of the oak.

Let India boast her plants, nor envy we
The weeping amber and the balmy tree,
While by our oaks the precious loads are borne,
And realms commanded which those trees adorn.

POPE.

The monarch oak, the patriarch of the trees,
Shoots rising up, and spreads by slow degrees:
Three centuries he grows, and three he stays
Supreme in state, and in three more decays.

DRYDEN.

THE ASH.

"THIS tree generally grows tall and elegant, and makes a graceful appearance, when contrasted with trees of greater bulk. It flourishes most in woods, but will also thrive well in good soils, upon open grounds.

There are few trees which excel the ash in utility; for its wood, next to that of oak, is employed for the greatest variety of purposes. It may be peculiarly termed the *husbandman's tree;* for it is one of the principal materials in making ploughs, harrows, carts, waggons, spokes, and felloes for wheels, and various other implements for rustic use. It is also employed by the turner for dairy utensils; and at sea it is used for oars and hand-spikes.

The toughness of its wood rendered it a favourite with the heroes of old, for the shafts of their potent spears; hence it is poetically termed "the martial ash." Homer arms his heroes with spears of ash.

> From Pelion's cloudy top an ash entire,
> Old *Chiron* fell'd, and sharp'd it for his sire.
>
> POPE'S HOMER.

THE BEECH.

THIS is one of the most stately of timber trees; large woods are wholly composed of it in some parts of this country. It particularly delights in a chalky soil, where it will flourish and arrive at a great size; although the land may have all the appearance of barrenness.

The wood of the beech is brittle, and apt to decay; but being of a fine grain, and easily wrought, it is used for a variety of domestic purposes. It is employed by the turner and cabinet-maker; the former using it for his larger ware, and the latter for common chairs, and other articles of furniture. It can be split so thin, that it is used for band-boxes, hat-cases, book-covers, and scabbards of swords.

THE ELM.

THE common elm is a large timber tree, of great beauty and utility. It grows to a great height, and at the same time, if permitted, expands some massy arms. It loves an open situation, and a black clayey soil.

The wood of the elm is hard and tough; and is used for a great variety of purposes. It is particularly serviceable in situations where it is kept constantly wet; and is therefore used for ship-planks, water-pipes, pumps, mill-wheels, &c. It is also employed for axle-trees, wheel-naves, gate-posts, chopping-blocks, &c.

THE SYCAMORE.

THIS tree is of quick growth; arrives at a large size, and flourishes best in open places, and sandy ground.

Its wood is soft and very white; hence it is proper for the use of the turner, who makes it into bowls, trenchers, and other domestic utensils. In consequence of its lightness, it is also occasionally wrought up as cart and plough-timber.

THE BLACK POPLAR.

THE name of *black* seems given to this tree in order to distinguish it from the *white* poplar; for its leaves are a beautiful green, and the tree has nothing dark in its appearance. It loves a rich and moist soil; it arrives at a great size, and is one of the tallest and most stately to be seen, when full grown.

The wood of the poplar is tougher and harder than fir; and is frequently used instead of it, for laths, packing-boxes, roofing, flooring, &c.

The white poplar does not arrive at so large a size as the black. It grows best in moist situations; and is very conspicuous from the whiteness of its foliage.

THE ALDER.

THE alder flourishes best in boggy situations, and by the sides of rivers, &c. It sometimes arrives at a large size; it is, however, chiefly used in this country for poles. Virgil, in his Georgics, tells as that by hollowing its trunk, it was anciently made into boats or canoes.

The wood of the alder will remain long sound under water. It is employed for pumps, water-pipes, piles, &c. It is likewise used for shoe-heels, clogs, and turners' work.

THE WEYMOUTH PINE.

THIS tree is a native of North America, where it is called the "white pine" It frequently arrives at the height of 100 feet; and is therefore preferred to the rest of the pine tribe for the masts of large ships. Our *men of war* are generally furnished with masts of this species of tree, from the timber-yards of Nova Scotia.

Lord Weymouth introduced it into this country; on which account it is generally known by his name; and has for some time, been a great favourite with planters.

THE FIR.

LINNÆUS considers all the fir tribe to be only different species of the pine. Some of them grow best on mountainous situations, others prefer bogs and swamps; and they alone often compose exten[illegible]e woods, clothing barren and desolate regions, unfit for human culture.

Many species of this tribe, are now become common in our plantations. Of these I shall only mention a few of the principal.

The Scotch Fir grows naturally in some parts of the Highlands of Scotland; and also upon the mountainous parts of Norway, Sweden, and Russia. The fir grown in the native forests of Scotland, is good, but affords a very scantly supply; consequently the greatest part of what is consumed by us, is brought from Norway, and the countries bordering upon the Baltic.

No wood is at present used amongst us in such quantities as the fir; which under the name of *deal*, is employed about buildings, for boards, planks, beams, rafters, joists, &c.

Fir is also used for the masts and yards of ships; and from it we obtain the important productions of turpentine, tar, and pitch. It may, therefore, be called the *sailors' tree*, with as much propriety as the oak.

The Spruce Fir is a native of Norway; and is said to afford the white deal. From the green tops of this tree is made the antiscorbutic beverage called "Spruce-beer."

The Silver Fir, grows in Norway and Germany; and from its trunk the yellow deal is said to be procured. It also yields great abundance of tar.

The Larch Fir is a native of the Alps and Appennines; and for beauty and durability of wood, it greatly surpasses the Scotch fir. It is now become a great favourite in this country, and thrives well on barren and sandy soils. In some countries ships are built of this wood, which are represented as proving very durable. The larch is remarkably resinous, and from it is obtained Venice turpentine.

THE BIRCH.

THERE are four species of birch. The common birch-tree may be cultivated upon barren land, where better trees will not grow; for there is no ground so bad as not to allow this to thrive in it. It will grow in moist springy land, or in dry gravel, or sand, where there is little surface; hence persons who are in possession of poor land, cannot employ it better than by planting it with these trees. They have sometimes been planted upon ground that produced nothing but moss; and in nine or ten years after planting, have been sold for ten pounds per acre; and the after-produce greatly increased.

The wood of this tree is used by broom-makers, hoop-benders, and turners.

THE YEW.

THE yew is a native of this country, and is found in rocky and mountainous situations. It is frequently planted in church-yards, probably on account of its being an ever-green. The leaves are of a poisonous nature; both horses, cows, and children have died in consequence of eating them.

This tree is remarkable for its toughness and elasticity; and is celebrated for the purposes to which its wood was anciently applied in making that most formidable weapon of our ancestors,—the *long-bow*.

> Th' elastic yew, whose distant wound
> With England's rivals heap'd the ground.

WILLIAM'S MISCELLANIES.

The wood of the yew is at present valued by cabinet-makers and inlayers on account of its beautiful red veins; and it is also a good material for flood-gates, axles, cogs for mill-wheels, and other works where strength and durability are required.

The yew seldom grows to a great height, but sometimes arrives at an amazing thickness of trunk.

In Darley church-yard, near Matlock, in Derbyshire, is a yew-tree, whose circumference is 33 feet.

THE HOLLY.

The holly is an ever-green, and a native of the woods in this country; where it sometimes arrives to the height of 20 or 30 feet. It is, however, more usually seen in gardens and hedge-rows, in the state of a shrub.

The wood of a full-grown holly is valuable. It is the whitest of all our hard woods, and is therefore used by inlayers, and it is sometimes stained black, to imitate ebony. It is also excellent for the uses of the turner, carver, and mill-wright; being extremely firm and durable.

THE BOX.

The box is another ever-green tree or shrub, which is sometimes met with here in a wild state; but more commonly in gardens and plantations. A strong shallow soil, of the lime-stone kind, seems to suit it best.

The wood of the box is of a pale yellow colour; and being very hard, smooth, and solid, it is much valued for various purposes. Mathematical and musical instruments are made of it; also knife-handles, combs, shuttles, &c. It is the only tree whose wood is serviceable to the engraver.

Having described the natures, properties, and uses of most of the trees found in our woods and plantations, I shall conclude with the following extract from Mr. John Tuke's Survey of the Agriculture of the North Riding of Yorkshire, a work that abounds with useful and just observations:—

"Most people, I think, concur in this point, that for the last half century, the wood in this kingdom has been terribly on the decline.

"That gloomy prospect is now become tremendous, and sufficiently visible to awaken the fears of every thinking person. The axe is often heard, but the planter is seldom seen. Let us cast our thoughts towards the future support and welfare of our navy—our sole protection!—and we must tremble at the continual disappearance of our oak. Some speedy method must be adopted to remedy this great, national evil; or, besides the danger from fierce external foes, we must determine to go barefoot: we should never think of looking to foreign countries for a constant supply of oak-bark to tan our leather. Let Britain help herself!

"Each nobleman and gentleman should insert, in the agreement with his tenants, a clause to compel them to plant and protect, in the corners of their fields, and upon pieces of waste ground, a certain number of good oak, elm, and ash-trees, annually. These trees should be found by the landlord; and he should enforce the performance of this clause as rigidly as the payment of the rent; then will the rising generation have cause to bless the wisdom and policy of the present age."

SECTION IV.

MISCELLANEOUS QUESTIONS

CONCERNING

SOLIDS.

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1. WHAT is the solidity of a cubical stone, whose diagonal is 30 inches? *Ans.* 9545.94154 *inches.*

2. The length of a parallelopipedon is 10, its breadth 6, and its depth 4 feet; what is the length of a similar parallelopipedon, whose solidity is three times as much? *Ans.* 14.14219 *feet.*

3. The altitude of a cylinder is 20, and its diameter 10 inches; what is the altitude of another cylinder whose solidity is twice as much, its diameter being 30 inches? *Ans.* $4\frac{1}{9}$ *inches.*

4. The diameter of a legal Winchester bushel is $18\frac{1}{2}$ inches, and its depth 8 inches; what is the diameter of that bushel whose depth is 7 inches? *Ans.* 19.77733 *inches.*

5. Two men bought a conical piece of timber, which is to be equally divided between them, by a plane parallel to the base; what will be the altitude of each part, the height of the cone being 21 feet, and the diameter of its base 3 feet 6 inches?

Ans. The altitude of the upper part is 16.66771 *feet; and that of the lower part* 4.33229 *feet.*

6. Each side of the greater end of a piece of timber, in the form of the frustum of a square pyramid, is 5 feet, each side of the less end 2 feet, and its perpendicular altitude 15 feet; it is required to divide it equally between two persons, by a plane parallel to the ends.

Ans. The height of the part next the less end is 10.25709, *and the height of the part next the greater end is* 4.74291 *feet.*

7. A gentleman has a bowling-green, 500 feet in length, and 300 feet in breadth, which he would raise 1 foot higher, by means of the earth to be dug out of a ditch with which he intends to surround it; what will be the depth of the ditch, if its breadth be every where 9 feet? *Ans.* 10.18744 *feet.*

8. What must be the diameter of the bore of a cannon, which is cast for an iron ball of 36lb. in weight, so that the diameter of the bore may be $\frac{1}{8}$ of an inch more than that of the ball? *Ans.* 6.4747 *inches.*

9. If a heavy sphere whose diameter is 8 inches, be put into a conical vessel, full of water, whose diameter is 10, and altitude 12 inches; it is required to determine how many cubic inches of water will run over. *Ans* 210 17722 *cubic inches.*

10. A cubical foot of brass is to be drawn into wire $\frac{1}{30}$ of an inch in diameter, what will be the length of the wire, allowing no waste in the metal? *Ans.* 31.25217 *miles.*

11. How many 4 inch cubes can be cut out of a 12 inch cube? *Ans.* 27.

12 The length of a piece of square timber is 12 feet, and its solidity 48 feet, what will be its solidity when it is made into the greatest cylinder possible? *Ans.* 37.6992 *feet.*

13. A farmer borrowed part of a hay-stack of his neighbour, which measured 8 feet every way, and paid him back by two equal cubical pieces, each side of which measured 4 feet: Query, was the lender fully paid? *Ans. He was paid only* $\frac{1}{4}$ *part.*

14 A piece of timber is 20 inches broad, and 16 inches thick, at what distance from the end must a section be made, so that the part sawn off may measure exactly 5 cubic feet? *Ans.* 27 *inches.*

15. A ship's hold measures 120 feet in length, 33 in breadth, and 6 in depth; how many bales of goods, 6 feet long, 4 feet broad, and 3 feet deep, may be stowed therein, leaving a *gang-way*, 3 feet broad, the whole length of the hold? *Ans.* 300.

16. The diameter of a circular island is 300 feet, and it is surrounded by an oblique-sided moat; how many cubic yards of earth were dug out of it, the breadth at the top being 10 feet, the breadth at the bottom 4 feet, and the perpendicular depth 8 feet?

Ans. 2019 93244 *cubic yards.*

17. Suppose it be found, by measurement, that a man of war, with its ordnance, rigging, and appointments, draws so much water as to displace 55000 cubic feet of sea-water; required the weight of the vessel.

Ans. $1571\frac{285}{116}$ *tons.*

18. The length of a cylindrical piece of timber is 15 feet, and its diameter 4 feet; what will be its solidity when hewn into a square prism? *Ans.* 120 *feet.*

19. How many bricks, each 8 inches long, 4 inches broad, and 3 inches thick, will build a wall 200 feet long, 10 feet high, and 1 foot thick?

Ans. 30000

20. What is the weight of a cast iron cannon-ball whose diameter is 4 inches? *Ans.* 9 *pounds*

21 The ball upon the top of St. Paul's church, in Leeds, is 3 feet in diameter, and it is covered with copper $\frac{1}{20}$ of an inch thick; required the value of the copper, at 2*s.* 3½*d.* per pound avoirdupois

Ans. £7 12*s.* 3¼*d*

22. What is the weight of a bomb-shell, or hollow sphere of cast iron, whose inside diameter is 9 inches, and the thickness of the metal an inch and a half?

Ans. 140 4746 *lb*

23. The length and breadth of the greater end of a mill-hopper, in the form of a prismoid, are 40 and 30 inches, the length and breadth of the less end 10 and 6 inches; and its perpendicular altitude 36 inches, how many Winchester bushels will it hold, 2150.42 cubic inches making a bushel? *Ans.* 8 53786 *bushels.*

24. A tree whose length is 28 feet, girts 45 inches in the middle, with a rope 1 inch in diameter; required its *true* girt, and likewise its solidity, according to the quarter-girt method of measuring timber; the length

of the rope when extended in a right-line, being equal to the circumference of its centre, when formed into a circle, in taking the girt of the tree.

Ans. The true girt is 41.858364 *inches, and the solidity* 21.293157 *feet; hence appears the propriety of girting trees with a small string, or with a measuring-tape.*

25. A leaden pipe 12 feet in length, weighs 100lb. avoirdupois; required the thickness of the lead, the diameter of the bore being 2 inches.

Ans. .2408 *of an inch.*

26. Required the thickness of the shell of a hollow sphere of copper, weighing 3 lb. avoirdupois, so that being put into common water, it may be just immersed by its own weight? *Ans.* .1042 *of an inch.*

27. The bore of a syringe which holds one pint, wine measure, is $1\frac{1}{2}$ inch in diameter; what is the length of the piston? *Ans.* 16.33986 *inches.*

28. If a balloon contains 1000 yards of silk, $\frac{3}{4}$ of a yard wide; what is its diameter, admitting it to be a perfect sphere? *Ans.* 15.45095 *yards.*

29. A hollow sphere of ebony, whose diameter is 18 inches, is found to sink just 6 inches in sea-water; required the thickness of the shell. *Ans.* .647333 *of an inch.*

30. One ev'ning I chanc'd with a tinker to sit,
Whose tongue ran a great deal too fast for his wit;
He talk'd of his art with abundance of mettle;
So I ask'd him to make me a flat-bottom'd kettle:
Let the top and the bottom diameters be,
In just such proportion as five is to three;
Twelve inches the depth I propos'd, and no more;
And to hold in ale-gallons seven less than a score.
He promis'd to do it, and straight to work went;
But when he had done it, he found it too scant;
He alter'd it then, but too big he now made it;
For though it held right, the diameters fail'd it:
Thus making it often too big, and too little,
The tinker at last, quite spoiled his kettle;
But he vows he will bring his said promise to pass,
Or he'll utterly spoil every ounce of his brass;

Now, to save him from ruin, I pray find him out
The diameter's length, for he'll ne'er do't I doubt.

Ans. The bottom diameter is 14.640098, *and the top diameter,* 24.400163 *inches.*

Note This ingenious question was, I believe, first proposed in the Ladies' Diary, for the year 1711.

PART V

ARTIFICERS' WORK

The Artificers whose works are here to be treated of, are Bricklayers, Masons, Carpenters, and Joiners, Slaters and Tilers, Plasterers, Painters, Glaziers, Plumbers, and Pavers.

THE contents of the works of all artificers, whether superficial or solid, must be found by the Rules given in the foregoing Problems, for the respective figures. This ought to be particularly attended to in taking the dimensions.

The following Rule, which should be got by heart, is better adapted for *Cross Multiplication,* than any other I have seen; and as 12 fourths make 1 third, 12 thirds 1 second, 12 seconds 1 inch, and 12 inches 1 foot; these numbers, when not too large, may be more expeditiously turned into the higher denominations by the *Pence Table,* then by dividing them by 12.

RULE.

Feet multiplied into feet give feet.
Feet multiplied into inches give inches.
Feet multiplied into seconds give seconds.
Inches multiplied into inches give seconds.
Inches multiplied into seconds give thirds.
Seconds multiplied into seconds give fourths.

The measures chiefly used by Artificers, are contained in the following table:

12 inches		1 lineal foot.
144 square inches		1 square foot.
9 square feet		1 square yard.
63 square feet	make	1 square rood.
100 square feet		1 square.
272¼ square feet, or 30¼ square yards		1 square rod, pole, or perch.

Note. As the number 272¼ is rather troublesome to divide by, it is customary, in practice, to divide by 272, omitting the ¼, but if this be not thought correct enough, convert the ¼ into decimals; or reduce the divisor to the improper fraction $\frac{1089}{4}$, and multiply the content in feet by 4, and divide the product by 1089, and the quotient will be the number of rods sought. Divide the remainder, if any, by 4, and the quotient will be the feet.

The rod of 272¼ square feet, is the square of 16½ feet: but in some places the customary rod is the square of 18 feet, viz. 324 square feet.

BRICKLAYERS' WORK.

Bricklayers generally compute their work by the rod of 272¼ square feet, and at the rate of a brick and a half thick; consequently, if a wall be more or less in thickness than this standard, it must be reduced to it, in the following manner: Multiply the superficial content of the wall, in feet, by the number of half bricks which it is in thickness; and ⅓ of the product will be the content sought.

In some places, however, brick-work is measured by the rood of 63 square feet; that is, 21 feet in length, and 3 feet high, and then no regard is paid to the thickness of the wall, in measuring, as the price of the workmanship is regulated according to the thickness.

Note 1. In taking the dimensions of a *building*, measure half round the outside and half round the inside, and the sum of these two will be the *true* compass of the building. Or, 4 times the thickness of the wall taken from the whole compass, on the outside, or added to the whole compass within, will give the true compass, which multiply by the height, and the product will be the superficial content of the walls, and if the breadth of the building be multiplied by the height of one gable-end, you will obtain the content of both the gable ends.

In some places it is customary to take the whole compass, on the outside, in order to make an allowance to the workmen, for the trouble of turning the corners; but this should not be done for both workmanship and materials, except specified in the agreement, but for workmanship only.

If the workmen be allowed so much per yard, lineal measure, for the corners, which is the custom in some places, the true compass of the building ought then to be taken for both workmanship and materials.

2 When the height of a building is unequal, measure a piece round the bottom, so as to make the upper part all of one height, and in doing this, holes must be dug in the ground, to enable you to take your dimensions to the foundation. Several altitudes of the bottom part must be taken, and their sum divided by their number, may be considered as a mean altitude

3 In most buildings of two or more stories, the walls decrease in thickness, towards the top, and this diminution generally consists of half a brick, in each story. The thickness is set off on the inside, and commonly in a place where the floor will be laid; a contrivance by which the *set off* is concealed

The stories that are of different thicknesses must be measured separately except by agreement, one price be allowed for the whole, which is not often the case

When the walls of a building are of different thicknesses, the upper rooms are broader than the lower ones, hence a *set-off* may be discovered, although the walls be plastered and the floors laid.

4 *Doors and windows* must always be deducted for materials, if there be no stipulation to the contrary, but for workmanship, these deductions are seldom made, except the doors and windows be very numerous, as in workshops, &c., or larger than the usual size, as in shop-fronts, &c

In these cases the Surveyor must exercise a discretionary judgment, for the windows of buildings vary so much in size and number, that no special Rule can be given.

5. If a *chimney* stand by itself, without any party wall being joined to it, take the girt, in the middle, for the length, and the height of the story for the breadth, but if the chimney-back be a party-wall, and the wall be measured by itself, you must girt the chimney round, to the wall, on each side, for the length, and take the breadth the same as before.

When a *chimney* is wrought upright from the *mantle-tree* to the *ceiling*, the thickness of the whole, is generally considered the same as that of the *jambs*, and no deduction is ever made for the vacancy between the floor and the *mantle-tree*, because of the gathering of the breast and wings, to make room for the hearth in the next story.

Chimney shafts above the roof, are measured by girting them in the middle, for the length, and taking the height for the breadth.

Their thickness is generally accounted half a brick more than it is in reality, in consideration of the plastering and scaffolding.

6 In some places double measure, for workmanship, is allowed for chimneys, in consequence of their being more troublesome to be made than the other parts of the building, and in others they are done at so much per yard, lineal measure, or at so much per piece.

It is also customary, in most places, for bricklayers, to charge so much extra for every arch they turn, and this charge is regulated by the size of the arch. They also make a difference in the price between an inside and an outside arch, charging less for the former than the latter

EXAMPLES.

1. The length of a wall is 86 feet 9 inches, its height 12 feet 6 inches, and its thickness 3 bricks; how many standard rods of brick-work does it contain?

By Decimals.

Feet
86·75 length.
12·5 height.
43375
17350
8675
1084·375
6
3)6506·250
272)2168·750(7 *rods, 264 ft. 9 in. the answer.*
1904
264 *remainder.*

By Cross Multiplication.

Feet Inches
86 9
12 6
1041 0
43 4 6″
1084 4 6
6
3)6506 3 0
272)2168 9 0(7 *rods, 264 ft 9 in the answer.*
1904
264 *remainder.*

2. In taking the dimensions of a cottage, I find that half the compass, on the outside, is 40 feet, half the compass within 37 feet, the height from the foundation to the eaves 9 feet, the height of the gable-end 5 feet, and its breadth 18 feet; how many roods of brick-work are contained in the building; deducting for a door, which measures 6 feet by 3, and a window whose height is 5, and breadth 4 feet?

Ans. 11 *roods*, 52 *feet*.

3. The circumference of a circular building at the iron foundry of Messrs. Fenton, Murray, and Wood, in Leeds, is 241 feet, and its height, from the bottom to the eaves, 49 feet 6 inches; how many roods are contained in the wall; deducting for a door whose breadth is 12, and height 14 feet: for 96 windows, each of which measures 6 feet 7 inches by 4 feet; and for 14 windows, whose heights are 9 feet 6 inches, and breadths 5 feet 4 inches? *Ans.* 135 *roods*, 19 *feet*.

4. The true compass of a building, two stories high, is 120 feet 6 inches; the height of the lower story is 10 feet 6 inches, and the thickness of the wall 2 bricks, the height of the upper story is 8 feet 3 inches, and the thickness of the wall $1\frac{1}{2}$ brick; the gable-end measures 20 feet in breadth, and 8 feet in height, and is $1\frac{1}{2}$ brick thick; what did the brick-work cost at £.14 12*s.* per standard rod; deducting for 6 windows in the lower story, each of which measures 5 feet 6 inches by 4 feet 3 inches, and a door whose height is 6 feet 6 inches, and breadth 3 feet 9 inches; and four windows in the upper story, whose dimensions are 4 feet 9 inches by 4 feet 3 inches? *Ans.* £ 136 6*s.* 9*d.*

MASONS' WORK.

All kinds of stone-work belong to Masonry; and the measures generally used are the lineal foot, the square foot, the square yard, the square rod of 63 feet, and the cubic foot.

Carved mouldings, &c. are generally measured by the lineal foot; and ornamental work, such as arches, architraves, friezes, cornices, chimney-pieces, &c. by the square foot. Also all *tooled* or *cleansed* work is measured by the square foot; viz. door-posts, window-jambs, flags, steps, &c.; but *rough* flagging is generally measured by the square yard.

Walls are sometimes measured by the square yard, and sometimes by the rood of 63 square feet.

Columns, pillars, blocks of marble or stone, &c. are measured by the solid foot; and sometimes the contents of walls are found in the same measure.

Solid measure is chiefly used for materials, and superficial for workmanship; in some places, however, masons are paid so much per rood, for workmanship and materials; and the price is regulated by the thickness of the wall.

Note 1 The dimensions of stone *buildings* are taken in the same manner as in Bricklayers' Work, and deductions must be made for *doors, windows, &c.*, except the agreement prohibits it. These deductions, however, ought only to be made for materials, as the workmen are fully entitled to receive pay for the whole as walling, in consequence of the trouble of fixing the window-jambs, &c.

In measuring *tooled or cleansed fronts*, the doors and windows must be deducted, as the price of the workmanship, in these cases, is too considerable for them to be included

2. The walls of the upper stories of buildings are, in general, not so thick as those of the lower stories, but the price for the workmanship is commonly the same, in the consideration of the trouble of scaffolding, and the labour of carrying up the materials.

In some places it is customary to measure *door-posts, window jambs, steps,* &c. by the cubic foot for the materials, and the superficial foot, for the workmanship, and in others, so much per superficial foot, is charged for workmanship and materials; and in taking the breadth, or girt, the *tape* is made to ply close over every part of the stone that has been *tooled*, except it appear that the workmen have *intentionally* tooled more than is necessary

3 The length of a circular window-head, or door-head, is found by taking half the sum of the greater and less arches.

4 In making the *notch* in the window-jambs, for the frame of the window, some workmen are in the habit of tooling or chiseling the jambs further on the inside, than the window-frame requires, in order to make the work measure as much as possible; few architects, will, however, allow more than 3 or 4 inches for both sides of the notch.

5. *In measuring a flight of steps*, make the *tape* ply close over them in the middle, from the top to the bottom, for the length and take the length of a step for the breadth. Or, if the steps be all of one size, multiply the area of one step by the number of steps.

The *ends* and the *landings* ought always to be measured by themselves.

6. All the parts of ornamental frontispieces must be measured separately viz. *plinths*, *dados*, *columns*, *pilasters*, *architraves*, *friezes*, *cornices*, *pediments*, &c. &c.

7 It is customary, in some places, to allow double measure for all kinds of circular work, such as *cylindrical or conical columns*, *circular pediments*, *arched door or window-heads*, &c., and also for *cornices*, *feathered gables*, &c, in other places, only the area and half the area are allowed; it is much better, however, to proportion the price to the workmanship, and take the *true* measurement

It is also customary, in most places, to pay the workmen so much per yard lineal, for the trouble of turning the corners of buildings and hewing the stones in a proper manner to form these corners.

If the corners be *formed* by tooled or cleansed *coins*, they must be measured separately, and added to the tooled or cleansed work. In this case no other charge ought to be made for the corners.

It may likewise be observed, that masons are generally allowed something extra, for the trouble of turning arches

EXAMPLES.

1. The length of a wall is 86 feet 9 inches, its height 10 feet 6 inches, and its thickness 2 feet 3 inches; required its superficies and solidity.

By Decimals.

Feet
86.75 length.
10.5 height.
43375
8675
910.875 superficies.
2.25 thickness.
4554375
1821750
1821750
2049.46875 solidity.

By Cross Multiplication.

Feet	*Inches.*			
86	9			length.
10	6			height.
867	6			
43	4	6″		
910	10	6		superficies.
2	3	0		thickness.
1821	9	0		
227	8	7	6‴	
2049	5	7	6	solidity.

2. The length of a flight of Yorkshire-stone steps is 10 feet 6 inches, and their breadth 3 feet 9 inches; the landing measures 5 feet 3 inches by 4 feet 6 inches; to what do they amount, at 1*s.* 9*d.* per square foot, for workmanship and materials? *Ans.* £5. 10*s.* 3*d.*

3. The side of a square pillar, of Portland-stone, measures 1 foot 9 inches, and its height 7 feet 6 inches; what is the value of 6 pillars of the same dimensions, at 5*s.* 9*d.* per cubic foot? *Ans.* £69. 12*s.* 5*d.*

4. The circumference of the base of a stone column measures 6 feet 10 inches, the circumference at the top 3 feet 8 inches, and the slant height 9 feet 3 inches; required the expense of *cleansing* or *polishing* the convex surfaces of 4 such columns, at 1*s.* 9*d.* per square foot. *Ans.* £16 19*s.* 11$\frac{1}{4}$*d.*

5. The threshold of a door measures 4 feet 6 inches in length, 11 inches in breadth, and 5 inches in thickness; the head is 4 feet 11 inches long, 10 inches broad, and 7 inches thick, each jamb is 6 feet 5 inches in height, 10 inches broad, and 7 inches thick; how many cubic feet of stone do they all contain?

Ans. 10 *ft.* 4 *in.* 2 *pa.*

6. A window-sole measures 5 feet 10 inches in length, and 15 inches in girt; the head is 5 feet 8 inches long,

and girts 11 inches; the height of each jamb is 6 feet 4 inches, and the girt, including 3 inches for the notch, 1 foot 2 inches; what will be the expense of tooling the soles, heads, and jambs of 8 such windows, at 7*d.* per square foot? *Ans.* £6. 7*s.* 2¾*d.*

7. The mantle-tree of a fire-place measures 6 feet 3 inches in length, 9 inches in breadth, and 6 inches in thickness; the height of each jamb is 4 feet 9 inches, the breadth 9 inches, and the thickness 6 inches; the length of the slab is 6 feet 3 inches, and its breadth 2 feet 6 inches; the two coves are each 4 feet 9 inches high, and 9 inches broad; the chimney-piece measures 6 feet 9 inches by 6 inches; what did the marble cost; the jambs and mantle being at £4. 10*s.* per cubic foot, and the coves, slab, and chimney-piece at 12*s.* 6*d.* per square foot? *Ans.* £42. 18*s.* 1½*d.*

8. Farnley Chapel, in the Parish of Leeds, measures 66 feet 8 inches in length, and 32 feet 7 inches in breadth, on the outside; its perpendicular height from the foundation to the ridge, is 32 feet, and from the foundation to the eaves, 22 feet 6 inches; and the thickness of the wall is 2 feet; how many roods of materials are contained in the building; taking its true compass, and deducting for 7 arched windows, whose heights, from the sole to the crown or middle of the arch are 11 feet 8½ inches, from the sole to the spring of the arch, or where it begins to turn, 9 feet 4 inches, and breadths 4 feet 9 inches; and for an arched door, whose height, from the threshold to the crown of the arch, is 9 feet 4½ inches, from the threshold to the spring of the arch, 7 feet, and breadth 4 feet 9 inches? *Ans.* 66 *roods*, 23 *feet.*

9. A cloth-mill, at Armley, in the Parish of Leeds, measures 192 feet in length, and 32 feet in breadth, on the outside; its perpendicular height, from the foundation to the eaves, is 35 feet, the height of the gable-end 8 feet, and the thickness of the wall 2 feet; how many roods does the building contain; taking the true compass, and making deductions for 2 doors, one of which measures 9 feet 6 inches by 6 feet, and the

other 9 feet 6 inches by 5 feet; for 69 windows, whose dimensions are 7 feet 6 inches by 4 feet, and 78 windows, each of which is 7 feet in height, and 4 feet in breadth? *Ans.* 179 *roods*, 20½ *feet.*

A DESCRIPTION

OF

Portland; Yorkshire, and Purbeck Stone;

AND ALSO OF

VARIOUS KINDS OF MARBLE.

THE learned and ingenious Mr. Robert Boyle observes, that a competent knowledge of the stones used in building, is of the greatest importance; one stone dug out of a quarry being found to moulder away in a few winters, while another will brave the weather for many ages. The same author adds, that although some stones will decay in a few years, others will not have attained their full hardness in half a century.

FREE-STONE is very much used in building. It is of a sandy nature, and may be cut freely in any direction, hence it derives its name. It is generally of reddish, yellowish, whitish, or greyish colour; and is sometimes mixed with small particles of mica, or vestiges of shells.

There are many excellent quarries of free-stone in England. Those in the peninsula of Portland, in Dorsetshire, and in the vicinity of Leeds, in Yorkshire, are the most celebrated.

PORTLAND-STONE is of a soft nature, when dug out of the quarry; but in process of time becomes harder. Some of the finest structures in London are built with this stone, as the piers and arches of Westminster Bridge, the magnificent cathedral of St. Paul's, &c. &c.

YORKSHIRE-STONE is harder when dug out of the quarry, than Portland; and it becomes still more hard by being exposed to the weather. Large quantities of this stone are sent to different parts of England. It is noted not only for its use in building, but also for its durability when laid under water; hence it is much used for bridges, docks, &c.

Kirkstall Abbey, situated about three miles from Leeds, is built of this stone; and most of the stones of which the outer walls are composed, are not only perfectly sound, but extremely hard. A few are in a state of decay; which prove the justness of Mr. Boyle's observations before quoted.

It is 667 years since this Abbey was built. Part of it was unroofed, at the dissolution of the house, in the reign of Henry VIII., and the whole is now completely in ruins.

PURBECK-STONE is dug out of quarries in the isle of Purbeck, in Dorsetshire; and is of a much harder nature than either Portland or Yorkshire-stone.

There are numerous sorts of this stone, the finest of which take a good polish, and are used for chimney-pieces, hearths, grave-stones, &c. The coarser kinds make excellent paving-stones.

MARBLE is a fine valuable stone, extremely hard and compact; and capable of receiving a very beautiful polish. It is chiefly used in works of ornament, as columns, statues, altars, tombs, chimney-pieces, &c &c

Of this stone there are many varieties, as white, black, purple, &c.

English white marble is generally veined with red. *Derbyshire marble* is variously clouded and diversified with brown, red, and yellow. That of *Devonshire* is either black with white veins, or red variegated with grey and orange. *Marble of Auvergne* in France, is of a pale red, mingled with violet, green, and yellow.

Various other kinds are denominated by the places from which they are broug

Languedoc, in l

CARPE

To this l

house; viz.

ting, &c. &c.

Flooring,

computed by

however, naked flooring and roofing are measured by the cubic foot for materials, and by the square of 100 feet, for workmanship.

Wainscotting, doors, window-shutters, &c. are commonly measured by the square foot; enriched mouldings and several other articles, by the lineal foot; and some things are done at so much per piece.

Note 1. *In measuring naked flooring*, take the length of the room added to the bearing of the *joists*, or [illegible] they are let into the wall, at each end, for one dimension; and the breadth of the room added to the bearing of the *girders*, for the other dimension.

If there be no girders, the length of the room must be taken for one dimension; and its breadth added to the bearing of the joists for the other.

The *girders* and *joists* of floors intended to bear great weights, ought to be let into the wall, at each end, $\frac{2}{3}$ of the wall's thickness; but in common flooring, the girders have seldom more than 9 or 10 inches bearing, and the joists about 5 or 6 inches.

For boarded flooring, take the length and breadth of the room; and deductions must always be made for hearths and well holes.

In naked flooring, no deductions are made for hearths, in consequence of the additional trouble and waste of materials, but well-holes must always be deducted.

If the materials of the *naked flooring* be charged by the cubic foot, multiply the solidity of one joist by the number of joists of the same dimensions: and when the *girders* are all of the same size, proceed with them in a similar manner, if not, find the solidity of each separately.

2 *Partitions* are measured from wall to wall for one dimension, and from floor to floor, or as far as they extend, for the other dimension, and deductions must be made for doors and windows, except the agreement includes them.

Strong partitions, made with framed timber, are generally measured in the same manner as flooring.

Weather boarding is sometimes computed by the square of 100 feet, the same as partitioning, sometimes by the square foot, the same as wainscotting; and sometimes it is measured by the lineal foot.

3 *In roofing*, the length of the house in the inside, added to $\frac{2}{3}$ of the thickness of each gable, is to be taken for the length; because the *purlins* ought to be let into the wall, at each end, $\frac{2}{3}$ of its thickness; and the breadth or girt will be equal to twice the distance measured from the ridge, down the principal *rafter*, over the end of the *tie-beam*, to the wall.

If the roof be covered with tiles or slates before the Joiner's Work be measured off, which is very often the case, you must endeavour to ascertain the *true* girt as nearly as possible. In general twice the distance between the top of the ridge and the extremity of the eaves, will be rather too much; this, however, depends very much upon the depth of the tie-beam.

In measuring roofing, whether for workmanship or materials, no deductions ought to be made for the holes of chimney-shafts, sky-lights, or luthern-lights, on account of their additional trouble, and the waste of materials.

When *angles*, running from the ridge to the eaves, are formed in a roof, that angle which bends outward is [illegible] that which bends inward a *valley*; and in roofing, it is customary to allow the Joiner some additional measurement, in consideration of the waste of timber, and extra trouble.

In most places the length of the hip or valley is multiplied by 2 feet, and the product added to the content of the roof.

The sides of the hipped roof of a rectangular building are trapezoids, and the ends triangles; hence, it is evident that if the contents of these figures be found separately, their sum will be the content of the whole roof.

Or, if the length of one side and the breadth of one end, taken half way between the ridge and the eaves, be multiplied by the girt of the roof, the product will be the content.

When the solidity of the *tie beams, rafters, king-posts, purlins, &c.* in the roof of a building, are required; they must be found in the same manner as directed for girders and joists, in naked flooring.

4. The pitches of roofs are frequently made according to the fancy of the designer Formerly they were made much higher than Architects, in general, make them at present; some regard, however, ought always to be paid to the covering they are to receive

When the length of the rafters is $\frac{3}{4}$ of the breadth of the building, within the walls, the roof is said to be of a true pitch. In this case, the angle formed at the ridge, by the rafters, is 83° 37'; and the perpendicular height of the gable $\frac{5}{9}$ of the breadth of the building, in the inside.

Some Architects use this pitch when the covering is pantiles, and others take no more than half the breadth of the building.

When the covering is slate, some persons take $\frac{1}{4}$ of the breadth of the building for the height of the gable; and others consider this pitch too low, and take $\frac{1}{3}$ of the breadth.

When the height of the gable is equal to $\frac{1}{2}$ the breadth of the building, the angle at the ridge is 90°. when it is equal to $\frac{1}{3}$, the angle is 112° 37', and when equal to $\frac{1}{4}$, the angle is 126° 52'.

5. *Wainscotting* is measured by taking the compass of the room, as it is upon the floor, for the length, and the height from the floor to the ceiling, or as far as the wainscot extends, for the breadth; and in doing this, you must gird over the swelling panels, &c., making the tape ply close into all the mouldings.

Cornices are sometimes done by the lineal foot, and sometimes by the square foot, they, however, must always be measured separately from the wainscotting.

Chimneys, window-seats, cheek-boards, soffits, casings, architraves, &c. must be measured by themselves; and deductions ought always to be made for doors, windows, fire-places, and other openings.

6. *In measuring a door*, it is usual to take its length, measured along the straight edge, and once its thickness for the length, and its breadth, pressing the tape into all the mouldings of the panels, and once its thickness for the breadth.

If a door be panelled on both sides, it is customary, in some places, to allow double measure for workmanship; but if one side only be panelled, the area and half the area are taken for workmanship. It is much preferable, however, to take the real measurement, and regulate the price according to the thickness of the door, the number of the panels, &c

This is the method followed in London, and ought to be adopted in every other place. (See Crosby's Builder's Price Book, by Mr. John Phillips, Surveyor, page 178)

For the architrave of a door, measure round the outermost edge for the length, and take the girt of its front and two edges, for the breadth.

Architraves are sometimes executed by the lineal foot

7. *Windows* are measured by taking the distance between the under side of the sill and the upper side of the top rail, for the height, and the distance between outside and outside of the jambs, for the breadth.

Sometimes windows are made at so much per piece.

Window-shutters, and the architraves of windows are measured in the same manner as doors, and their architraves.

8. *In measuring stair-cases*, make the *tape* ply close over the steps, from the top to the bottom, for the length; and take the length of a step for the breadth Or, if the steps be all of the same size, multiply the area of the *riser* and *tread* by the number of steps.

The ends and landings must be taken by themselves

The strings are measured either by the lineal or cubic foot, the *string boards* by the square foot; and the *brackets* are generally charged at so much per piece.

For the balustrade, take the whole length of the *hand rail* for the length, and the height of the *baluster* and *hand-rail*, upon the landing, for the breadth.

Formerly there was much more carved work in balusters than there is at present and it was then customary to allow double measure, but now the true measure is taken, and the price is proportioned according to the workmanship.

Hand rails are sometimes executed by the lineal foot, and the balusters charged at so much a piece and in some places both hand-rails and balusters are computed by lineal measure. In this case they must be taken separately.

9. *Centring for vaults* is measured by taking its length for one dimension, and girting over the arch for the other; but in *groin-centring*, it is customary, in some places, to allow double measure, on account of the additional trouble. It is a more proper method, however, to proportion the price to the workmanship. (See Crosby's Builder's Price Book, page 166)

10. The following articles are generally done by lineal measure; viz. *boxing to windows, skirting boards, beads, fillets, stops, cappings, astragals, an water trunks*

Examples in Flooring.

1. The length of a floor is 45 feet 7 inches, and the bearing of the joists is 6 inches at each end; its breadth is 22 feet 5 inches, and the bearing of the girders is 9 inches at each end; required the number of squares of naked and boarded flooring.

By Cross Multiplication.

NAKED FLOORING.

	Feet.	Inches.		
	22	5		
	1	6		
	23	11		breadth.
	45	7		
	1	0		
	46	7		length.
	23	11		breadth.
	151	5		
	92	0		
	42	8	5″	
1,00)	11,14	1	5	

Ans. 11 *squares and* 14 *feet.*

BOARDED FLOORING.

	Feet	Inches.		
	45	7		length.
	22	5		breadth.
	102	10		
	90	0		
	18	11	11″	
1,00)	10,21	9	11	

Ans. 10 *squares and* 21 *feet.*

2. In a common naked floor, there are 11 girders, whose lengths are 20 feet 9 inches, and *scantlings*, (viz. breadths and depths,) 1 foot by 7 inches; 144 joists, each 8 feet 6 inches in length, and their scantlings 5½ inches by 4 inches; required the solidity of the whole. *Ans.* 320 *ft.* 1 *in.* 9 *pa.*

3. In a naked floor, the girder is 1 foot 2 inches deep, 1 foot broad, and 25 feet long; there are 10 bridging joists, whose scantlings are 5 inches by 3 inches, and lengths 24 feet; 10 binding joists, whose scantlings are 10 inches by 5½ inches, and lengths 9 feet 6 inches. The ceiling joists are 24 in number, each 5 feet 9 inches in length, and their scantlings, 3 inches by 2½ inches; what is the value of the whole, at 4*s.* 9*d.* per cubic foot? *Ans.* £23. 3*s.* 9¼*d.*

4. A house of two stories, besides the ground floor, measures 45 feet 9 inches in length, and 21 feet 6 inches in breadth. There are 6 hearths whose dimensions are, two of 6 feet by 4 feet 6 inches; two of 6 feet by 4 feet 3 inches; and two of 5 feet 9 inches by 4 feet. The well-hole measures 12 feet 8 inches by 4 feet 3 inches; required the expense of the boarded flooring, at £6. 5*s.* per square. *Ans.* £168. 5*s.*

Examples in Partitioning.

1. The length of a partition between two rooms is 25 feet 6 inches, and its height 8 feet 10 inches; how many squares does it contain? *Ans.* 2 *squares,* 25 *feet.*

2. A partition measures 36 feet 4 inches in length, and 12 feet 3 inches in height; what did it cost, at £6. 15*s.* per square? *Ans.* £30. 0*s.* 9*d.*

Examples in Roofing.

1. How many squares are in the roof of a building whose length is 86 feet 10 inches, and girt 28 feet 6 inches? *Ans.* 24 *squares* 74$\frac{5}{4}$ *feet.*

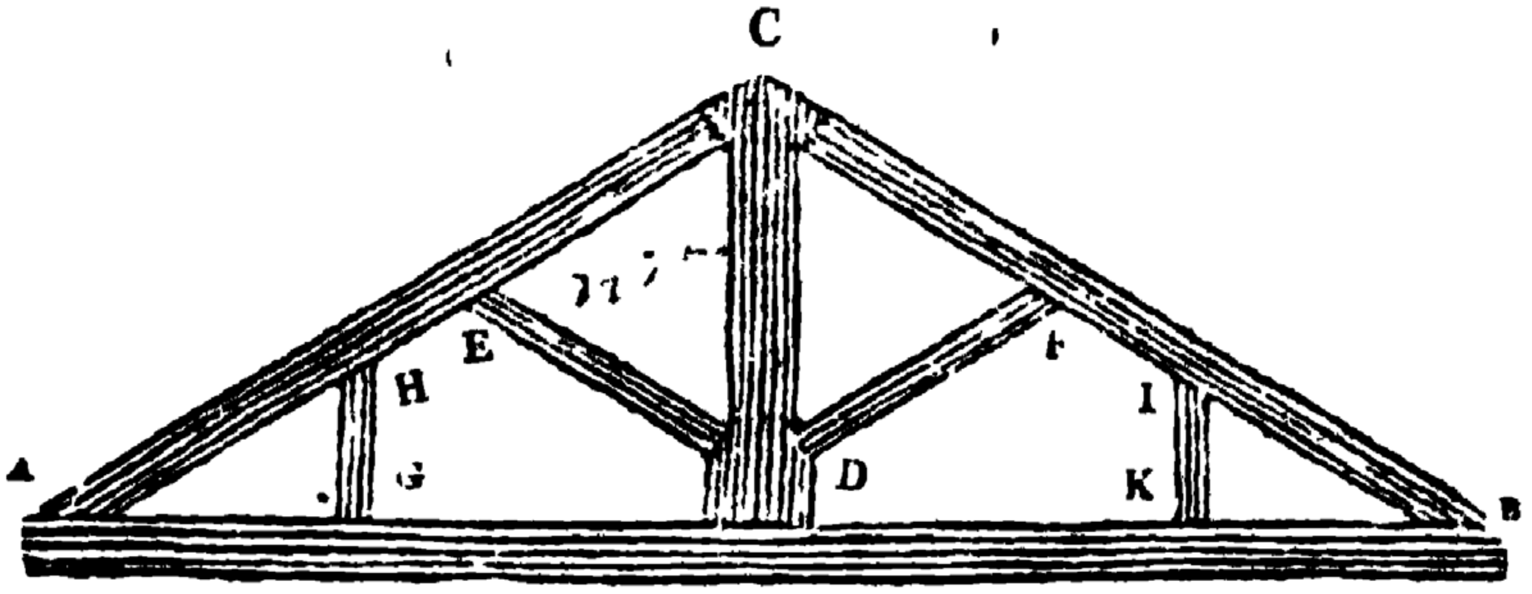
2. The length of a house, within the walls, is 32 feet 3 inches, and the purlins have 9 inches bearing at each end; the girt of the roof is 23 feet 4 inches; how many squares does it contain?

Ans 7 squares, 87½ feet.

3. The length of a hipped roof, at the eaves, is 46 feet 8 inches, and the breadth of the end 25 feet; the length of the ridge is 21 feet 8 inches, and its distance from the eaves, 18 feet 9 inches; each of the 4 hips measures 22 feet 6 inches in length; how many squares are contained in the roof; allowing 2 feet in breadth, for the hips? *Ans. 19 squares, 30 feet.*

4. Let the following figure represent a truss for the roof of a building; the tie-beam AB, is 28 feet long, 7 inches broad, and 12 inches deep; the principal rafters AC, BC, measure 16 feet 9 inches in length, 6 inches in breadth, and 10 inches in depth; the king-post CD, is 9 feet 3 inches in height, and its scantling at the bottom, 1 foot 4 inches by 6 inches; the braces DE, DF, are 5 feet 3 inches long, and their scantlings 6 inches by 4 inches; the punchins GH, KL, are 3 feet 3 inches in height, and their scantlings 6 inches by 4 inches; how many cubic feet are contained in the whole; deducting the two pieces cut out of the king-post, whose lengths are 6 feet, and breadths 3½ inches?

Ans. 37 ft. 6 in. 6''.



Note.—If the pieces, sawn out of the king post, in order to give the ends of the braces, what Joiners call a "square butment," be 2 inches or more, in breadth, and exceed 2 feet in length, they must be deducted from the solidity of the king post; but if they be of smaller dimensions than these, no deduction must be made, as the pieces cut out are considered to be of little or no value.

In measuring these pieces, the shortest lengths ought always to be taken, because the ends, in general, must be cut square, to render the pieces fit for use

Examples in Wainscotting, &c.

1. A room of wainscot, being girt downwards over the mouldings, is 12 feet 10 inches in height, and 98 feet 6 inches in compass, as upon the floor; what did it cost, at 1*s*. 3*d*. per square foot? *Ans*. £79. 0*s*. 1¼*d*.

2. A door measures 7 feet 2½ inches by 3 feet 10 inches; there are two architraves, one on each side of the door, whose lengths are 19 feet 7½ inches, and breadths 9½ inches; the lining-boards or casings round the door-way measure 17 feet 7 inches in length, and 11 inches in breadth. Now there are five doors with architraves and linings of the above dimensions, in the Court-House, in Leeds; required the content of these doors, in square feet, and also the contents of their architraves and linings.

Ans. The contents of the doors is 138 *ft.* 1 *in.* 11″; *the architraves* 155 *ft.* 4 *in.* 4″ 6‴; *and the linings* 80 *feet,* 7 *inches,* 1″.

3. The pulpit of a church measures 12 feet 6 inches by 4 feet 10 inches; the minister's reading-desk 12 feet 4 inches by 4 feet 3 inches; and the clerk's reading desk 10 feet 8 inches by 3 feet 6 inches. The divisions between the pews are 72 in number, each of which is 10 feet 6 inches in length, and 3 feet 8 inches in height; the doors and ends, taken together, measure 332 feet 3 inches in length, and 3 feet 8 inches in height; what did the whole cost, at 1*s*. 6*d*. per square foot? *Ans*. £310. 10*s*. 7½*d*.

4. A room of wainscot is 135 feet 6 inches in compass, and 14 feet 10 inches in height; the cornice girts 10 inches, and its length is equal to the compass of the room; the door is mahogany, and measures 7 feet 4 inches by 3 feet 9 inches; the length of the surrounding architrave is 20 feet 3 inches, and its girt 9 inches; the

casings round the door-way measure 17 feet 11 inches in length, and 1 foot 2 inches in breadth; there are three pair of window-shutters, each of which measures 7 feet 3 inches by 4 feet 9 inches; the soffit and cheek-boards of each window, are 21 feet 9 inches in length, and 15 inches in breadth; the fire-place measures 6 feet 3 inches by 4 feet 9 inches; what did the whole cost; the door being charged 18*s.* 6*d.* the cornice 2*s.* 3*d.* the architrave 1*s.* 9*d.* the window-shutters 1*s.* 6*d.* and all the other articles 1*s.* 2*d.* per square foot?

Ans. £161. 1*s.* 6¼*d.*

SLATERS' AND TILERS' WORK.

SLATING and TILING are generally computed either by the square yard, or by the square of 100 feet; and the length of the ridge is taken for one dimension, and the girt of the roof, from eaves to eaves, for the other dimension.

Note 1. *In measuring slating*, the tape is made to ply over the eaves, and returned up the under side till it meets the wall or eaves-board, in order to make an allowance for the double row of slate at the bottom; but in *tiling*, the distance between the extremities of the eaves is taken for the girt of the roof.

2. It is customary, in some places, instead of returning the tape to the wall or eaves-board, to add 3 feet to the girt of the roof, viz. 18 inches for each side; and when *hips* or *valleys* occur, either in slating or tiling, their length is generally added to the content of the roof; that is, an allowance of one foot in breadth, the whole length of the hips and valleys, is made for the waste of materials.

3. *Sky-lights and chimney shafts* are generally deducted, if they be large; but no deductions ought to be made for common luthern-lights, garret windows on the roof, and small chimney-shafts.

EXAMPLES.

1. How many square yards of slating are there in a roof whose length is 65 feet 10 inches, and breadth or girt 29 feet 6 inches?

By Cross Multiplication.

Feet.	*Inches.*	
65	10	
29	6	
609	2	
130	0	
32	11	
9)1942	1	
215	7	1

Ans. 215 *yards and* 7 *feet.*

2. What was the expense of covering the roof of a barn with pan-tiles, at *2l. 3s. 6d.* per square; the length being 37 feet 8 inches, and girt 25 feet 6 inches?

Ans. £20. 17s. $9\frac{3}{4}$*d.*

3. The roof of a school measures 56 feet 9 inches in length, and 32 feet 4 inches in girt: what did it cost covering with Tavistock slates, at *2l. 15s.* per square?

Ans. £50. 9s. $2\frac{1}{4}$*d.*

4. A roof measures 28 feet 4 inches in length, and 24 feet 6 inches in girt, exclusive of the allowance at the eaves, which is 18 inches on each side; what did it cost covering with Yorkshire slates, at *2s. 9d.* per square yard; deducting for two sky-lights, one of which measures 10 feet 8 inches by 6 feet 9 inches, and the other 6 feet 10 inches by 5 feet 6 inches?

Ans. £10. 4s. 7d.

PLASTERERS' WORK.

Plasterers' work is principally of two kinds; namely, plastering upon laths, called *ceiling*; and plastering upon walls or partitions made of framed timber, called *rendering*.

These different kinds must be measured separately, except they be done at the same price, which is not often the case.

The contents of ceiling and rendering are estimated either by the square yard, or by the square of 100 feet; and if cornices do not exceed 9 inches in girt, they are rated at so much per foot, lineal measure; but all above this girt are computed by the square foot.

Enriched mouldings are measured by the lineal foot.

Note 1. When Plasterers find materials, deductions must always be made for *fire-places, doors, and windows;* and the returns at the tops and sides of doors and windows must be measured separately; but for workmanship only, these deductions are generally omitted, the plastered returns being allowed to counterbalance them.

2. Deductions are never made for *cornices, enriched mouldings, festoons,* or other ornaments; because the spaces occupied by them are always plastered previously to such ornaments being made; and the length of a *cornice,* running round the top of a room, is always taken equal to the compass of the room, as upon the floor.

It is likewise customary to allow so much a piece extra, for every corner above four, in the cornice.

3 In measuring plastered timber partitions, in large warehouses, &c. where the *quarters* and *braces* project from the plastering, one-fifth part of the whole area is generally deducted; because the projecting timbers are not plastered. This work is commonly called "rendering between quarters."

4. *Whitewashing and colouring* are measured in the same manner as plastering; and in timber partitions, $\frac{1}{4}$ of the whole area is commonly added, for the sides of the quarters and braces.

EXAMPLES.

1. A ceiling measures 43 feet 10 inches in length, and 25 feet 6 inches in breadth; how many square yards does it contain?

By Cross Multiplication.

	Feet.	*Inches.*	
	43	10	
	25	6	
	235	10	
	86	0	
	21	11	
9)	1117	9	
	124	1	9

Ans. 124 *yards, and* 1¾ *foot.*

2. The compass of a room is 138 feet 8 inches, and its height 10 feet 3 inches; what will be the expense of plastering the wall with stucco, at 9*d.* per square yard, workmanship only? *Ans.* £5. 18*s.* 5¼*d.*

3. A partition which measures 85 feet 10 inches in length, and 15 feet 6 inches in height, is rendered between quarters, on both sides, and whitewashed; what did the whole cost, workmanship and materials; the lathing and plastering being charged 1*s.* 6*d.* per yard, and the whitewashing 2½*d.* per yard?

Ans. £21. 11*s.* 9*d.*

4. A room measures 32 feet 10 inches in length, 21 feet 6 inches in breadth, and 12 feet 3 inches in height; and the girt of the cornice is 11 inches. The ceiling cost 1*s.* 9*d.* per yard, the rendering 10*d.* per yard, and the cornice 1*s.* 4*d.* per foot; what was the expense of the whole? *Ans.* £19. 13*s.* 3¼*d.*

PAINTERS' WORK.

PAINTERS generally compute the contents of all large articles, such as wainscotting, doors, window-shutters, &c. by the square yard; and every part is measured upon which the colour is laid.

Cornices and enriched mouldings are estimated by the lineal foot; window-frames at so much a piece; and window-squares at so much per dozen, according to their size.

Deductions must always be made for fire-places and other openings.

Note 1. *In measuring wainscotting, doors, window-shutters, &c* painters always gird over the swelling panels, in taking both the length and breadth, and press the tape into all the mouldings.

2. *Balustrades* are generally measured by taking the length of the hand-rail, for one dimension, and twice the height of the baluster, upon the landing added to the girt of the hand-rail, for the other dimension.

3. *For lattice-work*, double the area of one side is generally taken for the measurement of both sides, and the area and half the area of one side, for

palisading; but no general rules can be given for these works, as they vary so much in the distance of their perpendiculars and horizontal parts.

4. Painters proportion their prices to the nature of the colouring, the number of coats the work receives, &c &c.

EXAMPLES.

1. If a room be painted, whose height is 15 feet 6 inches, and compass 98 feet 9 inches; how many yards does it contain?

By Decimals.

Feet
98.75
15.5
49375
49375
9875
9)1530.625
170.069 *Ans.*

By Cross Multiplication.

Feet.	*Inches.*		
98	9		
15	6		
501	3		
98	0		
49	4	6″	
9)1530	7	6	
170	0	7	6

Ans. 170 *yards.*

2. A door measures 7 feet 4½ inches by 3 feet 8⅝ inches; what did 4 such doors cost painting on both sides, with 3 coats, at 10*d.* per yard?

Ans. £1. 0*s.* 4¼*d.*

3. The length of the hand-rail of a stair-case is 21 feet 9 inches, and the girt of the hand-rail, and twice

the height of the baluster, upon the landing, 7 feet 10 inches; how many yards of painting are contained in the balustrade? *Ans.* 18 *yards*, 8 *feet*, 4½ *in.*

4. The lattice window of a dairy measures 4 feet 10 inches by 3 feet 6 inches; how many square yards of painting does it contain, taking the area of both sides? *Ans.* 3 *yards*, 6 *ft.* 10 *in.*

5. The compass of a wainscotted room is 118 feet 6 inches, and its height, to the under side of the cornice, 12 feet 8 inches; the door measures 7 feet 2 inches by 3 feet 6 inches; the lining round the door-way is 17 feet 4 inches long, and 9 inches broad; there are four pair of window-shutters, each of which measures 6 feet 10 inches by 4 feet 6 inches; the soffit and cheek-boards of each window, are 19 feet 6 inches in length, and 14 inches in breadth; the fire-place, which is to be deducted, measures 6 feet 6 inches by 5 feet 2 inches; the door and window-shutters are painted on both sides; required the expense of giving the whole 4 coats; the cornice, which is 117 feet in length, being charged 8*d.* per lineal foot, and all the other articles 1*s.* 10*d.* per square yard.

Ans. £21. 8*s.* 3*d.*

GLAZIERS' WORK.

Glaziers take their dimensions either in feet, inches, and parts, or in feet, tenths, and hundredths; and estimate their work by the square foot.

Note 1. The most general method of measuring a window is by taking the length and breadth, without making any deduction for the cross-bars between the panes, sometimes, however, the measurement of a window is found by multiplying the area of one pane by the number of panes.

2 If windows be circular or elliptical, they must be measured as if they were squares or rectangles; the greatest lengths and breadths always being taken, in order to make a compensation for the waste of glass in cutting the panes into proper shapes.

EXAMPLES.

1. How many feet of glass are contained in a window which measures 7 feet 10¾ inches by 4 feet 7½ inches?

By Cross Multiplication.

Ft.	*In.*	*Pa.*		
7	10	9		
4	7	6		
31	7	0		
4	7	3	3'''	
0	3	11	4	6''''
36	6	2	7	6 *Ans.*

2. A pane of plate glass measures 2 feet 8 inches by 1 foot 6 inches; what did 8 such panes cost, at 14*s.* 6*d.* per square foot? *Ans.* £23. 4*s.*

3. The diameter of a circular window is 2 feet 9½ inches; for how many feet must the glazier be paid, taking the window as a square? *Ans.* 7 *ft.* 9 *in.* 6¼ *pa.*

4. The base of a triangular sky-light measures 10 feet 6 inches, and the perpendicular 5 feet 8 inches; what did it cost glazing, at 1s. 10*d.* per square foot?
Ans. £2. 14*s.* 6½*d.*

5. There is a house with three tiers of windows, four in a tier; the height of the first tier is 7 feet 2 inches, of the second 6 feet 4 inches, and of the third 5 feet 8 inches; and the breadth of each window is 4 feet 6 inches. The height of a semi-circular window, above the door, is 2 feet; what did the whole cost glazing, at 2*s.* 6*d.* per square foot?
Ans. £14. 2*s.* 6*d.*

PLUMBERS' WORK.

Plumbers' work is generally done at so much per pound, or else by the hundred weight of 112 pounds; and the price is regulated according to the value of the lead at the time when the work is performed.

Note. Sheet lead, used in roofing, guttering, &c commonly weighs from 7 to 12 pounds per square foot; and leaden pipe varies in weight, per yard, according to the diameter of its bore in inches.

The first of the following Tables shews the weight of a square foot of sheet lead, in pounds, to each of the subjoined thicknesses, in tenths and hundredths, of an inch; and the second exhibits the general weight of a yard of leaden pipe, according to the diameter of its bore

TABLE I.

Thickness OF *Sheet Lead.*	*Pounds* TO A *Square Foot.*	*Thickness* OF *Sheet Lead.*	*Pounds* TO A *Square Foot.*
$\frac{1}{10}$	5.899	.15	8.848
.11	6.489	.16	9.438
$\frac{1}{9}$	6.554	$\frac{1}{6}$	9.831
.12	7.078	.17	10.028
$\frac{1}{8}$	7.373	.18	10.618
.13	7.668	.19	11.207
.14	8.258	$\frac{1}{5}$	11.797
$\frac{1}{7}$	8.427	.21	12.387

TABLE II.

Bore of LEADEN PIPE *in Inches.*	*Pounds* PER YARD.
$\frac{3}{4}$	10
1	12
$1\frac{1}{4}$	16
$1\frac{1}{2}$	18
$1\frac{3}{4}$	21
2	24

EXAMPLES.

1. A sheet of lead measures 18 feet 10 inches in length, and 5 feet 6 inches in breadth; required its weight, at 9½lb. to a square foot.

By Cross Multiplication.

Ft.	*In.*	
18	10	
5	6	
94	2	
9	5	
103	7	*content.*

As 1 *ft.* : 9 *lb.* 8 *oz.* :: 103 *ft.* 7 *in.* : 8 *cwt.* 3 *qu.* 4 *lb.* 0 *oz.* 10 *dr. the weight required.*

2. What is the weight of a sheet of lead, whose length is 15 feet 10 inches, breadth 4 feet 6 inches, and thickness ⅕ or .2 of an inch? *Ans.* 840.53625 *pounds.*

3. If I buy 150 yards of leaden pipe, whose bore is 1¾ inch; what will it cost me at 3¾*d.* per pound, admitting each yard to weigh 21 pounds?

Ans. £49. 4*s.* 4½*d.*

4. What cost the covering and guttering of the roof of a church with lead, at £1. 18*s.* per cwt.; the length of the roof being 82 feet 9 inches, and its girt 65 feet 3 inches; the length of the guttering 165 feet 6 inches, and its breadth 1 foot 9 inches; admitting the thickness of the lead to be ⅙ of an inch?

Ans. £948. 15*s.* 11½*d.*

PAVERS' WORK

PAVERS estimate their work by the square yard; and such dimensions must always be taken as will give the true area.

EXAMPLES.

1. A rectangle measures 85 feet 9 inches in length, and 43 feet 6 inches in breadth; how many square yards of paving does it contain?

By Decimals.

Feet.
85.75 length.
43.5 breadth.
42875
25725
34300
9)3730.125
414.4583 *yards, the answer.*

By Cross Multiplication.

Feet Inches.
85 9 length.
43 6 breadth.
287 3
340 0
42 10 6″
9)3730 1 6
414 *yds.* 4 *ft. Ans.*

2. The base of a triangle measures 76 feet 8 inches, and the perpendicular 42 feet 3 inches; what did it cost paving with Aberdeen granite, at 10*s.* 6*d.* per square yard? *Ans.* £94. 9*s.* 6*d.*

3. If the parallel sides of a trapezoid be 68 feet 7 inches, and 45 feet 3 inches, and their perpendicular distance 98 feet 6 inches; what will it cost paving with Guernsey pebbles, at 6*s.* 6*d.* per square yard?
Ans. £202. 8*s.* 11¾*d.*

4. How many yards of paving does the trapezium contain, whose diagonal measures 136 feet 8 inches,

and perpendiculars, 68 feet 2 inches, and 56 feet 4 inches? *Ans.* 945 *yds.* 2½ *ft.*

5. The length of a street is 538 feet 6 inches, and its breadth 65 feet 8 inches; what did it cost paving with Purbeck-stone, at 5*s.* 6*d.* per square yard? *Ans.* £1080. 9*s.* 9½*d.*

6. A rectangular court-yard measures 96 feet 9 inches in length, and 74 feet 6 inches in breadth. Across the middle and round the extremities of the yard, is a foot-way, 5 feet 3 inches broad; and paved with Guernsey granite, at 9*s.* 6*d.* per square yard. The rest is paved with Jersey pebbles, at 5*s.* 9*d.* per square yard; required the expense of the whole. *Ans.* £272. 8*s.* 3¾*d.*

VAULTED AND ARCHED ROOFS.

ARCHED *Roofs* are either *vaults, domes, saloons,* or *groins.*

Vaulted roofs are formed by arches springing from the opposite walls, and meeting in a line at the top.

Domes are made by arches springing from a circular or polygonal base, and meeting in a point at the top.

Saloons are formed by arches connecting the side walls to a flat roof, or ceiling, in the middle.

Groins are formed by the intersection of vaults with each other.

Vaulted roofs are commonly of the three following sorts:

1. *Circular roofs,* are those whose arch is some part of the circumference of a circle.

2. *Elliptical or oval roofs,* or those whose arch is an oval, or some part of the circumference of an ellipsis.

3. *Gothic roofs,* or those which are formed by two circular arcs, struck from different centres, and meeting in a point over the middle of the breadth or span of the arch.

Note. Domes and saloons are of various figures; they, however, seldom occur in the practice of measuring; but most cellars are covered either with vaults or groins.

PROBLEM I.

To find the content of the vacuity of a circular, an elliptic, or a gothic vaulted roof.

RULE.

Multiply the area of one end by the length of the roof or vault, and the product will be the content required.

Note 1. If the arch be the segment of a circle, the area of the end may be found by Problem 17, Part II.; if it be elliptical, multiply the span by the height, and the product by .7854, for the area of the end; but if it be a Gothic arch, the area of the end must be obtained by finding the areas of the two circular segments and the triangle of which the end is composed.

2. The upper sides of all arches, whether vaults or groins, are built up solid, above the haunches, to the same height as the crown of the arch

3. The solidity of the materials in any arched roof, may be found thus: find the content of the whole, considered as solid, from the spring of the arch to the upper side of the crown; find also the content of the vacuity, then the difference of these two contents will be the solidity required.

4 The whole arch, considered as a solid, will be a parallelopipedon, the content of which may be found by Problem 2, Part IV.

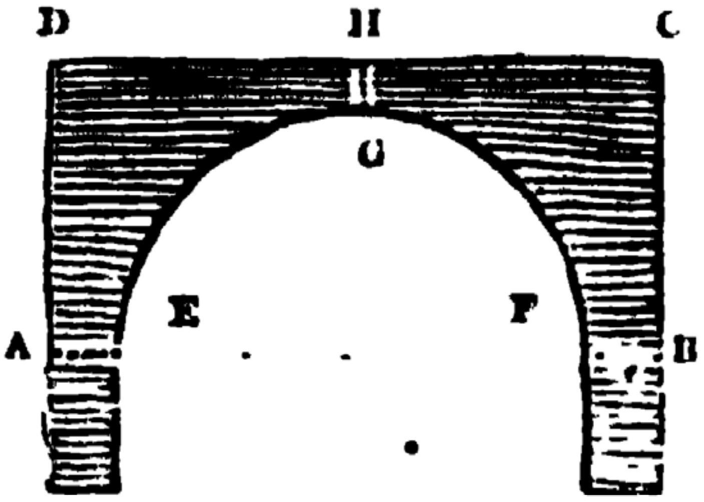
EXAMPLES.

1. Required the content of the vacuity of a semi-circular vault, the span or diameter of which is 20 feet, and its length 60 feet.

.7854
400 = the square of 20.
2)314.1600
157.08 the area of the end.
60 the length.
9424.80 *Ans.*

2. The span of an elliptical vault is 30 feet, its height 10 feet, and its length 50 feet; what is the content of the vacuity? *Ans.* 11781 *feet.*

3. Required the content of the vacuity of a Gothic vault, whose span is 30 feet, the chord of each arch 32 feet, the versed sine, or distance of each arch from the middle of these chords, 8 feet, and the length of the vault 35 feet 6 inches. *Ans.* 27755.0502 *feet.*



4. Let ABCD denote the end or upright section of a semi-circular roof. The span EF is 18 feet, the thickness of the wall AE or FB, at the spring of the arch, 3 feet, the thickness GH, at the crown of the arch, 2 feet, and the length of the vault 56 feet 9 inches; how many solid feet are contained in the roof?

Ans. 7761.425 *feet.*

PROBLEM II.

To find the concave or convex surface of a circular, an elliptic, or a gothic vaulted roof.

RULE.

Multiply the length of the arch by the length of the vault, and the product will be the superficies required.

Note. The convex length of an arch may be easily found by making a line ply close over it, but for the concave length, this method is not quite so applicable, for if care be not taken, the dimension will be made too short.

If the arch be the segment of a circle, its true length may be found by Problem 14, Part II.

EXAMPLES.

1. The span of a semi-circular vault is 30 feet, and its length 40 feet; what is its concave surface?

$$\begin{array}{r l}
3.1416 & \\
30 & \\
\hline
2)\overline{94.2480} & \\
\hline
47.124 & = \text{length of the arch.} \\
40 & \\
\hline
\textit{Ans. } 1884.960 & \textit{square feet.} \\
\hline\hline
\end{array}$$

2. The length of a vault is 62 feet 9 inches, and that of the arch 54 feet 6 inches; how many square yards are contained in the roof? *Ans.* 379.986 *yards.*

3. Required the concave surface of a bridge consisting of 5 circular arches; the span of each arch being 96 feet, the height, above the top of the piers, 36 feet, and the length 45 feet. *Ans.* 28955.232 *feet.*

Note Those who desire to make themselves acquainted with the essential properties, dimensions, proportions, and other relations of the various parts of a bridge, are referred to Dr. Hutton's Principles of Bridges

In this valuable little work, the learned Doctor proves, that the equilibrial arch, described in Prob V., is the most proper for a bridge of several arches Next to it, the elliptical arch claims the preference, after it the cycloidal arch, and lastly the arch of a circle

As for parabolic, hyperbolic, and catanarian arches, they ought never to be admitted into a bridge consisting of several arches, but may, in some cases be used for a bridge of one arch, which is to rise an unusual height.

PROBLEM III.

To find the solid content of a dome; its height, and the dimensions of its base being given.

RULE.

Multiply the area of the base by the height, and $\frac{2}{3}$ of the product will be the solidity.

EXAMPLES.

1. What is the solid content of a hemispherical dome; the diameter of the base being 40 feet?

.7854
1600 = square of 40.
4712400
7854
1256.6400 = area of the base.
20 = height.
25132.8000
2
3)50265.6000
16755.2000 = solidity.

2. Required the solidity of an octagonal dome; each side of the base being 20 feet, and the height 21 feet.
Ans. 27039.19176 *feet.*

PROBLEM IV.

To find the superficial content of a spherical dome.

RULE.

Multiply the area of the base by 2, and the product will be the superficies required.

Note. If the dome be elliptical, the product of the two diameters multiplied by 1.5708, will give the superficial content, sufficiently near for practical purposes.

EXAMPLES.

1. Required the superficies of a hexagonal spherical dome; each side of the base being 20 feet.

2.5980762 = tabular area, Prob. 12, Part II.
400 = square of 20.
1039.2304800 = area of the base.
2
2078.4609600 = superficies required.

2. What will an octagonal spherical dome cost painting, at 1*s.* 3*d.* per yard; each side of the base being 10 feet?
Ans. £6. 14*s.* 1¼*d.*

PROBLEM V.

To find the solid content or vacuity of a saloon

RULE.

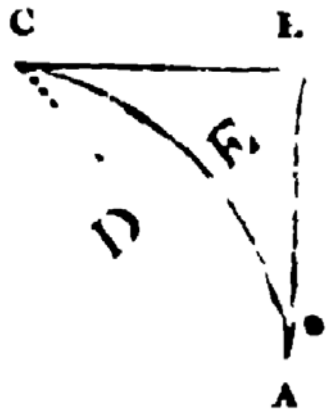
Multiply the area of a transverse section by the mean compass of the solid part of the saloon, subtract this product from the whole vacuity of the room, supposing the walls to go upright, all the height, to the flat ceiling, and the difference will be the answer.

Note 1. If the base of the saloon be a rectangle, the vacuity of the room, or the whole upright space, will be a parallelopipedon, the content of which may be found by Prob. 2, if the base be a regular polygon, the vacuity will be a prism, the content of which may be found by Prob. 3, and if the base be a circle, the vacuity will be a cylinder, the content of which may be obtained by Problem 4, Part IV.

2. For *practical purposes*, the mean compass of the solid part may be found by adding the compass taken at the middle of the arch to the compass of the room, taken within the walls, and dividing the sum by 2, but in solving the second and third examples, the mean compass of the solid part was found with mathematical accuracy.

EXAMPLES.

1. If the height AB of a saloon be 3.2 feet, the chord ADC of its face 4.5 feet, and the distance DE of its middle part from the arch be 9 inches; required the solidity, supposing the mean compass of the saloon to be 50 feet.



By Rule 2, Problem 17, Part II., we have $\overline{4.5 \times .75} \times \frac{2}{3} + \frac{.75^3}{4.5 \times 2} = 3.375 \times \frac{2}{3} \times \frac{.421875}{9} = 2.25 + .046875 = 2.296875$, *the area of the segment* ADCEA.

Again, by Problem 6, Part II., we have $AC^2 - AB^2 = 4.5^2 - 3.2^2 = 20.25 - 10.24 = 10.01$; *and* $\sqrt{10.01} = 3.163858 = BC$, *then* $BC \times \frac{1}{2} AB = 3.163858 \times 1.6 = 5.0621728$, *the area of the triangle* ARC; *consequently* $5.0621728 - 2.296875 = 2.7652978$, *the area of the transverse section,* AECBA.

Now, 2.7652978 × 50 = 138.26489 feet, the content of the solid part, which being taken from the whole upright space, will leave the content of the vacuity within the room.

2. What is the solid content of a saloon with a circular quadrantal arch of 2 feet radius, springing over a rectangular room of 20 feet long, and 16 feet broad?

Ans. 580 2065 cubic feet.

3. A circular building of 40 feet diameter, and 25 feet high to the ceiling, is covered with a saloon, the circular quadrantal arch of which is 5 feet radius; required the capacity of the room in cubic feet?

Ans. 30766.496 cubic feet.

PROBLEM VI.

To find the superficial content of a saloon

RULE.

Find its breadth by applying a line close to it across the surface; and its length by measuring along the middle of it, quite round the room; then the product of these two dimensions will be the surface required.

Note The area of the flat ceiling must be added to the area found by the above Rule, in order to obtain the *whole* surface of the saloon.

EXAMPLES.

1. The girt across the face of a saloon is 5 feet 3 inches, and its mean compass 94 feet 6 inches; what is the area of its surface?

Here 94.5 × 5.25 = 496.125 feet, the area required.

2. The mean compass of a saloon is 126 feet 10 inches, and the girt across its face 8 feet 6 inches; what is the area of its surface? *Ans.* 1078 *ft.* 1 *in.*

PROBLEM VII.

To find the solid content of the vacuity formed by a groin arch, either circular or elliptical.

RULE.

Multiply the area of the base by the height, and the product by .904, and it will give the solidity required.

Note Groins are sometimes measured as if they were solid, in consideration of the great trouble and waste of materials in forming the arches and intersections.

EXAMPLES.

1. What is the content of the vacuity formed by a circular groin, springing from the sides of a square base, each side of which is 14 feet?

14
14
56
14
196 = area of the base.
7 = height, or radius.
1372
.904
.5488
12348
1240.288 = solidity required.

2. What is the solid content of the vacuity formed by an elliptical groin; the side of its square base being 24 feet 6 inches, and its height 8 feet 3 inches?

Ans. 4476.6645 *feet.*

PROBLEM VIII.

To find the concave surface of a circular or an elliptical groin.

RULE.

Multiply the area of the base by 1.1416, and the product will be the superficies required.

Note. In measuring works where there are many groins in a range, the cylindrical pieces between the groins, and on their sides, must be taken separately.

EXAMPLES.

1. What is the concave surface of a circular groin arch; the side of its square base being 15 feet 6 inches?

Here 15.5 × 15.5 = 240.25, *area of the base; and* 240.25 × 1.1416 = 274.2694 *feet, the answer required.*

2. The base of a groin is a rectangle whose sides are 20 and 26 feet; required the concave surface of the arch. *Ans.* 593.632 *feet.*

GENERAL ILLUSTRATION.

Having gone through the Works of Artificers, and noted the methods of measuring buildings, and computing their contents; I now proceed to give a general illustration of the whole, by assigning the dimensions of a house, and from thence computing the contents of the works of the different Artificers employed in building it.

In performing this task, are shewn the methods of ruling the book, entering the dimensions, with the contents; then the method of abstracting the contents, and lastly, of forming the bills of expenses of the work and materials.

The building of which I have made choice, for the general illustration, consists of two stories, beside the cellars, and of two rooms upon each floor; which will be found quite sufficient to exemplify the methods of measuring the works of Artificers.

The whole length of the building, on the outside, is 53 feet 6 inches, and its breadth 24 feet. The walls of the cellars are 6 half bricks, or 24 inches in thickness, those of the lower story 20 inches; and those of the upper story 16 inches.

One of the lower rooms is considered to be the kitchen, and the other the parlour; and they are frequently distinguished by these denominations, in the following notes.

The plans of the different stories, and the elevation, could not be given without a folding plate; but it is presumed that the reader will find no difficulty in comprehending the dimensions without them.

Note The columns of numbers, in the following forms, are sufficiently explained by the titles at the tops of them; excepting the figures 2, 3, &c in the first column, which figures signify that there are more than one article of the same dimensions, consequently the contents arising from the dimensions to which these figures are prefixed, must be multiplied by 2, 3, &c and the products entered in the column of contents

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THE BRICKLAYER'S WORK.

	Dimensions		*Half Bricks thick*	CONTENTS.		TITLES.
	Ft	*In*		*Ft*	*In.*	
	147 10	0 6	6	1543	6	The cellar walls.
2	20 10	0 6	6	420	0	Middle walls of ditto.
2	20 19	0 0	5	760	0	Common arches over the two cellars.
	20 10	0 0	5	200	0	Ditto under the passage.
	147 13	0 0	5	1911	0	Outer walls of the ground story.
	147 12	0 6	4	1837	6	Ditto of the upper story.
	24 6	0 9	3	162	0	Gable-ends of the outer walls.
2	21 25	0 6	3	1071	0	Partition walls.
	21 6	0 9	3	141	9	Gable-ends of ditto.

	Dimensions.		*Half Bricks thick*	CONTENTS.		TITLES.
	Ft	*In.*		*Ft*	*In.*	
2	13 9	0 6	5	247	0	Chimney shafts of the ground story.
2	12 8	6 2	4	204	2	Ditto of the upper story.
2	6 6	9 8	3	90	0	Ditto of the gable-ends
2	6 3	10 6	6	47	10	*The deductions are as follow, viz.* Cellar doors.
	8 5	0 6	5	44	0	Front door.
	7 5	0 0	5	35	0	Back door.
2	6 5	6 0	[illegible]	65	0	Windows of the lower rooms.
3	5 4	8 6	4	76	6	Ditto of the upper rooms.
	6 5	10 0	4	34	2	Stair-case window.
4	6 3	8 6	3	93	4	Doors in partition walls.

In order to abstract the foregoing contents; that is, to collect them into one sum, make the deductions, and reduce the neat contents to the standard thickness of one brick and a half, proceed thus: Make only two columns for the whole contents, and two for the deductions of the same thickness; viz. one column for the contents that are one brick in thickness, and the other for the contents that are one brick and a half in thickness; and dispose of the superior denominations in one or both of these columns, by entering them more than once; thus, the contents that are 2 bricks in thickness must be set down twice in the one-brick column; those that are 2½ bricks in thickness, once in the one-brick column, and once in the brick-and-half column; and those contents whose thickness is 3 bricks, must be entered twice in the brick-and-half column; then add up all the columns, and reduce the sums in the one-brick columns to the standard thickness of one brick and a half, which add to the respective sums in the brick-and-half columns. Lastly, take one of the sums thus obtained, from the other, and the remainder will be the whole reduced content of the brick-work.

Abstract of the Brick-Work.

CONTENTS. 1½ *brick thk.*		CONTENTS. 1 *brick thick.*		DEDUCTIONS 1½ *brick thk*		DEDUCTIONS 1 *brick thick*	
Ft	*In*	*Ft*	*In*	*Ft.*	*In*	*Ft*	*In*
1543	6	760	0	47	10	44	0
1543	6	200	0	47	10	35	0
420	0	1911	0	44	0	65	0
420	0	1837	6	35	0	76	6
760	0	1837	6	65	0	76	6
200	0	247	0	93	4	34	2
1911	0	204	2			34	2
162	0	204	2	333	0		
1071	0			243	6	365	4
141	9	7201	4				2
247	0		2	576	6		
90	0					3)730	8
		3)14402	8				
8509	9					243	6
4800	10	4800	10				
13310	7						
576	6						
12734	1						

Then 12734 feet being divided by 272, we obtain 46 rods 222 feet of brick-and-half wall, for workmanship and materials.

Next find the contents of the corners or coins of the different stories of the house, to which add the deductions for doors and windows; and you will obtain the quantity to be charged for workmanship only.

Cellar walls.

Feet	*In.*	
2	0	thickness.
	4	multiply.
8	0	breadth.
10	6	height.
80	0	
4	0	
84	0	content.
	6	half bricks thick.
3)504	0	
168	0	reduced content.

Walls of the ground story.

Feet.	*In*	
1	8	thickness.
	4	multiply.
6	8	breadth.
13	0	height.
78	0	
8	8	
86	8	content.
	5	half bricks thick.
3)433	4	
144	5	reduced content.

Walls of the upper story.

Feet.	In	
1	4	thickness.
	4	multiply.
5	4	breadth.
12	6	height.
64	0	
2	8	
66	8	content.
	4	half bricks thick.
3)266	8	
88	10	reduced content.
168	0	ditto.
144	5	ditto.
401	3	total of the coins.
576	6	deductions.
272)977	9(3	rods 161 feet for workmanship only.
816		
161	rem.	

The Bricklayer's Bill.

Rods	Feet		L. s. d.
46	222	of brick-and-half wall, workmanship and materials, at —— per rod,	- - -
3	161	of ditto, for workmanship only, at —— per rod, ...	- - -
		£.	- - -

THE MASON'S WORK.

	Dimensions.		CONTENTS.		TITLES.
	Ft	*In*	*Ft*	*In.*	
	155 2	0 3	348	9	Stone base or plinth.
	53 0	6 9	40	1	Facia to the front of the house.
	14 2	6 0	29	0	Jambs of the front door.
	5 2	6 0	11	0	Head of ditto.
	5 1	6 3	6	10	Frize.
	6 1	8 6	10	0	Cornice.
	6 2	8 3	15	0	Threshold.
4	5 1	6 8	36	8	Steps to the front door.
2	8 2	0 4	37	4	Ends of ditto.
	7 3	6 6	26	3	Landing of ditto.
2	10 1	8 6	32	0	Front window jambs.

	Dimensions.		CONTENTS.		TITLES.
	Ft.	*In.*	*Ft.*	*In.*	
2	5	6	18	4	Soles of ditto.
	1	8			
2	5	0	13	4	Heads of ditto.
	1	4			
3	9	4	35	0	Upper window jambs.
	1	3			
3	5	0	22	6	Soles of ditto.
	1	6			
3	4	6	13	6	Heads of ditto.
	1	0			
	18	0	30	0	Back door jambs and head.
	1	8			
	5	6	8	3	Threshold of ditto.
	1	6			
	11	8	14	7	Stair-case window jambs.
	1	3			
	5	6	7	4	Sole of ditto.
	1	4			
	5	0	5	10	Head of ditto.
	1	2			
2	8	6	28	4	Jambs to the fire-places in the lower rooms.
	1	8			
2	5	6	16	6	Mantles to ditto.
	1	6			

	Dimensions		CONTENTS.		TITLES.
	Ft.	*In.*	*Ft.*	*In*	
2	5 2	6 4	25	8	Slabs or hearths to ditto.
4	4 1	3 4	22	8	Coves to ditto.
4	2 1	9 8	18	4	Back cover to ditto.
2	5 1	0 2	11	8	Chimney-pieces or cornices to ditto.
2	8 1	0 4	21	4	Jambs to the fire-places in the upper rooms.
2	4 1	6 4	12	0	Mantles to ditto.
2	4 1	0 0	8	0	Chimney-pieces or cornices to ditto.
2	4 2	6 0	18	0	Slabs to ditto.
4	4 1	0 2	18	8	Coves to ditto.
4	2 1	3 4	12	0	Back coves to ditto.
2	14 3	6 4	96	8	Chimney-tops of stone.
2	15 1	10 6	47	6	Bases and facias to ditto.

	Dimensions.		CONTENTS.		TITLES.
	Ft	In.	Ft.	In.	
	58 1	0 10	106	4	Cornice to the front of the house.
	55 1	6 6	83	3	Blocking course to ditto.
4	10 2	3 0	82	0	Tabling.
15	3 1	6 6	78	9	Steps down to the cellars.
	3 2	6 6	8	9	Landing of ditto.
2	20 18	0 0	720	0	Tooled flagging in the cellars.
	20 9	0 6	190	0	Ditto between the cellars.
	10 3	6 6	36	9	Well-hole of the cellar-stairs to be deducted.
	20 18	8 8	385	9	Polished flagging in the kitchen.
	20 9	8 6	196	4	Ditto in the passage.
	5 4	6 4	23	10	Kitchen fire-place to be deducted.

Abstract of the Masonry.

In making this abstract, all the articles of the same price must be collected together: thus, the *polished* work must be brought into one sum; the *tooled* work into another; the *cornices* into another, &c. &c.

Polished Work.

Feet.	Inches	
348	9	stone base or plinth.
40	1	facia to the front.
29	0	jambs to the front door.
11	0	head of ditto.
6	10	frize.
15	0	threshold.
36	8	steps to the front door.
37	4	ends of ditto.
26	3	landing of ditto.
32	0	front window jambs.
18	4	soles of ditto.
13	4	heads of ditto.
35	0	upper window jambs.
22	6	soles of ditto.
13	6	heads of ditto.
28	4	jambs to the fire-places in the lower rooms.
16	6	mantles to ditto
21	4	jambs to the fire-places in the upper rooms.
12	0	mantles to ditto.
96	8	chimney tops.
47	6	bases and facias to ditto.
83	3	blocking course to the front.
82	0	tabling.
1073	2	sum.

Tooled Work.

Feet.	Inches.	
30	0	back door jambs and head.
8	3	threshold of ditto.
14	7	stair-case window jambs.
7	4	sole of ditto.
5	10	head of ditto.
78	9	steps down to the cellars.
8	9	landing of ditto.
153	6	sum.

Cornices.

Feet.	Inches.	
10	0	cornice of front door.
11	8	chimney-pieces in the low rooms.
8	0	ditto in the upper rooms.
106	4	cornice to the front of the house
136	0	sum.

Slabs and Coves.

Feet.	Inches	
25	8	slabs of the low room fire-places.
22	8	coves of ditto.
18	4	back coves of ditto.
18	0	slabs of the upper room fire-places.
18	8	coves of ditto.
12	0	back coves of ditto.
115	4	sum.

Tooled Flagging.

Feet.	Inches	
720	0	flagging in the cellars.
190	0	ditto between the cellars.
910	0	sum.
36	9	deduction.
9)873	3	
97		square yards.

Polished Flagging.

Feet.	*Inches*	
385	9	flagging in the kitchen.
196	4	ditto in the passage.
582	1	sum.
23	10	deduction.
9)558	3	
62	0	square yards.

The Mason's Bill.

Ft.	*In.*			£ s d.
1073	2	of polished work,...... at —— per foot,		- - -
153	6	of tooled ditto, ... at —— per foot,		- - -
136	0	of cornices, ... at —— per foot,		- - -
115	4	of slabs and coves, ... at —— per foot,		- - -
97		yards of tooled flagging, at —— per yard,		- - -
62		ditto of polished ditto, ... at —— per yard,		- - -
			£.	- - -

THE CARPENTER'S AND JOINER'S WORK

	Dimensions.		CONTENTS.		TITLES.
	Ft	*In.*	*Ft*	*In.*	
	22 19	8 8	445	9	Naked flooring of the parlour; viz. girders, and binding and bridging joists
	20 18	8 8	385	9	Boarded flooring of ditto.
	5 4	6 4	23	10	Hearth to be deducted from the boarded flooring.
2	23 20	4 4	948	10	Naked flooring of the upper rooms.
2	21 19	4 4	824	10	Boarded flooring of ditto.

	Dimensions.		CONTENTS.		TITLES.
	Ft.	*In.*	*Ft.*	*In.*	
2	4	6	34	6	Hearths to be deducted from the boarded flooring.
	3	10			
	11	6	90	1	Naked flooring of the stair-case.
	7	10			
	9	6	74	5	Boarded flooring of ditto.
	7	10			
10	4	0	60	0	Deal steps to stair-case, first flight.
	1	6			
10	0	10	4	10	End of ditto.
	0	7			
	9	6	38	0	Foot-pace.
	4	0			
	9	6	5	6	Face of ditto.
	0	7			
10	4	0	60	0	Steps of the second flight.
	1	6			
10	0	10	4	10	Ends of ditto.
	0	7			
	26	6	92	9	Balustrade of the stair-case.
	3	6			
	39	6	26	4	String-board of ditto.
	0	8			
2	23	4	948	10	Ceiling-joists over the upper rooms.
	20	4			

Dimensions.		CONTENTS.		TITLES.
Ft.	In	Ft.	In.	
21 10	4 6	224	0	Ceiling-joists over the stair-case.
52 27	9 4	1441	10	Roofing.
7 4	6 8	35	0	Front door, 6 panelled.
21 0	6 8	14	4	Casings to ditto.
24 0	6 10	20	5	Architrave to ditto.
6 4	9 6	30	4	Back door, 4 panelled.
19 0	0 7	11	1	Casings to ditto.
6 3	0 6	21	0	Cellar door, 4 panelled.
6 3	6 2	20	7	Kitchen door, 6 panelled.
16 0	10 9	12	7	Casings to ditto.
18 0	10 8	12	6	Architrave to ditto.
6 3	6 2	20	7	Parlour door, 6 panelled.

	Dimensions.		CONTENTS.		TITLES.
	Ft	*In*	*Ft*	*In.*	
	16	10	12	7	Casings to parlour door.
	0	9			
	18	10	25	1	Architraves to ditto.
	0	8			
	6	6	41	2	Upper room doors, 6 panelled.
	3	2.			
	16	10	25	3	Casings to ditto.
	0	9			
	18	10	50	2	Architraves to ditto.
4	0	8			
	5	10	50	6	Window shutters to lower windows.
2	4	4			
	23	0	46	0	Casings to ditto, viz. cheek-boards, window-boards, and soffits.
2	1	0			
	27	0	15	9	Architrave to parlour window.
	0	7			
	23	8	23	8	Casings to stair-case window.
	1	0			
	27	8	16	1	Architrave to ditto.
	0	7			
	20	4	61	0	Casings to upper windows.
	1	0			
	24	4	42	7	Architraves to ditto.
	0	7			

	Dimensions.		CONTENTS.		
	Ft.	In.	Ft	In	
2	5 4	8 2	47	2	Sashes of the lower windows.
	6 4	2 4	26	8	Ditto of the stair-case window.
3	5 3	0 10	57	6	Ditto of the upper windows.

Abstract of the Carpenter's and Joiner's Work.

This abstract must be made in the same manner as that of the Masonry; viz. by collecting all the articles of the same price into one sum, making the proper deductions, &c. &c.

Naked Flooring.

	Feet	Inches	
	445	9	parlour floor.
	948	10	upper floors.
	90	1	stair-case.
1,00)	14,84	8	sum.

14 squares, 84 feet, 8 inches.

Boarded Flooring.

	Feet	Inches	
	385	9	parlour floor.
	824	10	upper floors.
	74	5	stair-case.
	1285	0	sum.
	58	4	hearths to be deducted.
1,00)	12,26	8	

12 squares, 26 feet, 8 inches.

Ceiling Joists.

Feet.	*Inches*	
948	10	upper rooms.
224	0	stair-case.
1,00)11,72	10	sum.

11 squares, 72 feet, 10 inches.

Stairs Steps.

Feet.	*Inches*	
60	0	first flight.
4	10	ends of ditto.
38	0	foot-pace.
5	6	face of ditto.
60	0	second flight.
4	10	ends of ditto.
26	4	string-board.
199	6	sum.

Doors, 6 panelled.

Feet	*Inches*	
35	0	front door.
20	7	kitchen door.
20	7	parlour door.
41	2	upper room doors.
117	4	sum.

Doors, 4 panelled

Feet.	*Inches.*	
30	4	back door.
21	0	cellar door.
51	4	sum.

Door and Window Casings.

Feet	*Inches.*	
14	4	front door.
11	1	back door.
12	7	kitchen door.
12	7	parlour door.
25	3	upper room doors.
46	0	lower room windows.
23	8	stair-case window.
61	0	upper room windows.
206	6	sum.

Architraves.

Feet	*Inches.*	
20	5	front door.
12	6	kitchen door.
25	1	parlour door.
50	2	upper room doors.
15	9	parlour window.
16	1	stair-case window.
42	7	upper windows.
182	7	sum.

Window Sashes.

Feet.	*Inches.*	
47	2	lower windows.
26	8	stair-case window.
57	6	upper windows.
131	4	sum.

The Carpenter's and Joiner's Bill.

Sqrs	Ft	In.		L s d
14	84	8	of naked flooring, ... at — per square,	- - -
12	26	8	of boarded flooring, at — per square,	- - -
11	72	10	of ceiling joists, ... at — per square,	- - -
14	41	10	of roofing, ... at — per square,	- - -
	109	6	of stair steps, ... at — per foot, ..	- - -
	92	9	of balustrade, ... at — per foot, ...	- - -
	117	4	of doors 6 panelled, at — per foot, ...	- - -
	51	4	of doors 4 panelled, at — per foot, ...	- - -
	206	6	of door and window casings, ... at — per foot, ...	- - -
	182	7	of architraves, ... at — per foot, ...	- - -
	131	4	of window sashes, ... at — per foot, ..	- - -
			£.	- - -

THE SLATER'S WORK.

The length of the roof, for the slating, is 52 feet 6 inches, and its girt, allowing 18 inches at each of the eaves, is 29 feet 10 inches, hence the content is 1566¼ square feet = 15 squares 66¼ feet, at — per square, £ - s - d.

THE PLASTERER'S WORK.

	Dimensions.		CONTENTS		TITLES.
	Ft	In	Ft	In.	
2	18 20	8 8	771	6	Ceilings of the lower rooms, 3 coats.
2	82 0	8 10	137	9	Cornices of ditto.

Dimensions.		CONTENTS.		TITLES.
Ft.	*In.*	*Ft.*	*In.*	
82	8	82	8	Enriched mouldings in the parlour, lineal measure.
9 7	6 10	74	5	Ceiling in the lower part of the stair-case, 3 coats.
34 0	8 10	28	10	Cornice of ditto.
82 10	8 6	868	0	Walls of the parlour, hard-finishing.
6 3	8 6	23	4	Door of ditto to be deducted.
6 5	6 0	32	6	Window to be deducted.
5 5	6 3	28	10	Fire-place to be deducted.
82 10	8 6	868	0	Walls of the kitchen, 2 coats.
6 3	8 6	23	4	Door to be deducted.
7 5	0 0	35	0	Back door to be deducted.
6 5	6 0	32	6	Window to be deducted.

	Dimensions.		CONTENTS.		TITLES.
	Ft	In.	Ft	In.	
	5 5	6 3	28	10	Fire-place to be deducted.
	60 10	4 6	633	6	Walls of the lower part of the stair-case, hard-finishing.
	60 9	4 3	558	1	Ditto of the upper part.
	8 5	0 6	44	0	Front door to be deducted.
2	6 3	8 6	46	8	Kitchen and parlour doors to be deducted.
	13 3	0 0	39	0	Space occupied by the stair-case, to be deducted.
	6 5	10 0	34	2	Stair-case window to be deducted.
	5 4	8 6	25	6	Window at the top of the stair-case, to be deducted.
2	6 3	8 6	46	8	Upper room doors to be deducted.
	60 9	4 6	573	2	Ceiling over the stair-case, 3 coats.
	60 0	4 10	50	3	Cornice of ditto.
2	21 19	4 4	824	10	Ceiling of the upper rooms, 3 coats.

Dimensions			CONTENTS		TITLES.
	Ft	*In.*	*Ft*	*In*	
2	85	0	141	8	Cornices of ditto.
	0	10			
2	85	0	1572	6	Walls of the upper rooms, 2 coats.
	9	3			
2	6	8	46	8	Doors to be deducted
	3	6			
2	5	8	51	0	Windows to be deducted.
	4	6			
2	5	0	45	0	Fire-places to be deducted
	4	6			

Abstract of the Plastering.

This abstract must be made by collecting all the ceiling of 3 coats into one sum; the hard-finishing into another, &c. &c.; and by making the proper deductions.

Ceiling.	RENDERING.		DEDUCTIONS.		CORNICES.
3 COATS.	HARD *Finishing.*	2 COATS.	HARD *Finishing.*	2 COATS.	
Ft In.	Ft. In	Ft In	Ft In	Ft In	Ft In
771 6	868 0	868 0	23 4	23 4	137 9
74 5	633 6	1572 6	32 6	35 0	28 10
573 2	558 1	2440 6	28 10	32 6	50 3
824 10	2059 7	262 4	44 0	28 10	141 8
9)2243 11	320 8	9)2178 2	46 8	46 8	358 6
Yds. Ft In	9)1738 11	Yds Ft In	39 0	51 0	
249 2 11	Yds Ft In	242 0 2	34 2	45 0	
	193 1 11		25 6	262 4	
			46 8		
			320 8		

Yds.	*Ft.*	*In.*	*The Plasterer's Bill.*	*£. s. d.*
249	2	11	of ceiling, 3 coats, at — per yard,	— — —
193	1	11	of rendering, hard-finishing, at — per yard,........................	— — —
242	0	2	of ditto, 2 coats, at — per yard,...	— — —
	358	6	of cornices, at — per foot,.........	— — —
	82	8	of enriched mouldings, at — per foot,............................	— — —
			£.	— — —

Note. The following Works will be found extremely useful to Bricklayers, Masons, Joiners, or any other persons who are desirous of obtaining a knowledge of Architecture.

Miller's Designs for Cottages, Farm-houses, Country-houses, Villas, Lodges for Park or Garden Entrances, with Plans of the Offices belonging to each Design, on 32 Quarto Plates, Price sewed, 10s. 6*d*

Rawlin's Designs of Houses for Gentlemen and Tradesmen, Parsonages and Summer Retreats; with Back-fronts, Sections, &c., together with Banqueting Rooms and Churches; to which is added the Masonry of semicircular and elliptical Arches, on 51 Royal Quarto Plates, Price, 1*l* 1*s*

The Student's Instructor, in drawing and working the Five Orders of Architecture, fully explaining the best Methods of striking regular and quirked mouldings, for diminishing and glueing of Columns and Capitals, for finding the true diameter of an Order to any given Height, for striking the Ionic Volute circular and elliptical, with finished Examples, on a large Scale, of the Orders, their Planceers, &c on 33 Octavo Plates, by *Peter Nicholson*, Price, bound, 20s. 6*d*.

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PART VI.

THE METHOD OF

MEASURING HAY-STACKS,

Drains, Canals, Marl-pits, Ponds, Mill-dams, Embankments, Quarries, Coal-heaps, and Clay-heaps.

HAY-STACKS.

THE contents of hay-stacks are found in order to ascertain their weights; which must, of course, vary according to the density of the hay

Some stacks will not weigh more than 8 or 10 stones per cubic yard, and others will weigh 15 or 16 stones; it is not, however, the measurer's province to determine the weight, but only the number of cubic yards which the stack contains; and leave the buyer and seller to settle about the weight as they think proper.

PROBLEM I.

To measure a hay-stack, having a circular base

CASE I.

When the stack is straight from the bottom to the eaves, and from the eaves to the top, as in the following figure, the upper part may be taken as a cone, and the lower part as a conical frustum.

RULE.

Multiply the square of the circumference at the bottom AB, by .07958, or for general practice, by .08; and the product will be the area of the base.

Find the area of a section at the eaves DC, in the same manner. To the sum of these areas, add the square root of their product; multiply this sum by the perpendicular height GF = HD, and $\frac{1}{3}$ of the product will be the solidity of the frustum ABCD.

Multiply the area of the section at the eaves, by the perpendicular height FE = DK, and $\frac{1}{3}$ of the product will be the solidity of the cone DCE.

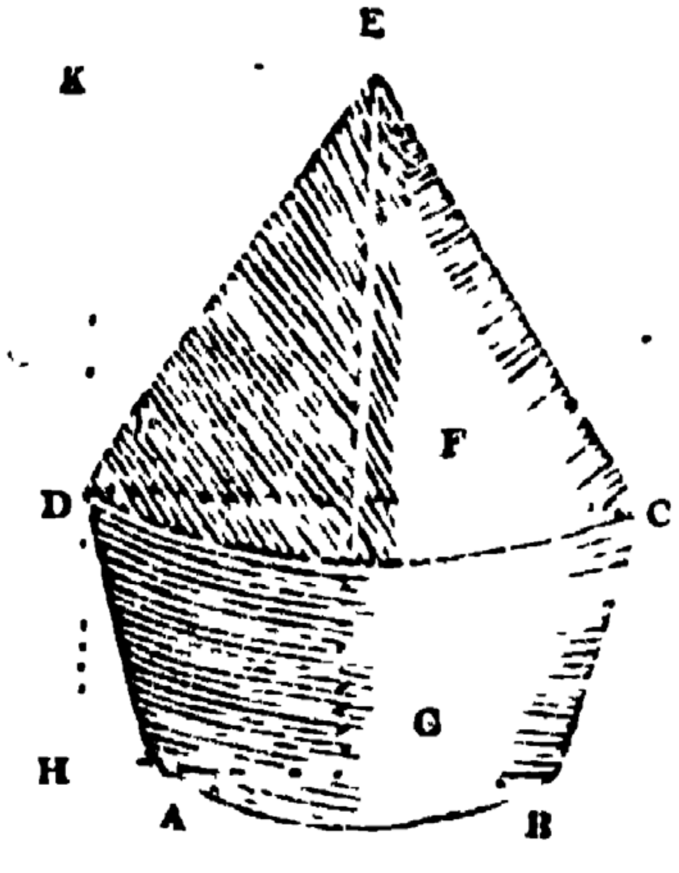
To the solidity of the frustum add that of the cone; and the sum will be the content of the whole solid ABCEDA.

Note 1. Some measurers take the dimensions of hay-stacks, canals, marl-pits, &c. with a tape divided into yards, tenths, and hundredths, but one divided into feet and tenths, is considered, by most practitioners, to be much preferable

2 When the dimensions are taken in feet, the content must be divided by 27, in order to reduce it to cubic yards.

EXAMPLES.

1. The circumference at the base AB, of the following figure, is 40, the circumference at the eaves DC 60, the perpendicular height GF = HD 15, and FE = DK 16 feet; how many solid yards does the stack contain?



CALCULATION.

Here 40 × 40 × .08 = 128, *the area of the base;* *and* 60 × 60 × .08 = 288, *the area of a section at the eaves; also,* $\sqrt{128 \times 288} = \sqrt{36864} = 192$, *the square root of their product; then* $\overline{128 + 288 + 192} \times \frac{15}{3} =$

$608 \times 5 = 3040$ *cubic feet, the content of the frustum* ABCD.

Again, $\frac{288 \times 16}{3} = \frac{4608}{3} = 1536$ *cubic feet, the content of the cone* CDE.

Lastly, $\frac{3040 + 1536}{27} = \frac{4576}{27} = 169$ *yards* 13 *feet, the answer required.*

2. The circumference of the base of a hay-stack is 52 feet 9 inches, the circumference at the eaves 75 feet 6 inches, the perpendicular height of the lower part 16 feet 3 inches, and that of the upper part 18 feet 6 inches, how many tons are contained in the stack; admitting each cubic yard to weigh 15 stones or 210 pounds? *Ans.* 28 *tons,* 10 *cwt.* 1 *qr.* 17 *lb.*

CASE II.

When the stack is bulged from the bottom to the top, as in the following figure, the equi-distant ordinate method, described in Problem 23, *Part II., must be adopted.*

RULE.

1. Find the areas of as many circular sections, taken at equal perpendicular distances from the bottom, as you judge sufficient, by multiplying the square of the circumference of each section by .08 Proceed with these areas in the same manner as if they were equidistant ordinates; and the result will be the solidity of the stack, from the bottom to the uppermost or last section.

2. Multiply the area of the base of the remaining part, at the top, which may be considered as a cone, by its perpendicular height; and ⅓ of the product will be the solidity.

3. Add these two solidities together, and the sum will be the content of the whole stack.

Note 1. Always make choice of an odd number of sections, in order that the number of parts into which the solid is divided, may be equal. Five or seven will, in general, be sufficient; of which one must be at the bottom, and another at the eaves, or as near to them as possible.

2. Great care must be taken to obtain the dimensions of the sections at equal perpendicular distances, for if the slanting distances be taken, it is evident that the content will be made too much.

SCHOLIUM.

The method of finding the areas of curvilineal figures, by means of equi-distant, perpendicular ordinates, was first demonstrated by the illustrious Sir Isaac Newton.

Mr. Robert Shirtcliffe, in his Theory and Practice of Gauging, appears to have been the first who applied it to finding the areas of curvilineal vessels used by Brewers, Distillers, &c.; and after him Mr. Samuel Farrer, in the Appendix to Overley's Gauging. Their Rules, however, were extremely tedious · and, though true to demonstration, were not general, but particular, according to the number of ordinates used.

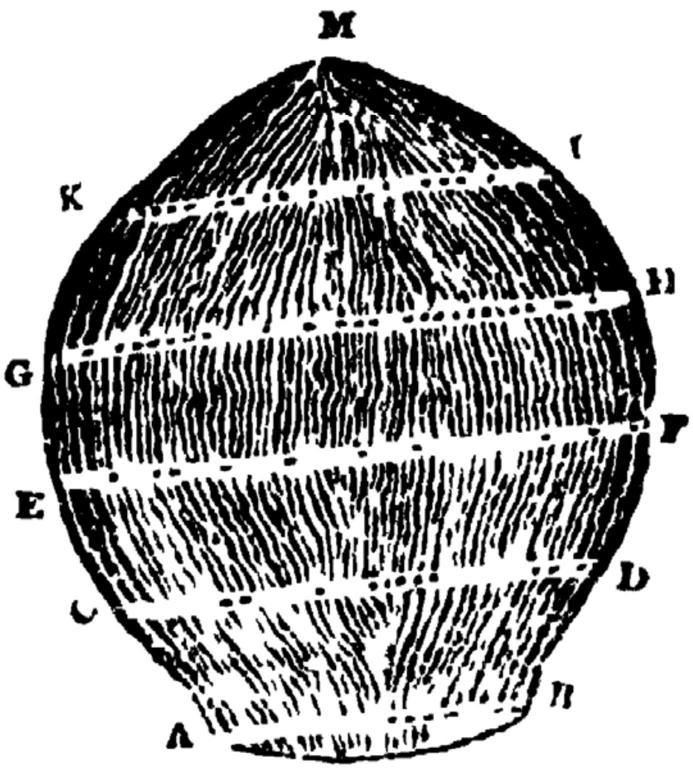
To obviate this inconvenience, the *general rule* given in Problem 23, Part II. of this Work, was deduced from SIMPSON'S Dissertations, page 109, by Mr. Thomas Moss, and demonstrated in his valuable Treatise of Gauging, page 235.

Dr. Hutton, in his Mensuration, Proposition I., Section II, Part IV., has also given an elegant demonstration of the same Rule; and adds, in a Corollary, that it will obtain for the contents of all solids, by using the areas of the sections perpendicular to the axe, instead of the ordinates.

The doctor particularly recommends it for the purpose of gauging and ullaging casks, hence, it is evident that it may be applied with propriety and success to the Mensuration of Hay-stacks, Canals, Marl-pits, and other irregular figures, as being the best approximation that has yet been, or perhaps ever can be given; for by taking an indefinite number of sections, the content of an irregular solid may be obtained to any degree of accuracy.

EXAMPLES.

1. Let the annexed figure represent a hay-stack, whose dimensions are as follow; viz. the girt at the bottom AB = 36, at CD = 54, at EF = 66, at GH = 58, and at KL = 37 feet. The perpendicular distance between each section is 5 feet, and the perpendicular height of the conical part KLM 4 feet 6 inches; how many cubic yards are contained in the stack?



CALCULATION.

Here $36 \times 36 \times .08 = 103.68$ *the area of the bottom or first section;* $54 \times 54 \times .08 = 233.28$, *the area of the second section;* $66 \times 66 \times .08 = 348.48$, *the area of the third section;* $58 \times 58 \times .08 = 269.12$, *the area of the fourth section; and* $37 \times 37 \times .08 = 109.52$, *the area of the fifth or last section; then by proceeding according to the Rule for equi-distant ordinates, we have* $A = 103.68 + 109.52 = 213.2$, $B = 233.28 + 269.12 = 502.4$, $C = 348.48$, *and* $D = 5$*; consequently* $\frac{A+4B+2C}{3}$ $\times D = \frac{213.2 + 2009.6 + 696.96}{3} \times 5 = \frac{2919.76 \times 5}{3}$ $= \frac{14598.8}{3} = 4866.26$ *feet, the solidity of the part* ABLK.

Again, $\frac{109.52 \times 4.5}{3} = \frac{492.84}{3} = 164.28$ *feet, the solidity of the conical part* KLM.

Lastly, $\frac{4866.26 + 164.28}{27} = \frac{5030.54}{27} = 186.31$ *cubic yards, the content required.*

2. The dimensions of a hay-stack are as follow; viz. the girt of the bottom or first section = 127.2, the girt of the second = 145.4, the girt of the third = 156.5, the girt of the fourth = 168.7, the girt of the fifth = 148.3, the girt of the sixth = 121.8, and the girt of the

seventh or last section = 68.6 feet. The perpendicular distance between each section is 8 feet, and the perpendicular height of the conical part, at the top, 7.4 feet, how many cubic yards are contained in the stack?

Ans. 2970 49641 *yards.*

PROBLEM II.

To measure a hay-stack, having a rectangular base.

CASE I.

When the stack is straight from the bottom to the eaves, and from the eaves to the top, as in the following figure; the lower part may be taken as a prismoid, and the upper part as a triangular prism.

RULE.

1. Multiply the mean length of the bottom by the mean breadth, and the product will be the area of the bottom. Find the area of a section at the eaves in the same manner. Multiply half the sum of the lengths of the bottom and eaves, by half the sum of the breadths, and the product will be the area of a section equally distant from the bottom and eaves. To the area of the bottom add the area of the section at the eaves, and four times the area of the middle section, multiply this sum by the perpendicular height, from the bottom to the eaves; and $\frac{1}{6}$ of the product will be the solidity of the lower part.

2. Multiply the breadth at the eaves by the perpendicular height from the eaves to the top, and half the product will be the area of the end, which being multiplied by the mean length, will give the solidity of the upper part.

3 Add these two solidities together, and the sum will be the content of the whole stack.

Note 1 Sometimes stacks are longer on one side than the other, and broader at one end than the other, in such cases, take half the sum of the lengths for a mean length, and half the sum of the breadths for a mean breadth

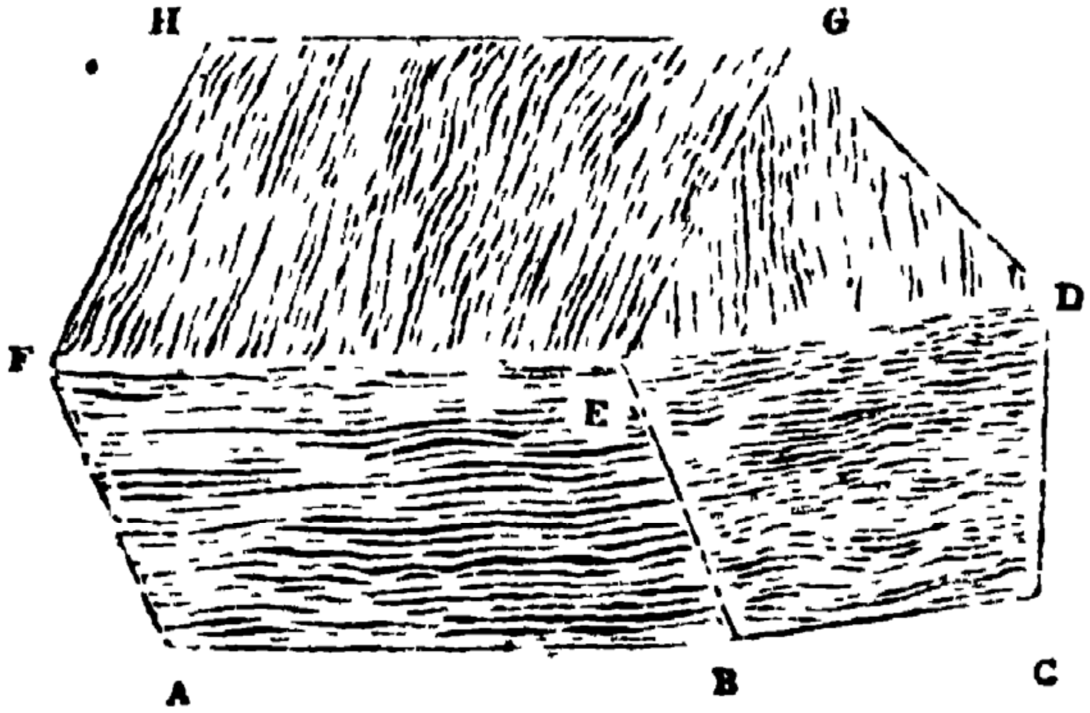
2 Some stacks are higher and broader at the ends than in the middle, when this is the case, a proper allowance must be made in taking the dimensions. Allowance also ought to be made for the thatch

3. If the ends of the upper part be not equal, find the area of both ends; and take half their sum for a mean area, which multiply by the length for the solidity.

4. If the length of the top or ridge GH, be more or less than the length of the base or eave EF, it is evident that the upper part of the stack is in the form of a cuneus or wedge, hence, its *true* content may be found by Prob. 9, Sect. 1 Part IV.

EXAMPLES.

1. Let the annexed figure represent a hay-stack, the dimensions of which are as follow; viz. the mean length at the bottom 36.8, and the mean breadth 18.3; the mean length at the eaves 44.6, and the mean breadth 25.9; the perpendicular height from the bottom to the eaves 18.6, and from the eaves to the top 15.5 feet; how many cubic yards does the stack contain; the mean length of the upper part being 43.7 feet?



CALCULATION.

Here $36.8 \times 18.3 = 673.44$, *the area of the base; and* $44.6 \times 25.9 = 1155.14$, *the area of the section at the eaves.*

Also, $\frac{36.8 + 44.6}{2} = \frac{81.4}{2} = 40.7$, *the length of the middle section; and* $\frac{18.3 + 25.9}{2} = \frac{44.2}{2} = 22.1$, *the breadth of the middle section; then* $40.7 \times 22.1 \times 4 = 3597.88$, *four times the area of the middle section; whence* $(673.44 + 1155.14 + 3597.88) + \frac{18.6}{6} = 5426.46 \times 3.1 = 16822.026$ *feet, the solidity of the prismoid* ABCDEF.

Again, $\frac{25.9 \times 15.5}{2} = \frac{401.45}{2} = 200.725$, *the area of the end* EDG; *and* $200.725 \times 43.7 = 8771.6825$ *feet, the solidity of the prism* FEDGH.

Lastly, $\frac{16822.026 + 8771.6825}{27} = \frac{25593.7085}{27} =$ 947.9151 *cubic yards, the answer required.*

2. How many cubic yards are contained in a hay-stack of the following dimensions; viz. the length at the bottom 49.4, and the breadth 24.6; the length at the eaves 58.8, and the breadth 34.2; the perpendicular height of the lower part 25.8; the perpendicular height of the upper part 21.7 feet, and its length the same as that at the eaves? *Ans.* 2335.143 *yards.*

CASE II

When a stack is bulged from the bottom to the eaves, and from the eaves to the top, as in the following figure, recourse must be had to the equi-distant ordinate method.

RULE.

1. Find the areas of as many equi-distant parallel sections as you think sufficient, with which proceed as if they were the equi-distant ordinates; and the result will be the solidity of the stack, from the bottom to the uppermost section.

2. Multiply the area of the end of the remaining piece, at the top, which may be considered as a triangular prism, by its length; and the product will be its solidity.

3. Add these two solidities together, and the sum will be the content of the whole stack.

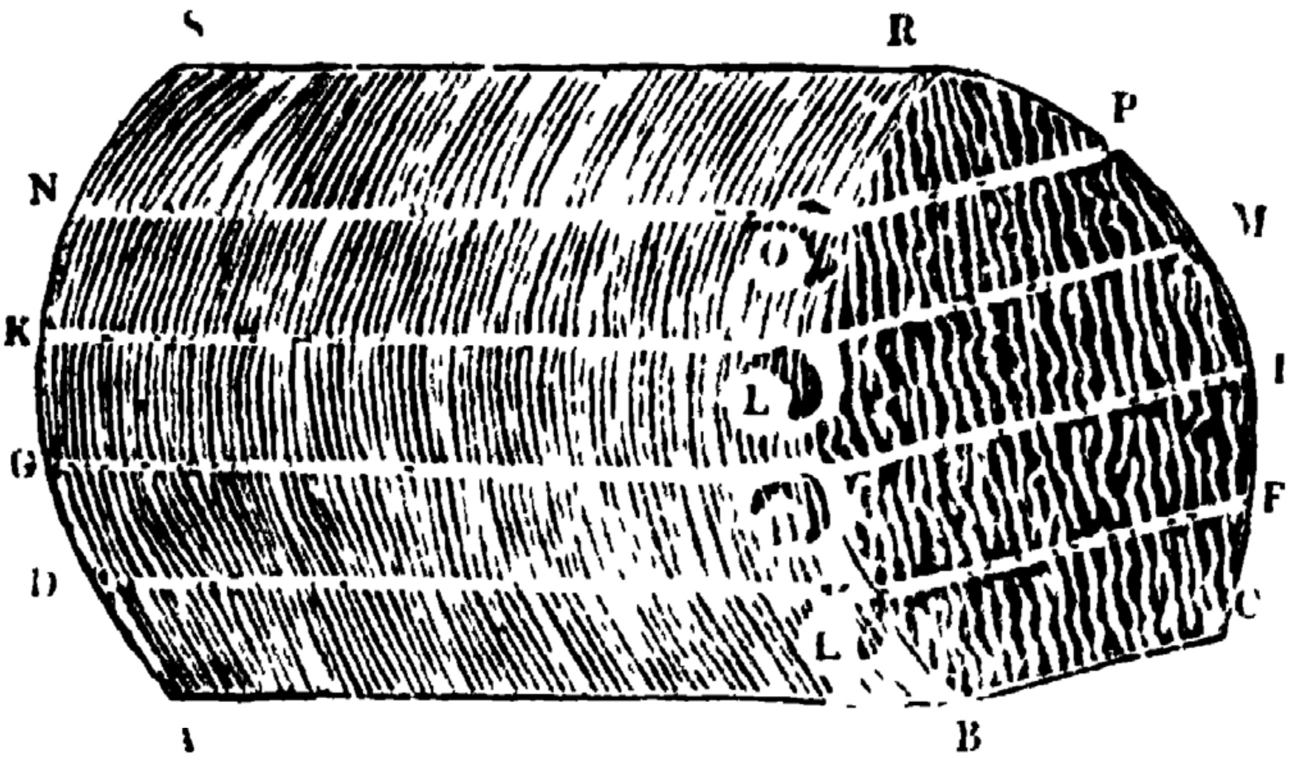
Note 1. The weight of a stack may be ascertained to a considerable degree of accuracy, in the following manner. Cut out a portion extending to the centre of the stack, from the top to the bottom, and weigh it, and also measure the vacuity from which it is taken; then say, as the content of this part is to its weight, so is the content of the whole stack, to its weight.

2. If the stack have a rectangular base, the portion to be weighed should not be cut off the end, but taken out about half way between the end and the middle, where it may be supposed that the hay is of a medium density.

EXAMPLES.

1. Let the annexed figure represent a hay-stack, the dimensions of which are as follow; viz. the length AB 38 feet, and breadth BC 13 feet; the length DE 41 feet, and breadth EF 18 feet; the length GH 45 feet,

and breadth HI 22 feet; the length KL 44 feet, and breadth LM 19 feet; the length NO 42 feet, and breadth OP 12 feet; the mean length of the triangular prism, at the top, 41 feet, and the perpendicular height of its end 5 feet; how many cubic yards are contained in the stack; the perpendicular distance between each parallel section being 6 feet?



CALCULATION.

Here 38 × 13 = 494, *the area of the first section*; 41 × 18 = 738, *the area of the second section*; 45 × 22 = 990, *the area of the third section*; 44 × 19 = 836, *the area of the fourth section*; *and* 42 × 12 = 504, *the area of the fifth or last section*; *then by proceeding according to the rule for equi-distant ordinates, we have* A = 494 + 504 = 998, B = 738 + 836 = 1574, C = 990, *and* D = 6; *consequently*, $\frac{A + 4B + 2C}{3} \times D = \overline{998 + 6296 + 1980} \times \frac{6}{3} = 9274 \times 2 = 18548$ *feet, the solidity of the part* ABCPON.

Again, $\frac{12 \times 5}{2} = \frac{60}{2} = 30$, *the area of the end* OPR: *and* 41 × 30 = 1230 *feet, the solidity of the prism* NOPRS.

Lastly, $\frac{18548 + 1230}{27} = \frac{19778}{27} = 732$ *yards*, 14 *feet, the answer required.*

2. The dimensions of a hay-stack are as folllow; viz. the length of the bottom or first section 70.8, and its breadth 20.2; the length of the second 75.4, and its breadth 28.7; the length of the third 81.2, and its breadth 32.4; the length of the fourth 86.7, and its breadth 36.8, the length of the fifth 84.9, and its breadth 34.6, the length of the sixth 83.4, and its breadth 30.9, the length of the seventh 82.7, and its breadth 16.5; the mean length of the triangular prism, at the top, 81.5, and the perpendicular height of its end 6.4 feet, how many cubic yards are contained in the stack; the perpendicular distance between each section being 8 feet? *Ans* 1668.8 *yards.*

3. What is the weight of a hay-stack which measures 425 cubic yards 18 feet; the weight of a piece cut out of the stack, from the top to the bottom, being 1 ton, 3 cwt 2 qrs. 15 lb., and the vacuity from which it is taken, 16 cubic yards 12 feet?

Ans. 30 *tons,* 11 *cwt.* 3 *qrs.* 1 *lb*

REMARK.

Sometimes hay is stored up in *barns* or *shades* When this is the case, the lower part of the *mow*, from the bottom to the eaves, will be a parallelopipedon, the content of which may be found by Problem 2, and the upper part, from the eaves to the ridge of the building, will be a triangular prism, the content of which may be obtained by Problem 3, Part IV.; then the sum of these two contents will be the whole content of the *mow*.

If the hay does not extend to the ridge, the upper part will be a trapezoidal prism, the content of which may be obtained by finding the area of the end by Problem 8, Part II.; and multiplying this area by the length of the prism. (See Problem 18, Part IV., on the method of measuring compound and irregular solids.)

Note. When hay-mows are irregular, mean dimensions must be taken

DRAINS AND CANALS.

A **Drain** is an artificial channel made to convey water from marshes, bogs, and other low grounds, for the purpose of making improvements in agriculture, &c.

A **Canal** is an artificial, navigable river, most commonly made for the purpose of transporting goods from one place to another, by water-carriage

Note 1 Drains and Canals are made with sloping sides, and are almost invariably dug by the cubic yard, hence, it is of the utmost moment to ascertain their contents with accuracy

The common method of measuring them is to take the breadths of the top and bottom, in different places, and their sum being divided by their number, the quotient is considered as a mean breadth. Several depths are likewise measured, in various places, and their sum divided by their number, is taken for a mean depth, then the length, breadth, and depth being multiplied continually together, the last product is taken for the content, but it is evident that this process must lead to very erroneous results.

2 In this country, many thousands of acres of bogs, marshes, and fens have of late been made fit for the purposes of agriculture, by means of drains, particularly in the counties of York and Lincoln, and improvements of this kind are still carried on in the latter county, with great spirit

PROBLEM III.

To find the number of cubic yards which have been dug out of a drain or canal.

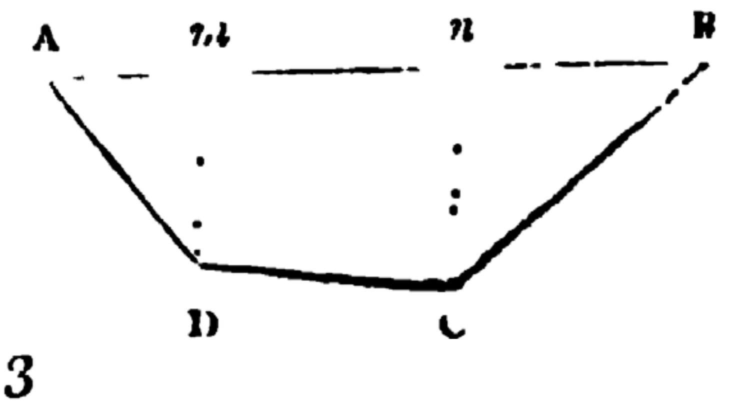
RULE.

Find the areas of as many equi-distant, perpendicular transverse sections, as you judge sufficient, with which proceed as if they were equi-distant ordinates; and the result will be the content between the first and last sections. (See Problem 23, Part II)

Note 1 The distance between the sections must be measured along the middle of the bottom of the drain or canal, and when the ground is very uneven, it will be necessary to take the sections nearer to each other, than when it is pretty level.

2. When a drain or canal is very long, divide it into several parts or lengths, and find the content of each separately by the above Rule, and the sum of these contents will be the content of the whole.

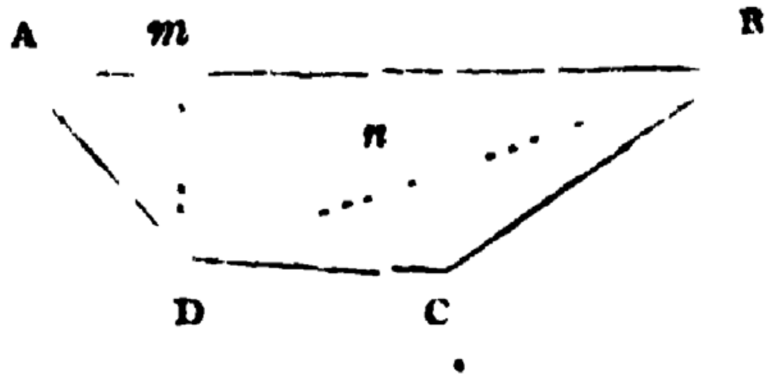
3. Let the annexed figure represent a perpendicular transverse section of a drain or canal, where the ground is even; then it is evident that if a line AB, be stretched across the top, it will be parallel to the bottom DC, admitting it to be



level, and the perpendicular D*m* will be equal to the perpendicular C*n*, which perpendiculars may be easily found by erecting a straight staff perpendicularly to the bottom DC, and so as to touch the line AB, at *m* and *n*.

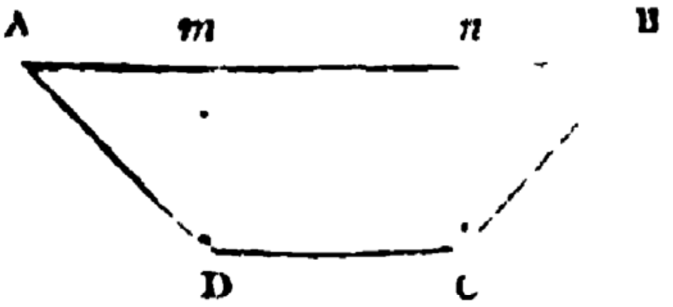
In this case, the section ABCD is a trapezoid, the area of which may be found by Problem VIII. Part II.

4. Let the subjoined figure denote a perpendicular, transverse section of a drain or canal, where the ground is uneven; then it is evident that the section is a trapezium, because AB is not parallel to DC.

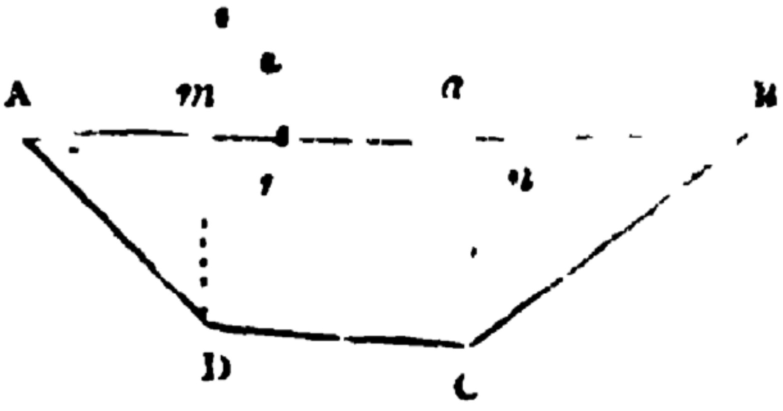


Now, as two perpendiculars cannot be taken upon either of the diagonals, the figure must be divided into two triangles in the following manner: Measure the diagonal BD, and at right-angles to it, the perpendicular C*n*; measure also the line AB, and at right-angles to it, the perpendicular D*m*, hence, the area of the section may be found by Problem VII., Part II.

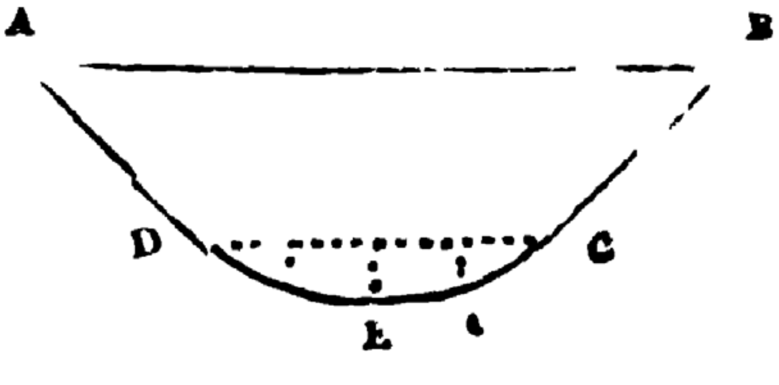
Or, the section may be divided into two right-angled triangles, and a trapezoid, as in the annexed figure, by measuring the line AB, and at right-angles to it, the two perpendiculars D*m* and C*n*; and hence the area may be found.



Or, the area of the section may be obtained by dividing it into the two triangles, AD*m*, BC*n*, and the trapezoid D*mn*C. In this case, D*m* and C*n*, which are at right-angles to DC, are the parallel sides of the trapezoid, and also the bases of the two triangles; and their perpendiculars are A*r* and B*a*.



5. If the bottom of a drain or canal be a curve line, as in the subjoined section; then the area of the quadrilateral figure ABCD, must be found by some of the foregoing methods; and the area of the part DEC, by the method of equi-distant ordinates, described in Problem XXIII., Part II.



Note The Rule given in Problem 23, Part II., being expressed in an algebraic form, is seldom perfectly comprehended by learners, but the following one may be easily understood, and committed to memory.

RULE.

To the sum of the first and last ordinates, add four times the sum of all the even ordinates, and twice the sum of all the odd ordinates, not including the first and last; multiply this sum by the common distance of the ordinates, divide the product by 3, and the quotient will be the area required.

EXAMPLES.

1. Let the subsequent figure represent part of a drain or canal; required the number of cubic yards that were dug out of it; the distance between each perpendicular, transverse section being 60 feet, and the dimensions of these sections, as follow.

First section ABCD.

Feet

$AB = 60.6$

$DC = 28.7$

$Dm = Cn = 15.4$

Second section EFGH.

Feet

$HF = 52.3$

$Gn = 13.5$

$EF = 63.4$

$Hm = 14.8$

Third section KLMN.

Feet.

$Km = 12.2$

$Nm = 13.5$

$mn = 27.4$

$Mn = 18.3$

$Ln = 23.8$

Fourth section PQRS

Feet

$Sm = 14.6$

$Pr = 15.2$

$SR = 29.7$

$Rn = 20.5$

$Qa = 17.3$

Fifth section TUVXW.

Feet

$TU = 61.5$

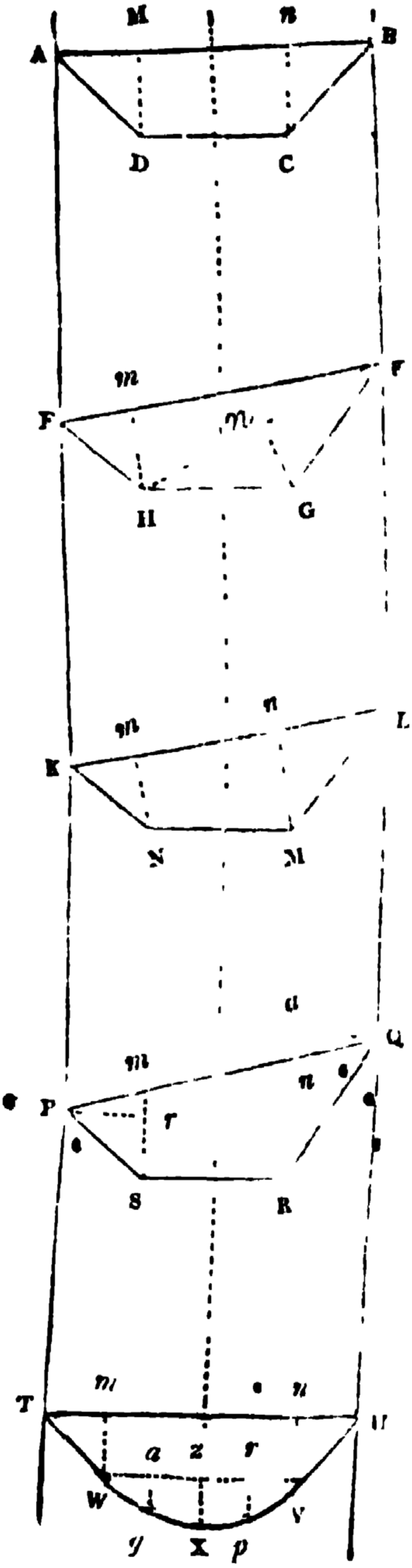
$WV = 38.4$

$Wm = Vn = 10.2$

$qa = 5.3$

$Xz = 7.4$

$pr = 5.8$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CALCULATION.

First Section.

Here $\overline{60.6 + 28.7} \times 15.4 = 89.3 \times 15.4 = 1375.22$; *and* $\frac{1375.22}{2} = 687.61$, *the area of the trapezoid* ABCD.

Second Section.

Here $52.3 \times 13.5 = 706.05$, *double the area of the triangle* FGH; *and* $63.4 \times 14.8 = 938.32$, *double the area of the triangle* EFH; *then* $\frac{706.05 + 938.32}{2} = \frac{1644.37}{2} = 822.185$, *the area of the trapezium* EFGH.

Third Section.

Here $12.2 \times 13.5 = 164.7$, *double the area of the triangle* KmN; *also* $\overline{13.5 + 18.3} \times 27.4 = 31.8 \times 27.4 = 871.32$, *double the area of the trapezoid* N$mn$M; *and* $23.8 \times 18.3 = 435.54$, *double the area of the triangle* LMn; *then* $\frac{164.7 + 871.32 + 435.54}{2} = \frac{1471.56}{2} = 735.78$, *the area of the trapezium* KLMN.

Fourth Section.

Here $14.6 \times 15.2 = 221.92$, *double the area of the triangle* SPm; *also* $\overline{14.6 + 20.5} \times 29.7 = 35.1 \times 29.7 = 1042.47$, *double the area of the trapezoid* S$mn$R; *and* $20.5 \times 17.3 = 354.65$, *double the area of the triangle* QRn; *then* $\frac{221.92 + 1042.47 + 354.65}{2} = \frac{1619.04}{2} = 809.52$, *the area of the trapezium* PQRS.

Fifth Section.

Here $\overline{61.5 + 38.4} \times 10.2 = 99.9 \times 10.2 = 1018.98$; *and* $\frac{1018.98}{2} = 509.49$, *the area of the trapezoid* TUVW;

and by the Rule for equi-distant ordinates, we have $A = 0$, $B = 5.3 + 5.8 = 11.1$, $C = 7.4$, *and* $D = \frac{38.4}{4} = 9.6$, *then* $\overline{A + 4B + 2C} \times \frac{1}{3}D = \overline{44.4 + 14.8} \times \frac{9.6}{3} = 59.2 \times 3.2 = 189.44$, *the area of the part* VXW; *then* $509.49 + 189.44 = 698.93$, *the area of the whole section* TUVXW.

To find the solid content

Here $A = 687.61 + 698.93 = 1386.54$, $B = 822.185 + 809.52 = 1631.705$, $C = 735.78$, *and* $D = 60$, *then* $\overline{A + 4B + 2C} \times \frac{1}{3}D = (1386.54 + 6526.82 + 1471.56) \times \frac{60}{3} = 9384.92 \times 20 = 187698.4$ *cubic feet; and* $187698.4 \div 27 = 6951.792$ *cubic yards, the answer required.*

2. Required the number of cubic yards dug out of part of a drain or canal, from the following dimensions, each perpendicular, transverse section being divided into two triangles in the same manner as the second section EFGH, in the foregoing figure; and the common distance of the sections 100 feet:

First Section.

Feet.
HF = 35.9
G*n* = 8.8
EF = 41.7
H*m* = 9.6

Second Section.

Feet.
HF = 36.1
G*n* = 8.9
EF = 39.8
H*m* = 9.1

Third Section.

Feet.
HF = 37.1
G*n* = 9.8
EF = 42.2
H*m* = 9.4

Fourth Section.

Feet.
HF = 36.5
G*n* = 9.2
EF = 41.5
H*m* = 10.2

Fifth Section.

Feet.
HF = 37.1
G*n* = 9.5
EF = 42.8
H*m* = 10.2

Sixth Section.

Feet.
HF = 38.6
G*n* = 9.4
EF = 43.2
H*m* = 9.8

Seventh Section.

Feet.

HF = 37.6

G*n* = 9.8

EF = 42.6

H*m* = 10.3

Ans. The area of the first section = 347.35; the area of the second = 341.735; the area of the third = 380.13, the area of the fourth = 379.55; the area of the fifth = 394.505; the area of the sixth = 393.1; the area of the seventh = 403.63, and the content = 225259$\frac{2}{3}$ cubic feet = 8342 cubic yards, 25$\frac{2}{3}$ feet, the answer required.

MARLPITS.

MARL is a kind of rich clay, and is used as manure for land, in Lancashire, Cheshire, Derbyshire, and other counties in England, and is commonly dug by the cubic yard.

Marlpits are of various forms. Sometimes they are laid out in the shape of a rectangle, sometimes in that of a trapezium, and sometimes as an irregular polygon.

In digging a marlpit the sides are always sloped, in order to prevent the upper edges from slipping in, and as the bottom is seldom perfectly level, there is generally a little variation in the depths.

PROBLEM IV.

To find how many cubic yards have been dug out of a marlpit.

CASE I.

When the top and bottom of a marlpit are rectangles, the pit may be considered as a prismoid.

RULE.

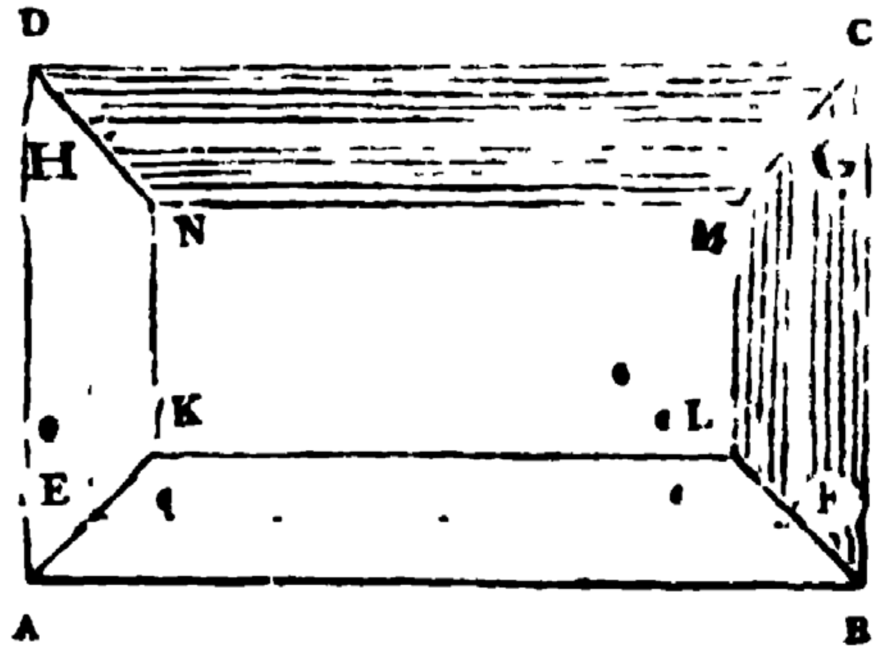
To the sum of the areas of the top and bottom, add four times the area of a section half-way between them; multiply this sum by the mean perpendicular depth, and $\frac{1}{6}$ of the product will be the solidity.

Note 1. When the sides and ends are regularly sloped, the length of the middle section will be equal to half the sum of the lengths of the top and bottom, and its breadth equal to half the sum of their breadths; but if the inclination of the sides and ends be not regular, the length and breadth of the middle section must be found by actual admeasurement.

2. A mean depth must be found by taking several depths, at equal distances from each other, and dividing their sum by their number.

EXAMPLES.

1. Let the subsequent figure represent a marlpit, the dimensions of which are as follow: viz. the length AB of the top = 58.6 feet, and the breadth BC = 36.2 feet; the length EF of the middle section = 55.7 feet, and its breadth FG = 33.4 feet; the length KL of the bottom = 52.8 feet, and breadth LM = 30.4 feet; how many cubic yards of marl were dug out of the pit, its mean perpendicular depth being 8.6 feet?



CALCULATION.

Here 58.6 × 36.2 = 2121.32, *the area of the top;* 55.7 × 33.4 × 4 = 1860.38 × 4 = 7441.52, *four times the area of the middle section; and* 52.8 × 30.4 = 1605.12, *the area of the bottom; then* (2121.32 + 7441.52 + 1605.12) × 8.6 = 11167.96 × 8.6 = 96044.456, *which being divided by* 6, *we obtain* 16007.409 *cubic feet* = 592.867 *cubic yards, the answer required.*

2. A marlpit measures 86.4 feet in length, and 36.8 feet in breadth at the top; the length of the middle

section is 82.2 feet, and its breadth 32.7 feet; the bottom is 78.3 feet in length, and 28.6 feet in breadth; how many cubic yards of marl were dug out of the pit, its mean perpendicular depth being 10.5 feet?

Ans. 1048.098 *yards.*

CASE II.

To find the number of cubic yards which have been dug out of a marlpit, the top and bottom of which are triangles, trapeziums, polygons, or any other figures whatever.

RULE.

Take such dimensions as will give you the area of the top, the area of the middle section, and the area of the bottom; proceed with these areas in the same manner as directed for a prismoid, and the result will be the content, *nearly.* (*See the* SCHOLIUM, *Prob. X. Part IV. Sect. I.*)

Note 1. The above Rule will give the content of the generality of marlpits very near the truth; if, however, a pit exceed 10 or 12 feet in depth, and the sides be not pretty regularly sloped, it will be more accurate to find the areas of 5 equi-distant, horizontal sections, and proceed with them in the same manner as if they were equi-distant ordinates (*See the SCHOLIUM, Problem I., Case 2, Part VI.*)

2. When the top and bottom of a marlpit are trapeziums approaching nearly to rectangles, and the inclination of the bottom from the plane of the horizon very considerable, so as to make the pit much deeper on one side than on the other, the content may be most correctly obtained by finding the areas of several equi-distant, perpendicular sections, and proceeding with them as directed in the last Problem.

The contents of the two pieces at the ends, resembling *wedges*, may be found by Problem IX., Part IV., Sect. I.

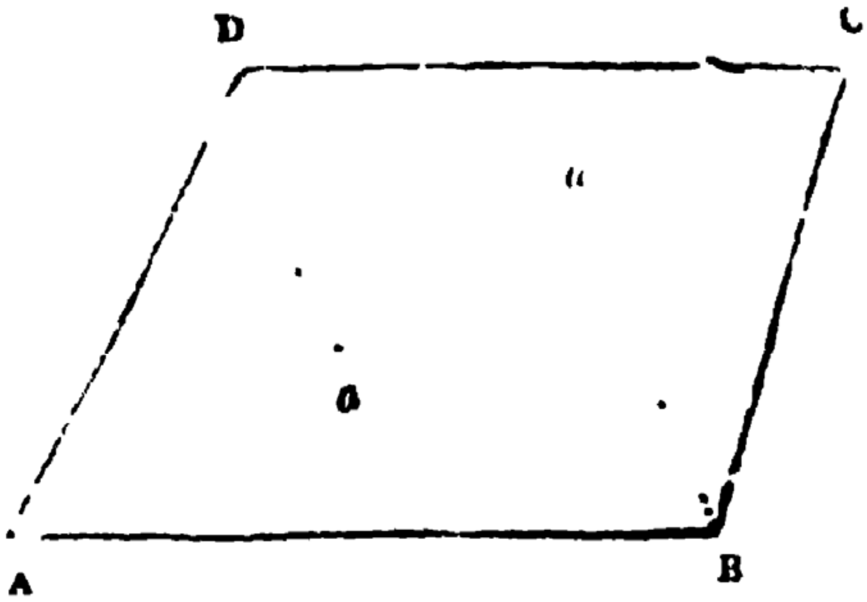
3. The content of the *space* or *road* leading into the pit, may be obtained by the last Problem.

The area of the first section, on that where the road commences, will evidently be nothing, and the distance between the first section and the middle of the slant height of the pit, must be taken for the length of the space.

4. When the edges of a pit give way, and slide in, these *slips* must be measured, if they be thrown out or removed by the labourers. The general method of doing this is by multiplying the mean length, the mean breadth, and the mean depth continually together, for the content. This method is certainly inaccurate; yet in such cases as these, it may, in general, suffice; and sometimes it is the only one that can be adopted with any probability of success.

EXAMPLES.

1. Let the following figure denote the uppermost, horizontal section of a marlpit, the diagonal AC of which measures 86.5, the perpendicular D*a* 26.8, and the perpendicular B*a* 32.6 feet. The diagonal of the middle section is found to be 80.6, and one of the perpendiculars 23.9, and the other 29.3 feet. The diagonal of the bottom measures 74.3, and the perpendiculars 20.7, and 26.5 feet; how many cubic yards were dug out of the pit, its mean perpendicular depth being 10.8 feet?



CALCULATION.

Here $\overline{26.8 + 32.6} \times 86.5 = 59.4 \times 86.5 = 5138.1$;

and $\frac{5138.1}{2} = 2569.05$, *the area of the top.*

Also, $\overline{23.9 + 29.3} \times 80.6 = 53.2 \times 80.6 = 4287.92$;

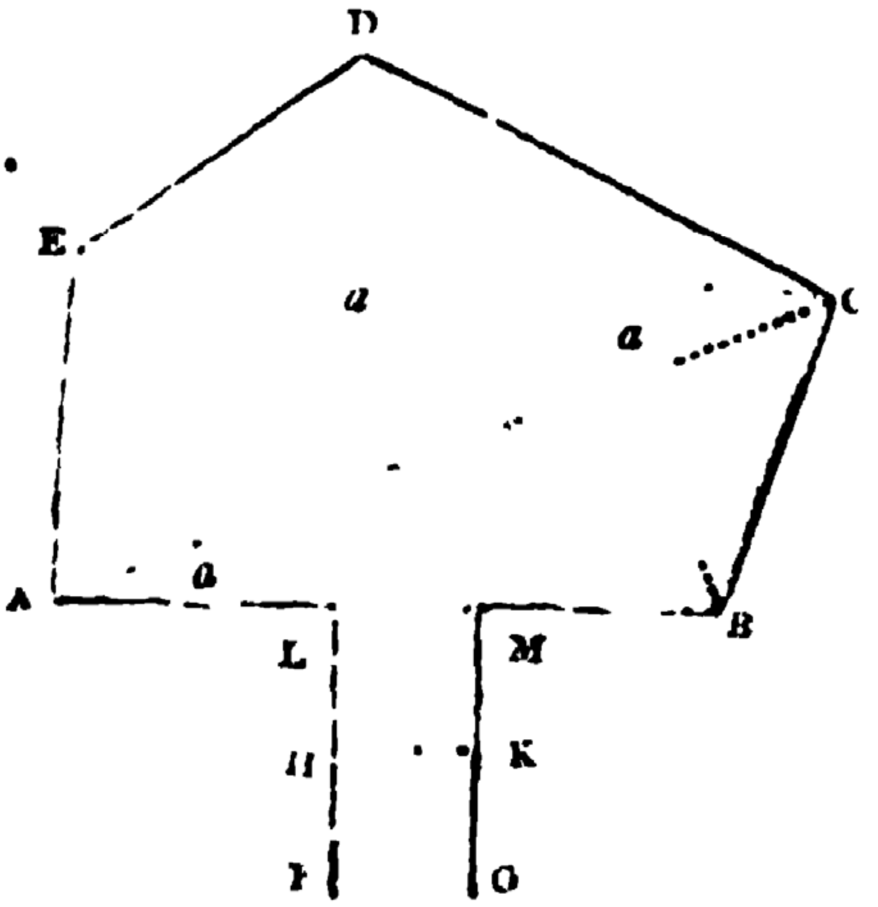
and $\frac{4287.92}{2} \times 4 = 2143.96 \times 4 = 8575.84$, *four times the area of the middle section.*

Again, $20.7 + 26.5 \times 74.3 = 47.2 \times 74.3 = 3506.96$;

and $\frac{3506.96}{2} = 1753.48$, *the area of the bottom.*

Lastly, $(2569.05 + 8575.84 + 1753.48) \times \frac{10.8}{6} =$ $12898.37 \times 1.8 = 23217.066$ *cubic feet* $= 859.8913$ *cubic yards, the answer required.*

2. Admit the following figure ABCDE to denote three horizontal sections of a marlpit; also the figure FLMG to represent the space or road leading into the pit, required the number of cubic yards that were dug out of each; the mean depth of the pit being 11.4 feet; the equi-distant, perpendicular sections at HK, LM, trapezoids; the whole length of the space 43.8 feet; and the other dimensions as below.



Horizontal sections of the pit.

Uppermost section.

Feet.
AC = 118.6
Ea = 43.4
Ba = 35.2
CE = 102.8
Da = 26.4

Middle section.

Feet.
AC = 111.7
Ea = 39.8
Ba = 31.6
CE = 95.9
Da = 22.6

Bottom section.

Feet.
AC = 104.8
Ea = 36.4
Ba = 28.3
CE = 88.6
Da = 19.5

Perpendicular sections of the space.

Section at HK.	Feet.	*Section at LM.*	Feet
AB =	12.4	AB =	12.8
DC =	8.5	DC =	7.9
$Dm = Cn$ =	5.7	$Dm = Cn$ =	11.2

(*See the figure in the last Problem, Note* 3.)

CALCULATION OF THE PIT.

Uppermost Section.

Here $\overline{43.4 + 35.2} \times 118.6 = 78.6 \times 118.6 = 9321.96$, *double the area of the trapezium* ABCE; *and* $102.8 \times 26.4 = 2713.92$, *double the area of the triangle* CDE; *then* $\frac{9321.96 + 2713.92}{2} = \frac{12035.88}{2} = 6017.94$, *the area of the whole section* ABCDE.

Middle Section.

Here $\overline{39.8 + 31.6} \times 111.7 = 71.4 \times 111.7 = 7975.38$, *double the area of the trapezium* ABCE; *and* $95.9 \times 22.6 = 2167.34$, *double the area of the triangle* CDE; then $\frac{7975.38 + 2167.34}{2} \times 4 = \frac{10142.72}{2} \times 4 = 5071.36 \times 4 = 20285.44$. *four times the area of the whole section* ABCDE.

Bottom Section.

Here $\overline{36.4 + 28.3} \times 104.8 = 64.7 \times 104.8 = 6780.56$, *double the area of the trapezium* ABCE; *and* $88.6 \times 19.5 = 1727.7$, *double the area of the triangle* CDE; *then* $\frac{6780.56 + 1727.7}{2} = \frac{8508.26}{2} = 4254.13$, *the area of the whole section* ABCDE.

To find the solid content.

Here $(6017.94 + 20285.44 + 4254.13) \times \frac{11.4}{6} = 30557.51 \times 1.9 = 58059.269$ *cubic feet* $= 2150.343$ *cubic yards, the content of the pit.*

CALCULATION OF THE SPACE.

Section at HK.

Here $\overline{12.4 + 8.5} \times 5.7 = 20.9 \times 5.7 = 119.13$; *and* $\frac{119.13}{2} = 59.565$, *the area of the trapezoid* ABCD.

Section at LM.

Here $\overline{12.8 + 7.9} \times 11.2 = 20.7 \times 11.2 = 231.84$; *and* $\frac{231.84}{2} = 115.92$, *the area of the trapezoid* ABCD.

To find the solid content.

Here $A = 115.92$, $B = 59.565$, $C = 0$, *and* $D = \frac{43.8}{2} = 21.9$; *then* $\overline{A + 4B + 2C} \times \frac{1}{3} D = (115.92 + 238.26) \times \frac{21.9}{3} = 354.18 \times 7.3 = 2585.514$ *cubic feet* $= 95.759$ *cubic yards, the content of the space.*

3. The diagonal of the first or uppermost horizontal section of a marlpit is 118.6, and the perpendiculars are 64.8, and 43.5 feet; the diagonal of the second section 112.4, and the perpendiculars 61.7, and 40.6 feet; the diagonal of the third section 106.3, and the perpendiculars 58.5, and 37.7 feet; the diagonal of the fourth section 100.8, and the perpendiculars 55.9, and 34.6 feet; the diagonal of the fifth or bottom section 94.3, and the perpendiculars 52.4, and 31.5 feet; required the number of cubic yards that were dug out of the pit, the sections being taken at equal distances

from each other, and the mean perpendicular depth of the pit 16 8 feet.

Ans. The area of the first section = 6422.19; *the area of the second*=5749.26; *the area of the third*=5113.03; *the area of the fourth* = 4561.2; *the area of the fifth* = 3955.885; *and the content* = 86584.365 *cubic feet* = 3206.828 *cubic yards, the answer required.*

DIRECTIONS FOR

MEASURING PONDS AND MILL-DAMS.

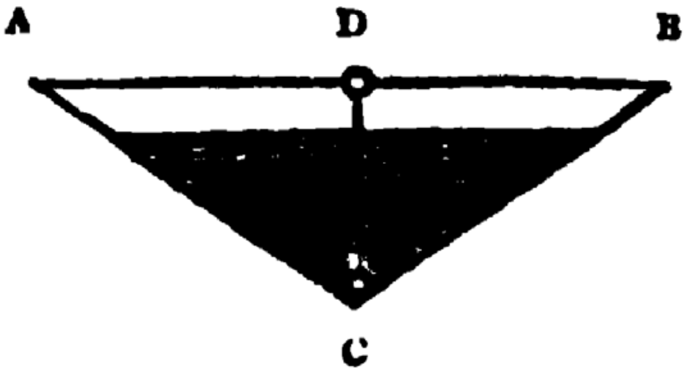
Ponds and mill-dams are commonly dug by the cubic yard; and assume a variety of shapes.

If the top and bottom of a pond or dam be rectangles, it may be treated as directed in the last Problem, Case I.; if the top and bottom be triangles, trapeziums, or polygons, the content may generally be found by Case II., of the last Problem, but if both these methods fail, you must proceed as directed in Problem III., for drains and canals In this case, the contents of the pieces at the ends, resembling wedges, must be found by Problem 9, Part IV., Sect I.

Mr. W. Putsey, Teacher of the Mathematics, at Pickering, informs me that the ponds made upon the wolds in Yorkshire, are generally of a conical shape; hence their contents may be found by Problem 7, Part IV., Sect. I.

Mr. P. who is a *practical* measurer, has also very kindly communicated the following method of taking the dimensions of a conical pond in which there is water.

EXAMPLE.



Let ABC denote a perpendicular section of a conical pond, whose dimensions are required.

Extend a cord over the pond, with which take the diameter AB; make a ring fast to the middle of the cord or diameter, as at D, through which put the end of the plumb-line ADC; let your assistant keep one end of the diameter at B; while you hold the other, and the plumb-line at A; permit the plummet to descend to the bottom of the pond, as at C; then draw back both the cords, and measure CD, which will be the perpendicular depth of the pond.

Note 1 When the top of a pond is not a perfect circle, measure two diameters at right angles to each other, and take half their sum for a mean diameter.

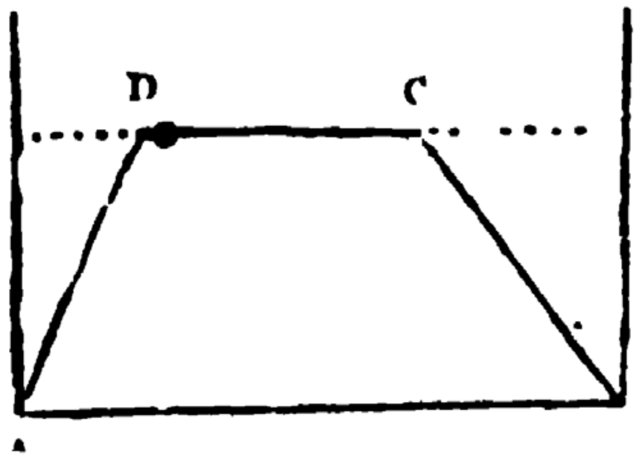
2. Cellars generally form parallelopipedons, and when they are dug by the cubic yard, their contents may be found by Prob. II., Sect I., Part IV.

DIRECTIONS FOR

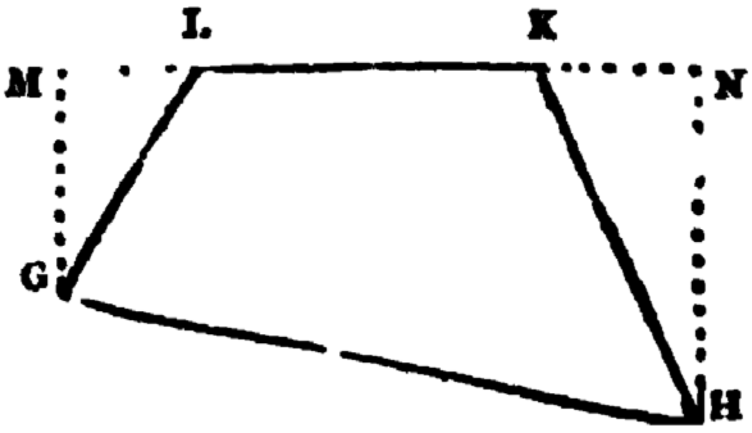
MEASURING EMBANKMENTS.

The most correct method of measuring embankments, is to proceed in the same manner as directed in Problem III., for drains and canals; and if the following observations be well understood, no difficulties will arise in taking the dimensions, and finding the areas of the sections.

Let ABCD represent a transverse perpendicular section of an embankment, made upon level ground; then if DC be parallel to AB, it is evident that the section is a trapezoid, the area of which may be found by Problem 8, Part II.



The distance of the parallel sides, or height of the section, may be obtained by placing a staff perpendicular to the horizon, at A or B, and producing DC to E or F; then will AE or BF be the perpendicular distance of the parallel sides AB, DC; and AB, the breadth of the bottom of the embankment, is evidently equal to ED + DC + CF.



Again, let GHKL, denote a perpendicular, transverse section of an embankment, made upon uneven ground; then it is evident that the section is a trapezium; and as the diagonals and perpendiculars cannot be measured, the content must be found in the following manner: Produce LK, the horizontal line of the top, both ways, to M and N, and let fall the perpendiculars MG and NH; then by Problem 8, Part II., half the sum of these parallel sides multiplied by their perpendicular distance MN, will give the area of the trapezoid GHNM, from which subtract the sum of the areas of the two right-angled triangles GML, HNK, and the remainder will be the area of the section GHKL.

Note Unskilful measurers affect to determine the contents of embankments by finding what they call mean breadths and thicknesses; but no person who has a scientific knowledge of mensuration, will have recourse to such an erroneous process, if the foregoing method can by any means be adopted. See Drains and Canals, Note 1.

DIRECTIONS FOR

MEASURING QUARRIES.

The baring of quarries is generally done by the cubic yard; and sometimes the stones themselves are got by the same measurement.

Quarries, in general, are very irregular; they may, however, commonly be measured by some of the methods already described for canals, marlpits, ponds, &c.

If a quarry be so irregular that none of these methods can be adopted, the general method is to take such dimensions as will give the area of a mean horizontal section; then this area being multiplied by the mean depth of the quarry; the product is taken for the content.

Note 1. Sometimes a quarry may be most correctly measured by dividing it into several parts; and taking the dimensions of each part separately.

2. When the stones that are left jutting out of the sides of the quarry, are measured with the vacuity, mean dimensions of these stones must be taken, and their contents subtracted, in order to obtain the true content of the vacuity

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DIRECTIONS FOR

MEASURING COAL-HEAPS.

When the stock of a colliery is to be taken, or an exchange of tenants takes place, it becomes necessary to ascertain the quantity of coals which are laid unsold at the pits; and as pit-heaps are generally very irregular, both in extent and thickness, it is no easy task to find their contents with a tolerable degree of accuracy.

The method generally adopted, is to take such dimensions as will give the area of a mean horizontal section of the heap, and to multiply this area by the mean thickness, for the content.

Note 1. The perpendicular height of any point at the extremity of a coal-heap, may be obtained in the following manner: Place a spirit or water-level upon the top of the heap, close by the edge, let your assistant hold a pole upon the ground, in a perpendicular direction, at the bottom of the heap; direct the level towards the pole, and note that point in it which is seen through the level, measure the distance between the bottom of the pole and this point, subtract the height of the level from the said distance; and the remainder will be the height of the heap, at the place where the observation is made.

2. When a coal-heap is upon level ground, the height or thickness of any place between the pit and the extremity of the heap, may be found thus. By Note 1, find the height of the heap where it is most elevated, then let your assistant place the pole upon that part of the heap the height of which you wish to obtain, and without removing the level from the highest point of the heap, direct it towards the pole, measure the distance between the bottom of the pole and the point seen through the level, from this distance take the height of the level, and if the remainder be subtracted from the greatest height of the heap, before found, you will obtain the height required.

3. If the ground on which the coal-heap is laid be level, and the upper surface of the heap a regular inclined plane, rising gradually from the pit to the utmost extremity of the heap, which is sometimes the case, it is evident that the mean thickness of the heap will be equal to half the sum of the two heights taken at that part of the heap adjoining the pit, and at the utmost extremity or highest point of the heap.

4. When a coal-heap is extremely irregular, it is generally necessary to divide it into several parts, and take the dimensions of each part separately. In this case, heights must be taken in such places as are most likely to give the mean thickness of each part; and when the ground is not level, a proper allowance must be made for this circumstance.

5. In some parts of England, 5 pecks, Winchester measure, or 2688 cubic inches, make a bushel of coals, and 36 bushels a chaldron; therefore, if the cubic inches in a coal-heap be divided by 2688, or the cubic feet by 1.555, the quotient will be the number of bushels contained in the heap, but as this measure is not general, every person who measures a coal-heap, ought to make himself acquainted with the customary measure of the place.—The content of the new imperial coal-bushel, is 2815 cubic inches. *(See the Author's Arithmetic, page 53, and 353.) (For the New Imperial Measures, see Part VIII. of this Work.)*

6. Notwithstanding what has been advanced on the subject of measuring quarries and coal-heaps, a great deal will always depend upon the ingenuity of the measurer; for it is impossible to give directions that will suit every particular case to be met with in the practice of measuring these irregular figures.

CLAY-HEAPS.

Clay is frequently dug out of pits, and laid upon the surface of the ground, generally in very irregular heaps; and then sold by the cubic yard or by the ton, for the purpose of making bricks.

When this is the case, it becomes necessary to ascertain its solid content, as nearly as possible; and this may be approximated by the following methods: Find the area of the base of the heap, and also the area of its top, in the most convenient manner, according to the rules and directions given in the different Problems of Part II.; and then take half the sum of these areas, for the mean area.—Also, take several depths, and divide their sum by their number, for a mean depth. Multiply the mean area by the mean depth, and the product will be the solid content; and, when this is in cubic feet, divide it by 27, to reduce it to cubic yards.

Note 1. When the base of a clay-heap, is very irregular, its area may be found by Problem 5, Part III.

2. When the dimensions are taken in yards, tenths, and hundredths, the solid content will be in cubic yards, and decimals of a yard; but if the dimensions be taken in feet, tenths, and hundredths, the content will be in cubic feet, and decimals of a foot.

3. When a clay-heap is not very irregular, the mean area may sometimes be found by taking the dimensions of the top of the heap, and increasing each dimension, as you take it, so as to give, (as nearly as you can judge,) the dimensions of the middle horizontal section.—This method of measuring, is generally called "*Giving and Taking*," as the dimensions thus obtained, are smaller than those of the base of the heap, but larger than those of the top.

4 When a clay-heap assumes something of the form of a parallelopipedon, (Problem 2, Part IV.) some measurers take lengths, in two or three different places and divide their sum by their number, for a mean length. A mean breadth and a mean depth are found in the same manner, and the continued product of these three dimensions, is taken for the solid content.

5 When a heap is very large and irregular, it is frequently advisable to divide it into two or three parts, and take the dimensions, and find the content of each part separately.

6 When a clay-heap resembles a prismoid, or the frustum of a cone, the rule given in Problem 10, Part IV, may be applied with propriety. For other useful Rules and Observations, the reader is referred to Problem 18, Part IV, and also to the methods of measuring marl-pits, and coal-heaps.

7 A cubic foot of clay, according to the table of specific gravities, page 173, weighs 2160 ounces avoirdupois, consequently, a cubic yard will weigh 1.627 tons, therefore, if we multiply the number of cubic yards, in a clay-heap, by 1 627, the product will be the weight in tons.

8 By experience, it has been found, that when clay is very compact, a cubic yard will weigh 1¾ tons, consequently, the multiplier will be 1 75; but when it is less dense, a cubic yard will not weigh more than 1¼, or 1½ tons, in these cases, the multiplier will be 1.25 or 1.5, to bring cubic yards into tons.

9. A measuring tape divided into feet, tenths, and half tenths, is best adapted for taking the dimensions; as we then have no trouble in reducing the inches to the decimals of a foot; for, in this case, the tenths are given by the tape; and the half-tenths are 5 hundredths of a foot; hence we have always the dimensions in feet and tenths, or in feet, tenths, and hundredths of a foot; thus, 28.6½ feet = 28.65 feet, &c.

10. The rules and directions given for clay-heaps, will apply to all *compost* and *manure* heaps; when it is required to determine their solid contents for the purposes of sale, &c.

EXAMPLE.

Required the number of cubic yards, in two clay-heaps, resembling parallelopipedons, from the following dimensions; and also the weight of the clay, supposing each cubic yard to weigh 1.627 tons.

No.	LENGTHS.	BREADTHS.	DEPTHS.
	Feet	*Feet*	*Feet*
1	93.85	56.55	9.65
	86.75	48 45	8.85
	74.65	39.35	7.70
2	62.25	43.25	8.35
	58.30	39.70	
	46.45		

Content by the third and fourth Notes.

No. I.

Length.	Breadth.	Depth.
Feet.	*Feet.*	*Feet*
93.85	56.55	9.65
86.75	48.45	8.85
74.65	39.35	7.70
3)255.25	3)144.35	3)26.20
85.083 *mean.*	48.116 *mean.*	8.666 *mean.*

Then, 85.083×48.116=4093.853628 *square feet, the mean area; and* 4093.853628×8.666=35477.335540248 *cubic feet, the solid content of No. I.*

No. II.

Length.	Breadth.	Depth.
Feet.	*Feet.*	*Feet.*
62.25	43.25	8.35
58.30	39.70	
46.45	2)82.95	
3)167.00	41.475 *mean.*	
55.666 *mean.*		

Then, 55.666 × 41.475 = 2308.74735 square feet, the mean area; and 2308.74735 × 8.35 = 19278.0403725 cubic feet, the solid content of No. II.

Lastly, 35477.335 + 19278.040 = 54755.375 cubic feet, the solid content of both heaps; then, 54755.375 ÷ 27 = 2027.9768 cubic yards, the whole content.

Weight by the seventh Note.

Here, 2027.9768 × 1.627 = 3299.5182536 tons = 3299½ tons, the weight required.

REMARK.

Having, in the preceding pages, treated of the Mensuration of Drains and Canals, it is presumed that a short account of a few of the principal ones, will not be unacceptable to the young reader; as it will tend to give him some idea of the great Improvements that Agriculture and Commerce have received, and are daily receiving by the use of them; and will inform him what stupendous Works have been effected by the Ingenuity, Perseverance, and united Efforts of men.

A DESCRIPTION
OF SOME OF THE
PRINCIPAL CANALS
In England, Scotland, France, and China.

CANALS are to be met with in every civilized country; and perhaps it will not be going too far to say, that the internal commerce of no nation has received greater improvements by them, than that of Great Britain.

THE DUKE OF BRIDGEWATER'S CANAL, is a work that begins at Worsley, seven miles from Manchester, where, at the foot of a mountain, composed of coal, a basin is cut, containing a great body of water, which serves as a reservoir to the navigation. The canal runs under a hill, by a subterraneous passage from this basin, nearly three quarters of a mile, to the

Duke's coal-works. In some places the passage is cut through the solid rock, and in others arched over with brick; and at certain distances, air funnels, some of which are thirty-seven yards in perpendicular height, are cut through the rock to the top of the hill. At Barton-bridge, three miles from the basin, is an aqueduct, which, for upwards of 200 yards, conveys the canal across a valley and the navigable river Irwell. There are three arches over this river; the middle one is 60 feet wide, and 38 feet above the water in the Irwell; and will admit the largest barges that navigate that river, to pass under with masts and sails standing. At Longford bridge, the canal turns to the right, and crossing the Mersey, passes near Altringham, Dunham, Grappenhall, and Kaulton, into the tideway of the Mersey, at Runcorn Gap, where barges can come into the canal from Liverpool at low water.

This canal is 29 miles in length. It was begun in 1758, and finished in 1763, under the direction of the great mechanic and engineer, Brindlev.

Note. From this canal, at Worsley, there is a branch cut to the town of Leigh; and thence to Wigan, where it communicates with the Leeds and Liverpool canal.—From the Duke's canal, there is also another branch which terminates at Manchester; and thus unites this great *emporium* with the maritime town of Liverpool.

THE GRAND JUNCTION CANAL, is a work that joins several other canals in the centre of the country, which thence form a communication between the rivers Thames, Severn, Mersey, and Trent, and, consequently, an inland navigation to the four principal sea-ports, London, Bristol, Liverpool, and Hull.

This canal commences at Braunston, on the west-borders of Northamptonshire, passes by Daventry, to Stony Stratford, in Buckinghamshire, thence on the confines of Bedfordshire, west of Leighton Buzzard, to Tring, Berkhamstead, and Rickmansworth, in Herts, and through Middlesex, by Uxbridge, to Brentford, where it enters the Thames, 12 miles, by that river, above London. Its whole length is upwards of 90 miles.

THE GRAND TRUNK CANAL, is a work that forms a communication between the rivers Mersey, and Trent, and, of course, between the Irish sea and the German ocean. Its length is 92 miles, from the Duke of Bridgewater's canal at Preston on the Hill, in Cheshire, to Wildon-ferry, in Derbyshire, where it communicates with the Trent. The canal is carried over the river Dove, in an aqueduct of 23 arches, and over the Trent, by an aqueduct of 6 arches. At Preston on the Hill, it passes under ground 1241 yards; at Barton, and in the neighbourhood, it has two subterraneous passages; and, at Hardcastle Hill, in Staffordshire, it is conveyed under ground 2880 yards. In the neighbourhood of Stafford, a branch is made from this canal, to run near Wolverhampton, and to join the Severn near Bewdley: from this again other branches cross Warwickshire to Braunston, where commences the Grand Junction Canal to the Thames at Brentford.

This canal was begun in 1766, by Brindley, who died before its completion; but it was finished by Mr. Henshall, his brother-in-law, in 1777.

THE LEEDS AND LIVERPOOL CANAL. "The first act for this canal passed in 1770, by which the undertakers, a body corporate, by the name of "*The Company of Proprietors of the Canal Navigation from Leeds to Liverpool*," were empowered to raise the sum of 320,000 l. in shares, deemed personal estate.

The work was prosecuted with spirit; and in a few years there were finished 33½ miles on the Yorkshire side, viz. from Leeds to Holme Bridge, near Gargrave; and 28 miles on the Lancashire side, viz. from Liverpool to Newbrough; but the money subscribed being all expended, the works were discontinued for several years.

In 1790 another act was obtained, to enable the company to vary the line from the north to the south side of the river Calder; and to raise a further sum of money: in consequence of which the works were again resumed.

In 1794 another act passed, to enable the company to make a considerable alteration in the line, so as to pass by the market-towns of Burnley, Blackburn, Chorley, and Wigan, where it joins the head of the Douglas Navigation; by means of which it unites the Lancashire side of this canal at Newbrough.

The whole length of the canal, according to this arrangement, is 129 miles.

The summit of this canal is at a village, called Foulridge, near Colne, where it passes under ground 1630 yards, by a tunnel, which is 18 feet high, and 17 feet wide. The fall eastward, from the summit to the navigable river Aire at Leeds, is 409½ feet, and westward to the basin, at Liverpool, 431 feet, which basin is about 52 feet above the river Mersey, at low water. Between the summit and Leeds are 44 locks, viz. 15 to Holme Bridge, (from which place to near Bingley, there is a level pool of 17 miles in length) 11 from Bingley to the Junction with the Bradford canal, and 18 from thence to Leeds. Between the summit and Liverpool are 47 locks.

There are several large aqueduct bridges on this canal, particularly those over the river Aire below Bingley, and above Gargrave.

It has a communication with the town of Bradford, by means of a branch, about three miles in length, called "Bradford Canal"

The general advantages which the public will derive from this canal, are more than the limits of this paper will allow to be explained. It will tend greatly to encourage trade and manufactures; will bring into extensive use inexhaustible rocks of lime-stone, slate, flags, and free-stone, excellent mines of coal; and great quantities of timber for building ships, houses, &c.

This canal is 42 feet wide at the top, with 5 feet depth of water; and the dimensions of the locks upon it, are about 70 feet in length, and 15½ feet in width.

It was completed in the latter part of the year 1816; and *now* forms a most eligible communication between the port of Liverpool and the town of Leeds; and from thence, by the river

Aire, and other canals and river navigations, to the port of Hull. Thus the object of the original promoters of this noble undertaking is obtained, viz. "*A Navigable Communication between the East and West Seas.*"

Note. Mr. Thomas Beeston, of the Canal office, Leeds, and Mr. John Bottomley, of the Canal office, Bradford, very kindly favoured me with the above description of the Leeds and Liverpool Canal.

THE GREAT CANAL, in Scotland, is a work that forms a junction between the Forth and Clyde. Its length is 35 miles from the influx of the Carron, at Grangemouth, to the junction with the Clyde, six miles above Dumbarton. In the course of this navigation, the vessels are raised to the height of 155 feet above the level of the sea; and passing afterward upon the summit of the country for 18 miles, they descend into the river Clyde, and have from thence free access to the Atlantic ocean. This canal is carried over 36 rivers and rivulets, and 2 great roads, by 38 aqueducts of hewn stone. In some places it passes through mossy ground, and in others through solid rock.

The road from Edinburgh to Glasgow passes under it near Falkirk, and over it by means of a drawbridge, six miles from Glasgow. In the course of this inland navigation are many striking scenes; particularly the romantic situation of the stupendous aqueduct over the Kelvin, near Glasgow, 420 feet in length, carrying a great artificial river over a natural one, where large vessels sail at the height of 65 feet above the bed of the river below. The utility of this communication between the German and Atlantic oceans, to the commerce of Great Britain and Ireland, in their trade with Norway, Sweden, and the Baltic, must be strikingly evident; as it shortens the nautical distance in some instances 800, and in others 1000 miles.

This canal cost about 300,000*l.* It was begun in 1768, under the direction of Mr. Smeaton, and finished in 1790, by Mr. Whitworth; Mr. Smeaton having resigned, in consequence of the bad state of his health.

THE ROYAL CANAL OF LANGUEDOC, in France, is a work that effects an inland communication between the Mediterranean and Atlantic. From the port of Cette, in the Mediterranean, it crosses the lake of Thau; and, below Toulouse, is conveyed by three sluices into the Garonne. At St. Ferreol, near Revel, between two rocky hills, is a grand basin, above 1000 feet in diameter, into which the rivulet Laudot is received; and hence three large cocks of cast brass open, and discharge the water, which then goes under the name of the river Laudot, and continues its course to the canal called Rigole de la Plaine. Thence it is conveyed to another reservoir near Naurouse, out of which it is conveyed by sluices, both to the Mediterranean and Atlantic, as the canal requires it; this being the highest point between the two seas. Near Beziers are eight sluices, which form a regular and grand cascade, 936 feet long, and 66 feet high, by which vessels cross the river Orb, and continue their voyage on the canal. Above it, between Beziers and Capestan, is the Mal-Pas, where the canal is conveyed, for the length of 720 feet, under a mountain. At Adge is a round sluice, with three openings, three different depths of water meeting here; and the gates are so contrived, that vessels may pass through by opening which sluice the master pleases. The canal has 37 aqueducts, and its length from Toulouse to Beziers, where it joins the river Orb, is 152 miles.

This canal was begun in 1666, and completed in 1681, under the direction of the engineer Riquet.

THE IMPERIAL CANAL, in China, is a stupendous work, extending nearly through the whole empire, from north to south; and serves to convey goods between Pekin and Canton; being only interrupted by a mountain, in the province of Kiang-si, of about one day's journey. This canal was finished about the year 980; and 30,000 men were employed 43 years in completing it; its whole length being 825 miles. The traffic upon it is exceedingly great; and it is, in various other respects, an object of wonder and admiration to Europeans.

There are in England, as well as in other nations, many more Canals, far too numerous to be mentioned here; those who desire to see a more circumstantial account, may consult Mr. Phillips's *General History of Inland Navigation*, which is a work replete with useful information.

AN ACCOUNT OF SOME OF THE

Principal Drains in the County of York,

AND THE

AGRICULTURAL IMPROVEMENTS

MADE BY THEM.

EXTENSIVE Drainages have of late years been executed in this county; and little land, which is capable of improvement by such means, now remains undrained.

The Drainage known by the name of the "Holderness Drainage," lies chiefly adjoining to and on the east side of the river Hull. It extends from north to south about eleven miles; and contains, by survey, 11211 acres. Previously to the execution of this undertaking, much of the level was usually covered with water for above half of the year; and on some parts were extensive cars or lakes.

This Drainage, though still imperfect, has made a great improvement in the adjoining lands, many of which from having been of little or no value, are now let from fifteen to thirty shillings per acre; and this has been accomplished at an expense of somewhat less than *5l* per acre.

The Act of Parliament for executing this Drainage was obtained in 1762; but it was not until the year 1775, that an assessment was levied for 24,000*l.* to carry it into effect. Since that time, engineers have been consulted on the further improvement of this Drainage; and the consent of the proprietors has been obtained for raising further supplies; in consequence of which the sum of 30,000*l.* has been advanced, in order to complete the work.

The winter of 1763-4 being unusually wet, the water on the west-side of the river Hull was higher than common, and occasioned a breach in the bank of the river, near the upper end of this level, by which the whole of it was overflowed, together with part of the townships of Stoneferry, Marfleet, Bilton, Preston, Southcoats, and Drypool. The turnpike road between Hull and Headon, for nearly four miles, stood from two to four feet deep in water, as did likewise that leading from Whitecross to Beverley, and persons going to market were obliged to pass in boats; but it is now believed that these low-lands will never be flooded again, if the banks of the river Hull can be preserved strong enough to resist the water of floods.

The Beverley and Barmston Drainage lies parallel to the Holderness Drainage, but on the contrary side of the river Hull; and extends from Barmston, along the course of that river, nearly to Kingston-upon-Hull, a distance of about twenty-four miles.

The Act of Parliament for executing this Drainage, was obtained about the year 1792. It is divided into two parts, the upper or more northern of which, containing 2,136 acres, extends from the township of Foston to Barmston, where it has an outfall into the German ocean. The lower or southern part commences at the same point, and has its outlet into the river Hull, at a place called Wincolmlee, near the town of Hull. It contains 10,432 acres.

This Drainage is very effectual, as the water in the drains commonly stands three or four feet below the surface of the land.

The expenses hitherto incurred, amount to 135,000*l.* or about ten pounds fifteen shillings per acre; and the average improvement of the land is about fourteen shillings per acre per annum; but as the annual expense of keeping the drains and works in repair, amounts to between twelve and thirteen hundred pounds, the clear interest upon the capital expended, is reduced to little more than 5*l.* per cent. per annum; hence it appears not to have been a very profitable undertaking to the proprietors. The public, however, has been benefited by land being brought into cultivation, which before was of little value.

The Keyingham Drainage is divided into two parts, having distinct outlets. The eastern part commences at Sand-le-Mere, and discharges itself into the Humber at a place called "Stone Creek;" and is in length about fourteen miles. The other part lying a little to the north and west of the former, commences at Aldborough, and empties itself also into the Humber at a place called "Headon Haven;" being in length about eleven miles.

The Act of Parliament for executing this Drainage was obtained in 1802; and the level contains 5,505 acres of land.

The whole expenses of the act and the completion of the works, amounted to 30,000*l.* or about five pounds nine shillings per acre. Prior to the commencement of this Drainage, the Commissioners made a valuation of the low grounds; and subsequently to the completion of it, they made an estimate of the improvements which had been effected, amounting to 2,475*l.* per annum, which is rather more than 8*l.* per cent.; hence it appears that this has been a profitable undertaking.

The Harford and Derwent Drainage contains upwards of 10,500 acres, of which nearly 4,500 are situated in the East Riding, and the remainder in the North Riding.

The Act of Parliament for this Drainage, was obtained in

1800; and it has been completed at a total expense of 41,612*l.* which is nearly at the rate of 4*l.* per acre; but as the land has been benefited to the amount of 17*s.* per acre; after paying the annual assessment, the disbursements are paying an interest to the proprietors of about 21*l.* per cent., per annum; hence it appears that this has been a very advantageous undertaking.

Spulding-Moor and Walling-Fen Drainage was completed about thirty years ago; and has greatly improved about 20,000 acres of land which were before subject to be flooded. It commences at Market Weighton, and enters the Humber near a place called "Fossdyke Clough," situated upon the banks of that river.

This Drain also serves as a *navigable* Canal; and turns out greatly to the advantage of the land-owners and proprietors, not only in draining the lands, but by facilitating the carriage of the produce and manufactures of a large extent of country, which before were conveyed at a very great expense by land-carriage.

Note. Those who desire to see more on this subject are referred to the excellent Agricultural Survey of the East Riding of Yorkshire, by H. E. Strickland, Esq.

AN ACCOUNT OF

AGRICULTURAL IMPROVEMENTS

Made in the County of Lincoln,

BY MEANS OF DRAINS.

THE extensive Drainages which have been executed in different parts of the county of Lincoln are far too many to be enumerated here; but the following extract from Mr. Young's Agricultural

Survey of that County, will give the reader some idea of the great improvements that have been made by them: "It is very probable that no county in the kingdom has made equal exertions in this very important work of draining. The quantity of land thus added to the kingdom has been great; fens of water, mud, wild fowl, frogs, and agues, have been converted to rich pasture and arable ground, worth from 20*s.* to 40*s.* an acre: health improved, morals corrected, and the community enriched. These, when carried to such an extent, are great works; and reflect the highest credit on the good sense and energy of the proprietors.

"Without going back to very remote periods, there cannot have been less than 150,000 acres drained and improved, on an average, from 5*s.* an acre to 25*s.*; or a rental created of 150,000*l.* a year. But suppose it only 100,000*l.* and that the profit has, on an average, been received during the period of thirty years; the rental has in that time amounted to three millions, and the produce to nearly ten; and when with the views of a political arithmetician, we reflect on the circulation that has attended this creation of wealth through industry; the number of people supported, the consumption of manufactures; the shipping employed; the taxes levied by the State; and all the classes of the community benefited, the magnitude and importance of such works will be seen; and the propriety well understood of giving all imaginable encouragement and facility to their execution.

"Early in the days of republican France, decrees were issued for draining marshes: I do not ask what progress has been made? but I would demand, if any drainages equal to this have been executed in that kingdom during a century? From Bourdeaux to Bayonne, in one of the finest climates in Europe, nearly all is marsh. What Frenchman has been so actuated by the blessings of republican security, as to lay out one louis on that or any other marsh or bog?

"Undertakings of this kind prove the reliance of the people on the secure possession of what their industry creates; and an Englishman must examine his native land with a cold heart, who does not pray for the continuance of a system of legislation which has tended so powerfully to adorn, improve, and cultivate the country, and to diffuse prosperity and happiness through the whole of society."

MR. JOSEPH ELKINGTON'S

Method of Draining Land,

APPROVED OF, AND RECOMMENDED BY THE BOARD OF AGRICULTURE.

SEVERAL gentlemen have favoured the public with Treatises on the subject of draining land; but Mr. Elkington's Method appears to be now almost universally adopted. Mr. Elkington was a Warwickshire farmer, and practised the art of draining land with unexampled success. The publication of this art was represented, to the Board of Agriculture, to be one of the greatest means of promoting the improvement of this country that could be suggested. In consequence of this, the House of Commons, on the 10th of June, 1795, voted an Address, "That His Majesty would be graciously pleased to give directions for issuing to Mr Joseph Elkington, as an inducement to discover his Mode of Draining, such sum as his Majesty, in his wisdom, shall think proper, not exceeding the sum of 1000*l.* sterling."

Mr Elkington's health being extremely precarious, there was a risk that the public might lose the benefit of the knowledge he had acquired, by the experience of above thirty years, in a species of improvement, which, in these kingdoms, ought to be considered as the basis of every other. To prevent so unfortunate a circumstance, the Board resolved to send Mr. John Johnstone, to visit, in company with Mr. Elkington, the principal Drainages he was then executing; and to take Drawings and Sections of the same.

The result of that journey was drawn up by Mr. Johnstone; and has been published under the title of "An Account of the

Mode of Draining Land, according to the System practised by Mr. Joseph Elkington." This Work has been well received by the public; and has already been of great service in promoting agricultural improvements. It not only points out the best mode of draining *bogs* and *marshes*; but also treats very extensively of the most approved methods of making *under-drains*; hence it will be found of infinite service to every person who is troubled with springy grounds.

The following extracts from some of the Agricultural Reports of those counties in England, where Mr. Elkington executed the most remarkable Drainages, will serve to shew the superiority of his method over every other.

County of Warwick, by Mr. John Wedge.

"Draining is, without doubt, the first step towards the improvement of all wet land. It has been practised with much success in this county for several years; but more particularly so, since Mr Elkington, a farmer of this district, introduced a method of draining boggy lands, by making deep drains, and boring at the bottom or sides of them, through the different under-strata, so as to *tap* the springs, and thereby, in many instances, cure large tracts of land with very few drains."

County of Leicester, by Mr. John Monk.

"The most capital improvements have been made under the direction of a Mr. Elkington, who is supposed to be the first in that line in the world. After forming the drain, by beginning at the fall, and working upwards, he makes use of a *borer* to find the spring, with which he generally succeeds, which has a wonderful effect in draining the land."

County of Worcester, by Mr. W. J. Pomeroy.

"In speaking of under-drains, it may be thought right to mention, that various experiments have been made at Ewell-Grange, the seat of the Earl of Plymouth, and in that neighbourhood; but that by boring, after Mr. Elkington's method, deserves to be most particularly noticed."

County of Derby, by Mr. Thomas Brown.

"Every other method seems to bend to that practised by Mr. Elkington, whose practice is becoming every day more extensive, and seems to me the most effectual of all others, for carrying off subterraneous waters. He lays a stone drain from three to six feet below the surface, in such a direction as to cut the source of the spring, and with such a declivity as to scour itself. Wherever he finds the source of the spring below the level of his drain, he bores, and with such judgment, that, to a stranger, his auger seems possessed of the virtue of that rod with which Moses struck the rock; for the water immediately gushes out, and perhaps lays land that before was too wet to carry a sheep, sufficiently dry to carry the heaviest ox. This method is certainly effectual against springs."

Some readers may perhaps think that I have expatiated too much on the excellency of Mr. Elkington's mode of Draining, but I am of opinion that this science, which tends so powerfully to promote the general welfare, cannot be too much recommended.

PART VII.

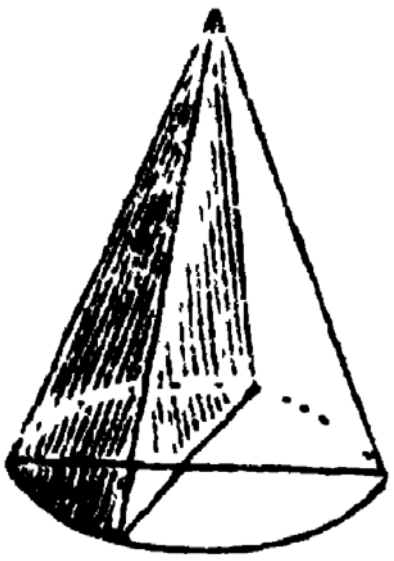
CONIC SECTIONS AND THEIR SOLIDS.

DEFINITIONS.

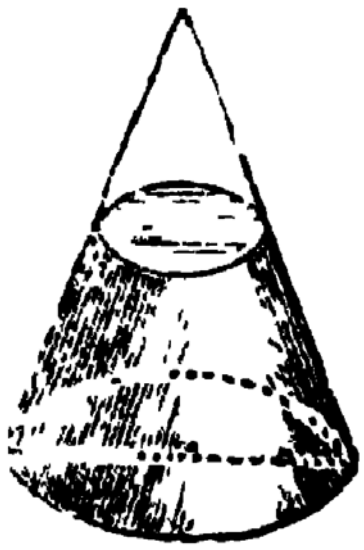
1. CONIC SECTIONS are plane figures formed by cutting a cone.

According to the different positions of the cutting plane, there will arise five different figures or sections.

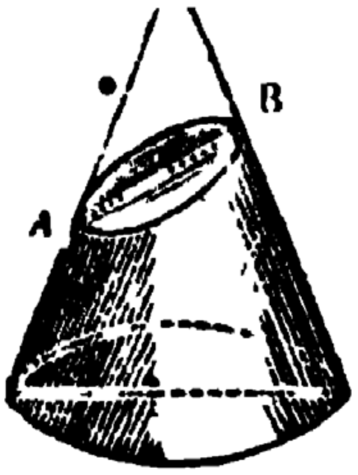
2. If the cutting plane pass through the vertex of the cone, and any part of the base, the section will be a *triangle*.



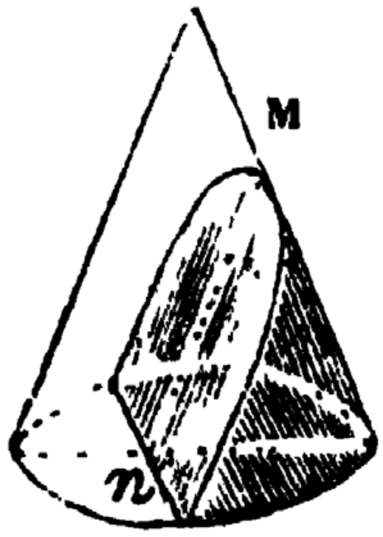
3. If a cone be cut into two parts, by a plane parallel to the base, the section will be a *circle*.



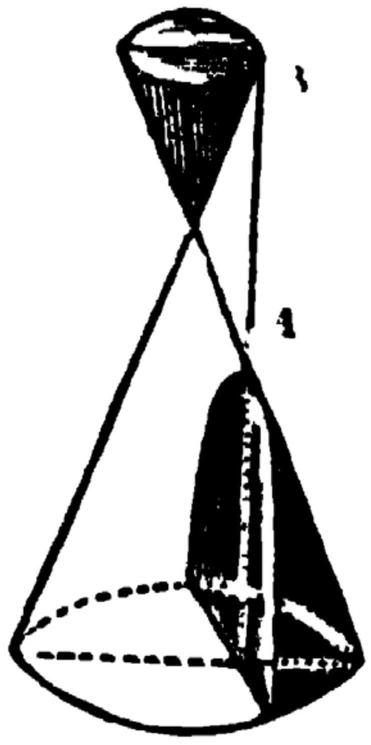
4. If a cone be cut by a plane passing through its two slant sides, in an oblique direction, the section will be an *ellipse*, or ellipsis.



5. If a cone be cut by a plane, which is parallel to either of its slant sides, the section will be a *parabola*.



6. If a cone be cut into two parts, by a plane, which, if continued, would meet the opposite cone, the section is called a *hyperbola*.



Note All the figures that can possibly be formed by cutting a cone, are mentioned in the preceding definitions, and are five in number, viz. a *triangle*, a *circle*, an *ellipse*, a *parabola*, and a *hyperbola*, but the last three only, are usually denominated CONIC SECTIONS.

7. The *vertex* of a conic section, is that point where the cutting plane meets the opposite sides of the cone, as at A and B; hence the ellipse and the hyperbola have, each of them, two vertices; but the parabola has only one *vertex*.

8 The *axis* or transverse diameter of an ellipse or a hyperbola, is the distance AB between the vertices; and the axis of a parabola is a right line drawn from the vertex, so as to divide the figure into two equal parts, as M*n*.

9. The *conjugate diameter* of an ellipse or hyperbola is a right line drawn through the centre of the figure, perpendicularly to the transverse; hence the semi-conjugate diameter of a hyperbola is the perpen-

dicular distance from the centre of the figure, or middle of the transverse diameter AB, to the vertices of the opposite cones.

10. An *ordinate* is a right line drawn from any point in the curve, perpendicularly to the axis.

11. An *absciss* is that part of the axis which is contained between the vertex and the ordinate.

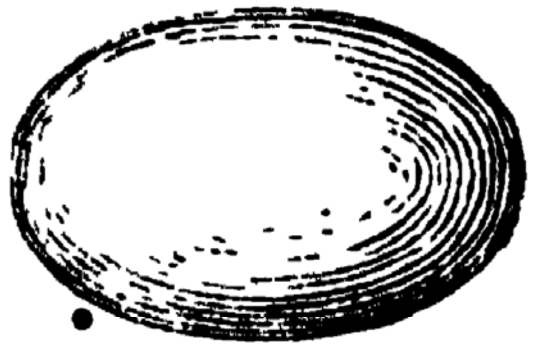
12. The *parameter* of any diameter is a third proportional to that diameter and its conjugate.

The parameter is sometimes called the *latus rectum.*

13. The *focus* is a point in the axis where the ordinate is equal to half the parameter.

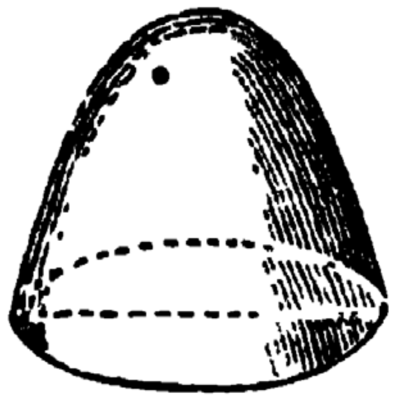
14. The ellipse and hyperbola have each two *foci*: but the parabola has only one *focus.*

15 A *spheroid* or *ellipsoid* is a solid generated by the revolution of an ellipse about one of its diameters. If the revolution be made about the transverse diameter, the solid is called a *prolate spheroid;* but if about the conjugate diameter, an *oblate* spheroid.



16. A *conoid* is a solid formed by the revolution of a parabola, or hyperbola, about its axis; and is accordingly called *parabolic,* or *hyperbolic.*

The parabolic conoid is also called a paraboloid; and the hyperbolic conoid, a hyperboloid.



17. An *elliptic,* a *parabolic,* or a *hyperbolic spindle,* is a solid formed by the revolution of a segment of an ellipse, a parabola, or a hyperbola, about its double ordinate, which remains fixed.

PROBLEM I.

In an ellipse, to find the transverse, or conjugate, or ordinate, or absciss; having the other three given.

CASE I.

When the transverse, conjugate, and absciss are given, to find the ordinate.

RULE. [illegible]

As the transverse diameter is to the conjugate, so is the square root of the product of the two abscisses, to the ordinate.

Note 1. An ellipse may be constructed by Problem 15, Part I., its area found by Problem 21, Part II., and its circumference by Note 3, of the same Problem.

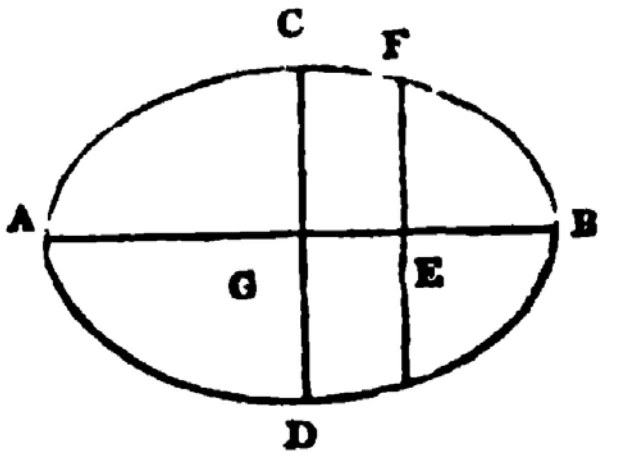
Also, the area of an elliptical segment may be obtained by Problem 22, Part II.

2. An ellipse may also be constructed in the following manner. Having found the foci F, f, as directed in Problem 15, Part I., take a thread equal in length to the transverse diameter AB, and fasten its ends, with two pins, in the points F, f, then stretch the thread to its greatest extent, and by moving a pencil round, within the thread, keeping it always tight, you will trace out the curve of the ellipse.

This construction is founded on the second property of the ellipse, given at the end of this Problem.

EXAMPLES.

1. In the ellipse ABCD, the transverse diameter AB is 60, the conjugate diameter CD 20, the absciss BE 12, and the absciss AE 48; what is the length of the ordinate EF?



As $60 : 20 :: \sqrt{48 \times 12} : \frac{20}{60}\sqrt{48 \times 12} = \frac{1}{3}\sqrt{576}$ $= \frac{1}{3} \times 24 = 8 =$ EF, *the ordinate required.*

2. If the transverse diameter be 100, the conjugate 75, and the less absciss 20; what is the ordinate? *Ans.* 30.

CASE II.

When the transverse, conjugate, and ordinate are given, to find the absciss.

RULE.

As the conjugate diameter is to the transverse, so is the square root of the difference of the squares of the semi-conjugate and ordinate, to the distance of that ordinate from the centre. Then this distance being added to the semi-transverse, will give the greater absciss; but if subtracted from it, the remainder will be the less absciss.

EXAMPLES.

1. The transverse diameter AB is 60, the conjugate diameter CD 20, and the ordinate EF 8; what are the abscisses AE and BE?

As $20 : 60 :: \sqrt{10^2 - 8^2} : \frac{60}{20}\sqrt{10^2 - 8^2} = 3\sqrt{100 - 64}$ $= 3\sqrt{36} = 3 \times 6 = 18 =$ EG, *the distance from the centre.*

Then $30 + 18 = 48 =$ AE, *the greater absciss;* *and* $30 - 18 = 12 =$ BE, *the less absciss.*

2. What are the two abscisses to the ordinate 30, the diameters being 100 and 75? *Ans.* 20 *and* 80.

CASE III.

When the conjugate, ordinate, and absciss are given, to find the transverse.

RULE.

From the square of the semi-conjugate subtract the square of the ordinate; and extract the square root of the remainder. Add this root to the semi-conjugate, if the less absciss be given; but when the greater absciss is given, subtract this root from the semi-conjugate; and reserve the sum or difference. Then say, as the square of the ordinate is to the rectangle of the conjugate and absciss, so is the reserved sum or difference to the transverse diameter.

EXAMPLES.

1. The conjugate diameter CD is 20, the ordinate EF 8, and the absciss BE 12; required the transverse diameter AB.

Here $\sqrt{10^2-8^2} = \sqrt{100-64} = \sqrt{36} = 6$, *and* $10+6=16$; *then, as* $8^2 : 20\times 12 :: 16 : \frac{20\times 12\times 16}{8^2}$ $= \frac{20\times 12\times 2}{8} = \frac{5\times 12\times 2}{2} = 5\times 12 = 60$, *the transverse diameter required.*

2. If the conjugate diameter be 75, the ordinate 30, and the greater absciss 80; what is the transverse?

Ans. 100.

CASE IV.

When the transverse, ordinate, and absciss are given, to find the conjugate.

RULE.

As the square root of the product of the two abscisses is to the ordinate, so is the transverse diameter to the conjugate.

Note. The transverse diameter may be found from the conjugate, in the same manner; using the abscisses of the conjugate, and their ordinate perpendicular to the conjugate.

EXAMPLES.

1. The transverse diameter AB is 60, the ordinate EF 8, and the absciss BE 12; required the conjugate diameter CD.

Here $60 - 12 = 48 =$ AE, *the greater absciss, and* $\sqrt{48 \times 12} = \sqrt{576} = 24$; *then, as* $24 : 8 :: 60 : \frac{8 \times 60}{24} = \frac{60}{3} = 20$, *the conjugate diameter required.*

2. If the transverse diameter be 100, the ordinate 30, and the absciss 20; what is the conjugate diameter? *Ans.* 75.

PROPERTIES OF THE ELLIPSE.

1. The square of the distance of the focus from the centre, is equal to the difference of the square of the semi-axes.

2 The sum of two lines drawn from the foci, to meet at any point at the curve, is equal to the transverse axis.

3. The parameter, or double ordinate at the focus, is equal to the square of the conjugate diameter divided by the transverse.

4. The rectangles of the segments of any diameter, are as the squares of their ordinates.

5 All the parallelograms circumscribed about an ellipse, are equal to one another, and each equal to the rectangle of the two axes

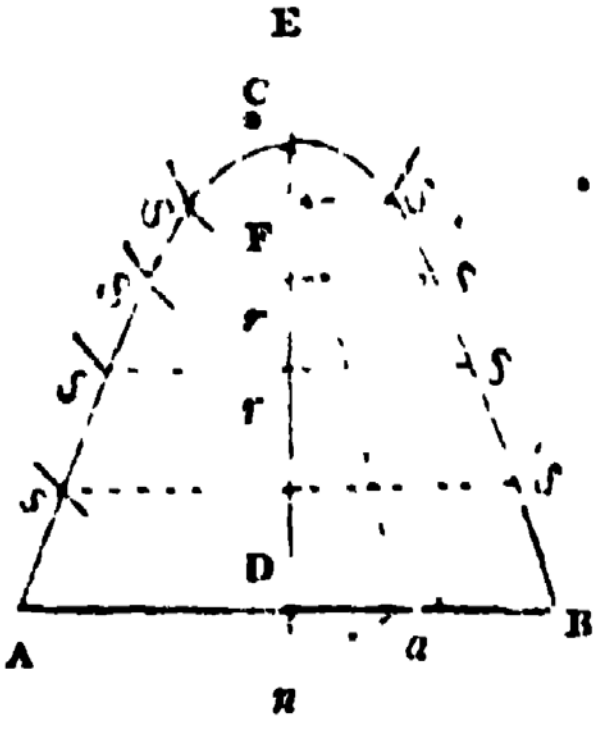
6 An ellipse is to the rectangle of the two axes, as any circle is to the square of its diameter.

7. The areas of ellipses are to one another, as the rectangles of their transverse and conjugate axes.

8 As the transverse axis of an ellipse is to the conjugate, so is the area of a circle whose diameter is the transverse, to the area of the ellipse

PROBLEM II.

To describe a parabola, the absciss CD, *and the ordinate* BD *being given.*



Construction. Bisect the ordinate BD in *a*; join C*a*, and draw *a n* perpendicular to it, meeting CD produced in *n*.

Make CE and CF each equal to D*n*; and F will be the focus of the parabola.

Take any number of points, *r*, *r*, &c. in the axis, through which draw the indefinite, double ordinates, *srs*, *srs*, &c. perpendicularly to CD.

With the radii EF, E*r*, &c. and the centre F, describe arcs cutting the corresponding ordinates in the points, *s*, *s*, &c.; then draw the curve through all the points of intersection, and ACB will be the parabola required.

Note. Various methods of constructing parabolas and hyperbolas, may be seen in Emerson's Conic Sections.

PROBLEM III.

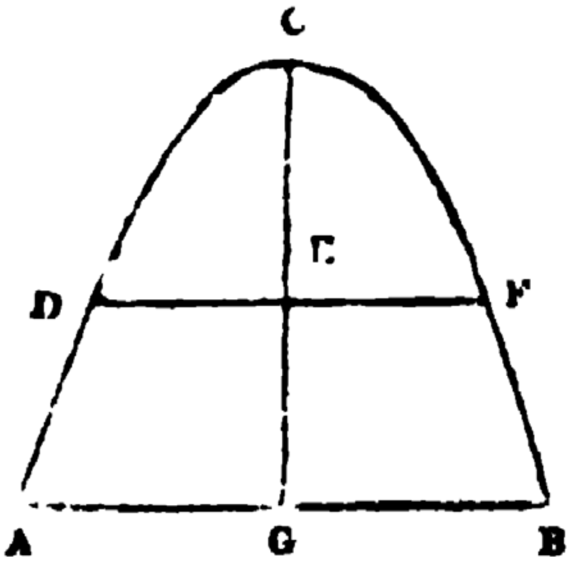
To find any parabolic absciss or ordinate

RULE.

As any absciss is to the square of its ordinate, so is any other absciss to the square of its ordinate. Or, as the square root of any absciss is to its ordinate, so is the square root of any other absciss to its ordinate; and *vice versâ*.

EXAMPLES.

1. The absciss CE is 9, and its ordinate DE 6, what is the ordinate AG, whose absciss CG is 16?



As $9 : 6^2 :: 16 : \frac{36 \times 16}{9} = \frac{576}{9} = 64$; *and* $\sqrt{64} = 8$ = AG, *the ordinate required.*

Or, as $\sqrt{9} : 6 :: \sqrt{16} : \frac{6 \times \sqrt{16}}{\sqrt{9}} = \frac{6 \times 4}{3} = \frac{24}{3} = 8$ $=$ AG, *the same as before.*

2. The ordinate AG is 8, the ordinate DE 6, and the absciss CG 16; what is the absciss CE?

As $8^2 : 16 :: 6^2 : \frac{16 \times 6^2}{8^2} = \frac{16 \times 36}{64} = \frac{576}{64} = 9 =$ CE, *the absciss required.*

Or, as $8 : \sqrt{16} :: 6 : \frac{\sqrt{16} \times 6}{8} = \frac{4 \times 6}{8} = \frac{24}{8} = 3$; *and* $3 \times 3 = 9 =$ CE, *the same as before.*

3. If an absciss be 12, and its ordinate 15; required the ordinate whose absciss is 27. *Ans.* $22\frac{1}{2}$.

4. If an absciss be 18, and its ordinate 12, what is the absciss whose ordinate is 16? *Ans.* 32.

PROBLEM IV.

To find the length of the arc of a parabola, cut off by a double ordinate.

RULE.

To the square of the ordinate add $\frac{4}{3}$ of the square of the absciss; multiply the square root of the sum by 2, and the product will be the length of the curve required.

Note. This rule will not hold good, when the absciss is greater than half the ordinate, that is, when the ordinate falls below the focus.

EXAMPLES.

1. If the absciss CE be 6, and the ordinate DE 12; what is the length of the curve DCF?

Here $12^2 + \overline{6^2 \times \frac{4}{3}} = 144 + \frac{36 \times 4}{3} = 144 + \frac{144}{3} = 144 + 48 = 192$; *and* $\sqrt{192} \times 2 = 13.8564 \times 2 = 27.7128$, *the length of the curve required.*

2. What is the length of a parabolic curve, whose absciss is 7, and ordinate 15? *Ans.* 34.0782.

PROBLEM V.

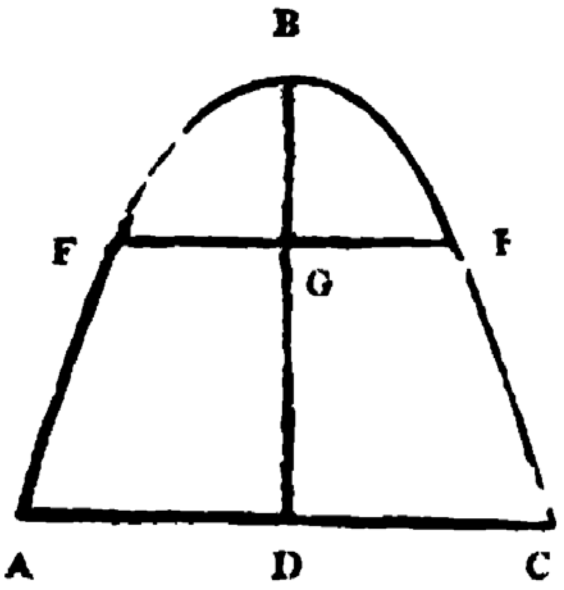
To find the area of a parabola, its base and height being given.

RULE.

Multiply the base by the height, and $\frac{2}{3}$ of the product will be the area.

EXAMPLES.

1. What is the area of the parabola ABC, whose height BD is 18, and base or double ordinate AB 24?



Here $24 \times 18 \times \frac{2}{3} = \frac{24 \times 18 \times 2}{3} = 8 \times 18 \times 2 =$ 288, *the area required.*

2. The absciss is 24, and the double ordinate 72; what is the area of the parabola? *Ans.* 1152.

PROBLEM VI.

To find the area of a parabolic frustum.

RULE.

Divide the difference of the cubes of the two ends of the frustum, by the difference of their squares; multiply the quotient by $\frac{2}{3}$ of the altitude, and the product will be the area.

EXAMPLES.

1. What is the area of a parabolic frustum AEFC, the end AC being 24, EF 16, and the altitude DG 10?

Here $\overline{24^3-16^3} \div \overline{24^2-16^2} = \overline{13824-4096} \div \overline{576-256} = 9728 \div 320 = 30.4$; and $30.4 \times \frac{10\times 2}{3}$ $= \frac{30.4\times10\times2}{3} = \frac{608}{3} = 202\frac{2}{3}$, *the area required.*

2. Required the area of a parabolic frustum, the greater end of which is 20, the less end 12, and the height 6.

Ans. 98.

PROPERTIES OF THE PARABOLA.

1 THE distance of the focus from the vertex is equal to the square of any ordinate divided by 4 times its absciss.

2. The parameter, latus rectum, or double ordinate at the focus, is equal to the square of any ordinate divided by its absciss.

3 The distance between the focus and the vertex is equal to ¼ of the parameter, or ½ of the ordinate at the focus.

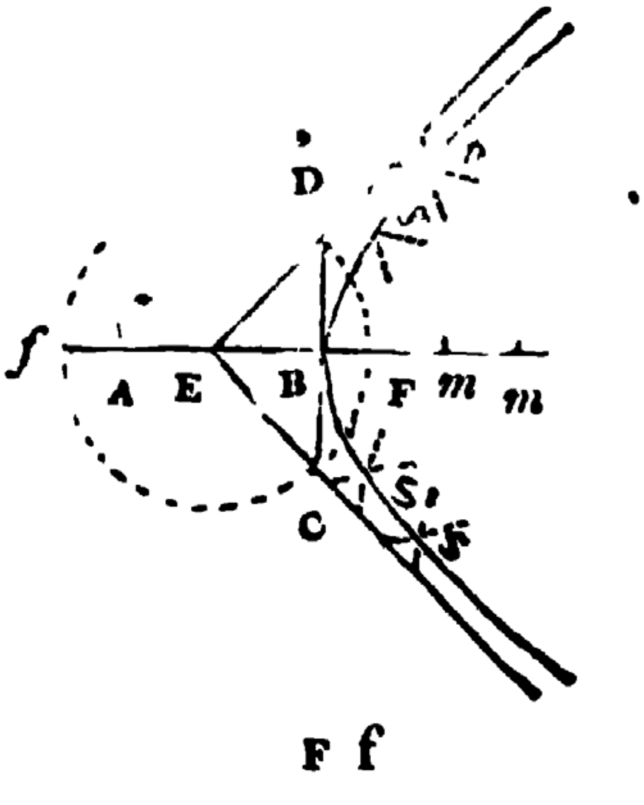
4. As the parameter is to the sum of any two ordinates, so is the difference of those ordinates to the difference of their abscisses.

5 All parabolas are similar to each other.

6. The area of a parabola is equal to ⅔ of the area of the circumscribing parallelogram.

PROBLEM VII.

To construct a hyperbola, having the transverse and conjugate diameter given.



Construction. Draw CD equal to the conjugate diameter; and bisect it perpendicularly with a transverse diameter AB. From E, the middle of the transverse diameter, with the radius EC or ED, describe the circle C*f*DF cutting AB produced in F and *f*, which points will be the foci of the hyperbola. In AB produced, take any number of points *m*, *m*, &c.; and from F and *f*, as centres, with the distances B*m*, A*m*, as radii, describe arcs cutting each other in the points *s*, *s*, &c. Through these points draw the curve *s*B*s*, and it will be the hyperbola required.

Note. If the straight lines EC, ED, be drawn from the point E, through the extremities of the conjugate axis CD, they will be the asymptotes of the hyperbola; the property of which is to approach continually to the curve, but not to meet it, until they be infinitely produced. (*See Emerson's Conic Sections, Book II. Prob.* 75.)

PROBLEM VIII.

In a hyperbola, to find the transverse, or conjugate, or ordinate, or absciss.

CASE I.

The transverse and conjugate diameters, and the two abscisses being given, to find the ordinate.

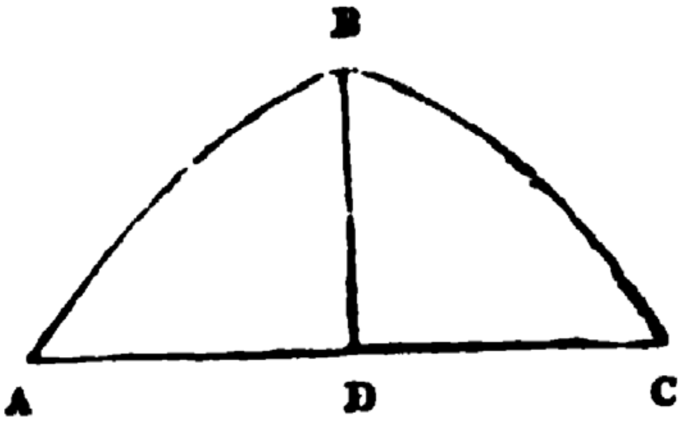
RULE.

As the transverse diameter is to the conjugate, so is the square root of the product of the two abscisses to the ordinate.

Note. The less absciss added to the transverse diameter, will give the greater absciss.

EXAMPLES.

1. In the hyperbola ABC, the transverse diameter is 30, the conjugate 18, and the absciss BD 10; what is the ordinate AD?



As $30 : 18 :: \sqrt{40 \times 10} : \frac{18 \times \sqrt{40 \times 10}}{30} = \frac{3 \times \sqrt{400}}{5}$ $= \frac{3 \times 20}{5} = 3 \times 4 = 12$, *the ordinate required.*

2. The transverse diameter is 48, the conjugate 42, and the less absciss 16; what is the ordinate?

Ans. 28.

CASE II.

The transverse and conjugate diameters, and an ordinate being given, to find the abscisses.

RULE.

To the square of half the conjugate, add the square of the ordinate; and extract the square root of the sum. Then say, as the conjugate diameter is to the transverse, so is the said square root to half the sum of the abscisses, or the distance between the ordinate and the centre.

Then this distance being added to half the transverse diameter, will give the greater absciss; and their difference will be the less absciss.

EXAMPLES.

1. The transverse diameter is 30, the conjugate 18, and the ordinate 12; what are the two abscisses?

Here $\sqrt{9^2 + 12^2} = \sqrt{81 + 144} = \sqrt{225} = 15$; *then, as* $18 : 30 :: 15 : \frac{30 \times 15}{18} = \frac{5 \times 15}{3} = 5 \times 5 = 25$, *half the sum of the abscisses; hence,* $25 + 15 = 40$, *the greater absciss; and* $25 - 15 = 10$, *the less absciss.*

2. If the transverse diameter be 48, the conjugate 42, and the ordinate 28; what are the two abscisses? *Ans.* 64 *and* 16.

CASE III.

The transverse diameter, the two abscisses, and the ordinate being given, to find the conjugate.

RULE.

As the square root of the product of the two abscisses is to the ordinate, so is the transverse diameter to the conjugate.

EXAMPLES.

1. The transverse diameter is 30, the ordinate is 12, and the two abscisses are 40 and 10; required the conjugate diameter.

Here $\sqrt{40\times10} = \sqrt{400} = 20$, *the square root of the product of the two abscisses; then, as* $20 : 12 :: 30 : \frac{12\times30}{20}$ $= \frac{12\times6}{4} = 3\times6 = 18$, *the conjugate required.*

2. The transverse diameter is 48, the ordinate 28, and the abscisses are 64 and 16; required the conjugate diameter. *Ans.* 42.

CASE IV.

The conjugate diameter, the ordinate, and the two abscisses being given, to find the transverse.

RULE.

To the square of half the conjugate add the square of the ordinate, and extract the square root of the sum. To this root add half the conjugate, when the less absciss is used, but subtract it when the greater absciss is used; and reserve the sum or difference. Then say, as the square of the ordinate is to the product of the absciss and conjugate, so is the sum or difference, above found, to the transverse required.

EXAMPLES.

1. The conjugate diameter is 18, the ordinate is 12, and the less absciss 10; required the transverse diameter.

Here $\sqrt{9^2+12^2} = \sqrt{81+144} = \sqrt{225} = 15$; *and* $15 + 9 = 24$. *Also,* $12 \times 12 = 144$, *the square of the ordinate;* *and* $18 \times 10 = 180$, *the product of the absciss and conjugate;* *then, as* $144 : 180 :: 24 : \frac{180 \times 24}{144} = \frac{15 \times 24}{12} \times 15 \times 2 = 30$, *the transverse required.*

2. The conjugate diameter is 42, the greater absciss is 64, and the ordinate 28; what is the transverse diameter? *Ans.* 48.

PROBLEM IX.

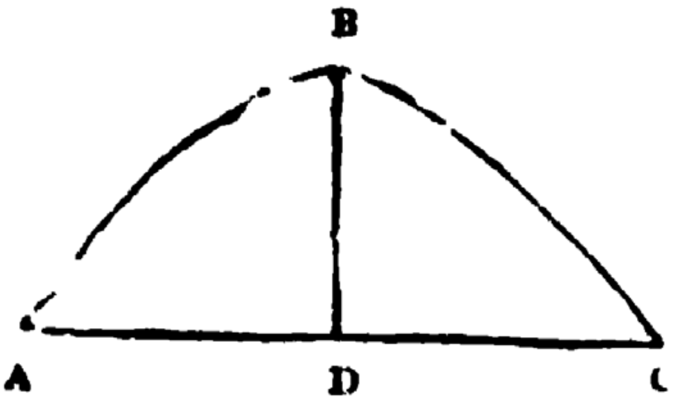
To find the length of a hyperbolic curve.

RULE.

To 21 times the square of the conjugate, add 9 times the square of the transverse; and to the said 21 times the square of the conjugate, add 19 times the square of the transverse; and multiply each sum by the absciss. To each of these two products add 15 times the product of the transverse multiplied by the square of the conjugate; then say, as the less of these sums is to the greater, so is the double ordinate to the length of the curve, *nearly*.

EXAMPLES.

1. The transverse diameter of the hyperbola ABC, is 30, the conjugate 18, the absciss BD 10, and the double ordinate AC 24; what is the length of the curve ABC?



Here $\overline{18^2 \times 21} + \overline{30^2 \times 9} = \overline{324 \times 21} + \overline{900 \times 9} = 6804 + 8100 = 14904 = 21$ *times the square of the conjugate, added to* 9 *times the square of the transverse;* *and* $6804 + \overline{900 \times 19} = 6804 + 17100 = 23904 = 21$ *times the square of the conjugate, added to* 19 *times the square of the transverse; then* $14904 \times 10 = 149040$, *and* $23904 \times 10 = 239040$.

Again, $324 \times 30 \times 15 = 145800 = 15$ *times the product of the transverse multiplied by the square of the conjugate; and* $149040 + 145800 = 294840$, *and* $239040 + 145800 = 384840$; *then, as* $294840 : 384840 :: 24 : 31.326007$, *the length of the curve required.*

2. The transverse diameter of a hyperbola is 45, the conjugate 27, the absciss 15, and the double ordinate 36; what is the length of the curve?

Ans. 46.989

PROBLEM X.

To find the area of a hyperbola, the transverse, conjugate, and absciss being given.

RULE.

Multiply the transverse by the absciss; to the product add $\frac{5}{7}$ of the square of the absciss, and multiply the square root of the sum by 21. Add the product, last found, to 4 times the square root of the product of the transverse and absciss; and divide the sum by 75. Divide 4 times the product of the conjugate and absciss by the transverse; then this quotient being multiplied by the former, will give the area, *nearly*.

EXAMPLES.

1. Required the area of a hyperbola, whose transverse diameter is 30, the conjugate 18, and the absciss 10.

Here $21\sqrt{30 \times 10 + 10^2 \times \frac{5}{7}} = 21\sqrt{300 + 100 \times \frac{5}{7}} = 21\sqrt{300 + 500 \div 7} = 21\sqrt{300 + 71.428571} = 21\sqrt{371.428571} = 21 \times 19.272 = 404.712$.

Again, $(4\sqrt{30 \times 10} + 404.712) \div 75 = (4\sqrt{300} + 404.712) \div 75 = (4 \times 17.3205 + 404.712) \div 75 = (69.282 + 404.712) \div 75 = 473.994 \div 75 = 6.3199$.

Lastly, $\frac{18 \times 10 \times 4}{30} \times 6.3199 = \frac{18 \times 4}{3} \times 6.3199 = 6 \times 4 \times 6.3199 = 24 \times 6.3199 = 151.6776$, *the area required.*

2 The transverse diameter of a hyperbola is 50, the conjugate 30, and the absciss 25; what is its area?

Ans. 805.0902.

PROPERTIES OF THE HYPERBOLA.

1. THE square root of the sum of the squares of the semi-axes, is equal to the distance of the focus from the middle of the transverse axis.

2. If half the transverse axis be subtracted from the distance between the focus and the middle of the said axis, the remainder will be the distance of the focus from the vertex.

3. The difference of two lines drawn from the two foci to any point in the curve, is equal to the transverse axis.

4 The parameter, latus rectum, or double ordinate at the focus, is equal to the square of the conjugate divided by the transverse.

Note Those who desire to obtain a complete knowlenge of the properties of Conic Sections, are referred to the works of Emerson, Hutton, and Simson, on that subject.

PROBLEM XI.

To find the solidity of a spheroid.

RULE.

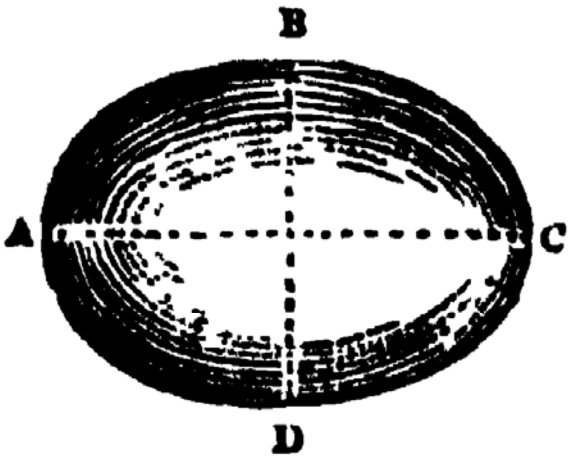
Multiply the square of the revolving axis by the fixed axis, and the product thence arising by .5236; and the last product will be the solidity.

Note 1. The multiplier .5236 is $\frac{1}{6}$ of 3.1416.

2. A semi-spheroid is equal to $\frac{2}{3}$ of a cylinder, or to double a cone of the same base and altitude.

EXAMPLES.

1. What is the solidity of the prolate spheroid ABCD, whose transverse or fixed axis AC is 50, and the conjugate or revolving axis BD 30?



Here $30^2 \times 50 \times .5236 = 900 \times 50 \times .5236 = 45000 \times .5236 = 23562$, *the solidity required.*

2. What is the content of an oblate spheroid, whose axes are 50 and 30? *Ans.* 39270.

3. Required the solidity of a prolate spheroid, whose axes are 45 and 95. *Ans.* 100727.55.

PROBLEM XII.

To find the solidity of the segment of a spheroid.

CASE I.

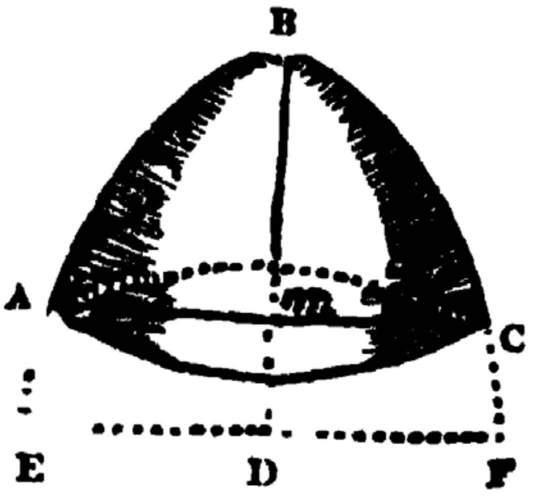
When the base is circular, or parallel to the revolving axis.

RULE.

From triple the fixed axe, take double the height of the segment; multiply the difference by the square of the height, and the product thence arising by .5236; then say, as the square of the fixed axe is to the square of the revolving axe, so is the last product to the solidity required.

EXAMPLES.

1. Required the solidity of the segment ABC of a prolate spheroid, the height B*m* being 5, the fixed axe, 2 BD 50, and the revolving axe EF 30.



Here $(\overline{50 \times 3 - 5 \times 2}) \times 5^2 \times .5236 = \overline{150 - 10} \times 25 \times .5236 = 140 \times 25 \times .5236 = 3500 \times .5236 = 1832.6$; *then, as* 2500 : 900 :: 1832.6 : 659.736, *the solidity required.*

2. The axes of a prolate spheroid are 100 and 60; what is the solidity of that segment whose height is 10, and its base parallel to the revolving axe?

Ans. 5277.888.

3. The axes of an oblate spheroid are 50 and 30; what is the solidity of that segment whose height is 6, and its base parallel to the revolving axe? *Ans.* 4084.08.

CASE II.

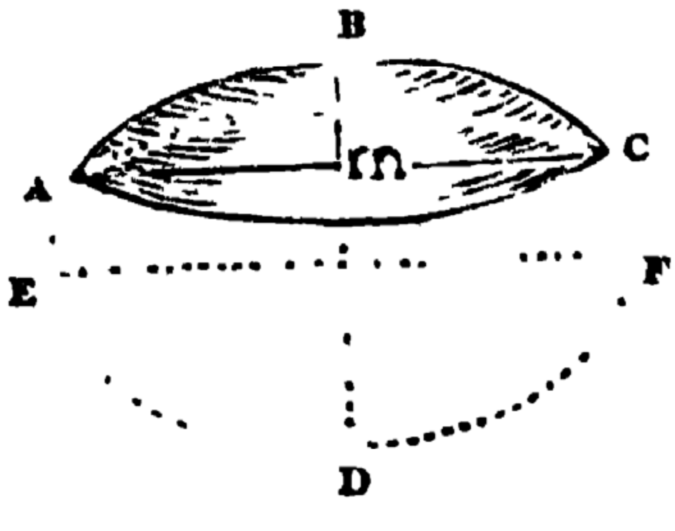
When the base is elliptical, or perpendicular to the revolving axe.

RULE.

From triple the revolving axe, take double the height of the segment; multiply the difference by the square of the height, and the product thence arising by .5236, then say, as the revolving axe is to the fixed axe, so is the last product to the solidity required.

EXAMPLES.

1. What is the solidity of the segment ABC of a prolate spheroid, the height Bm being 6, the fixed axe EF 50, and the revolving axe BD 30?



Here $(\overline{30 \times 3} - \overline{6 \times 2}) \times 6^2 \times .5236 = \overline{90 - 12} \times 36 \times .5236 = 78 \times 36 \times .5236 = 2808 \times .5236 = 1470.2688$; *then, as* 30 : 50 :: 1470.2688 : 2450.448, *the solidity required.*

2. The axes of an oblate spheroid are 50 and 30; what is the solidity of that segment whose height is 6, and its base perpendicular to the revolving axe?

Ans. 1560.74688.

PROBLEM XIII.

To find the content of the middle frustum of a spheroid.

CASE I.

When the ends are circular or parallel to the revolving axe.

RULE.

To twice the square of the middle diameter, add the square of the diameter of one end; multiply the sum by the length of the frustum, and the product thence arising by .2618, for the solidity.

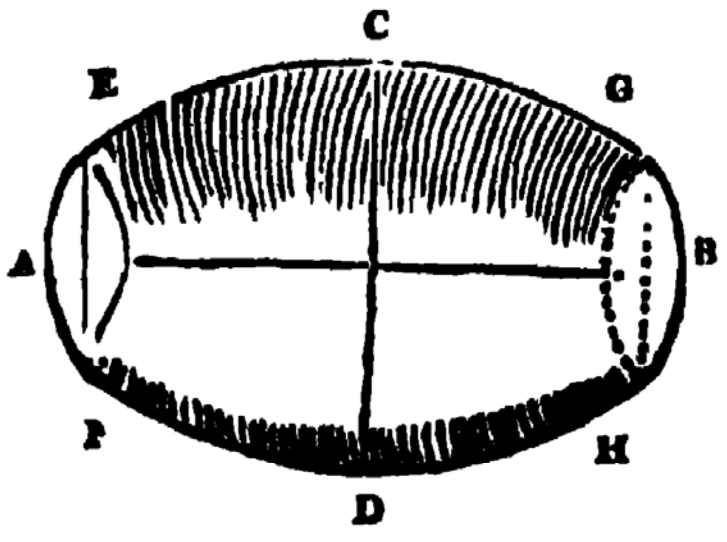
Note 1. A cask in the form of the middle frustum of a prolate spheroid is by Gaugers, called a cask of the *first variety*, hence this Rule is useful in cask gauging.

2. The old ale gallon contains 282, the old wine gallon 231, and the old Winchester bushel 2150.4 cubic inches.—The new imperial gallon contains 277.274, and the new imperial bushel 2218.192 cubic inches. *(See the New Imperial Tables, Part VIII.)*

3. If the content of any vessel, in cubic inches, be divided by 277.274, and 2218.192, the respective quotients will be the content in imperial gallons and bushels. *(See Gauging, Prob. 1, Part VIII.)*

EXAMPLES.

1. What is the solidity of the middle frustum EGHF of a prolate spheroid; the middle diameter CD being 30, the diameter of each end EF or GH 18, and the length AB 40?



Here $(\overline{30^2 \times 2} + 18^2) \times 40 \times .2618 = (\overline{900 \times 2} + 324) \times 40 \times .2618 = (1800 + 324) \times 40 \times .2618 = 2124 \times 40 \times .2618 = 84960 \times .2613) = 22242.528$, *the solidity required.*

2 What is the solidity of the middle frustum of an oblate spheroid; the diameter of each end being 40, the middle diameter 50, and the length of the frustum 18? *Ans.* 31101.84.

3. The length of a spheroidal cask is 30, the bung diameter 24, and the head diameter 18 inches; what is its content in imperial gallons?

Ans. 41.088 *gallons.*

CASE II.

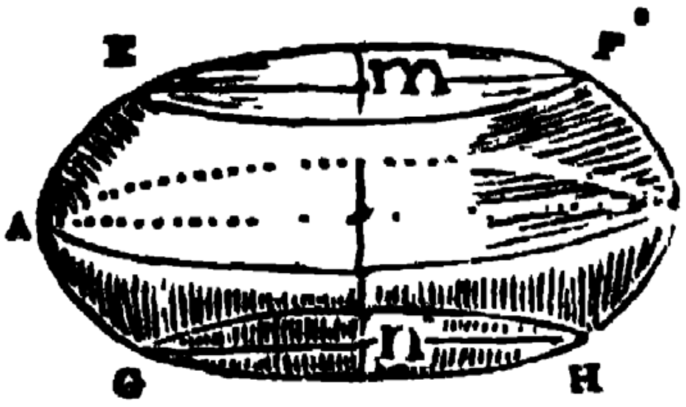
When the ends are elliptical, or perpendicular to the revolving axe.

RULE.

To twice the product of the transverse and conjugate diameters of the middle section, and the product of the transverse and conjugate of one end; multiply the sum by the height of the frustum, and the product thence arising by .2618, for the solidity.

EXAMPLES.

In the middle frustum EFGH, of an oblate spheroid, the diameters of the middle section AB are 50 and 30; those of the end EF or GH 40 and 24; and its height mn 18; what is the solidity?



Here $(\overline{50 \times 30 \times 2} + \overline{40 \times 24}) \times 18 \times .2618 =$

$(3000 + 960) \times 18 \times .2618 = 3960 \times 18 \times .2618 = 71280 \times .2618 = 18661.104$, *the solidity required.*

2. What is the solidity of the middle frustum of a prolate spheroid; the diameters of the middle section being 100 and 60, those of each end 80 and 48, and its height 36? *Ans.* 149288.832.

PROBLEM XIV.

To find the solidity of an elliptic spindle.

RULE.

From 3 times the square of the middle or greatest diameter take 4 times the square of the diameter at $\frac{1}{4}$ of the length, or equally distant between the middle and one end; and from 4 times this last diameter take 3 times the middle diameter; and $\frac{1}{4}$ of the quotient arising from dividing the former difference by the latter, will give the central distance.

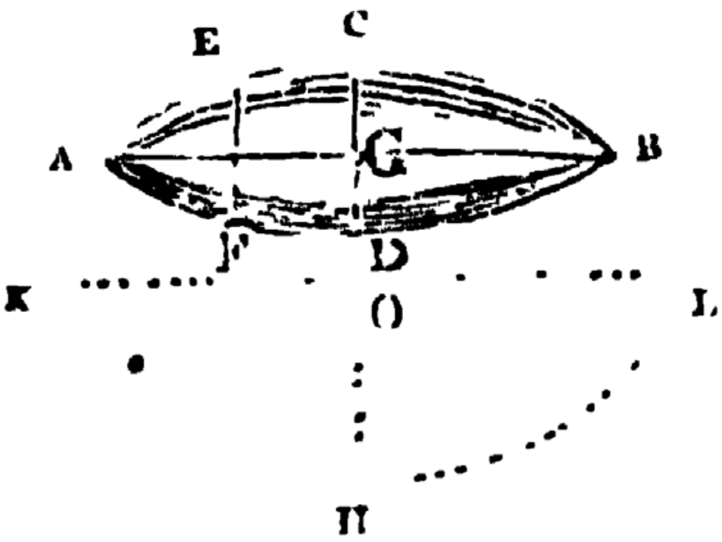
Find the axes of the ellipse; and also the area of the generating segment.

Divide 3 times this area by the length of the spindle; from the quotient subtract the middle diameter, and multiply the remainder by 4 times the central distance, before found.

Subtract this product from the square of the middle diameter; multiply the remainder by the length of the spindle, and the product thus obtained by .5236, for the solidity.

EXAMPLES.

1. What is the solidity of the elliptic spindle ACBDA, the length AB being 40, the middle diameter CD 12, and the diameter EF, at $\frac{1}{4}$ of the length, 9.49546?



Here $(\overline{12^2 \times 3 - 9.49546^2 \times 4} \div \overline{9.49546 \times 4 - 12 \times 3}) \times \frac{1}{4} = (\overline{432 - 360.0546} \div \overline{37.98184 - 36}) \times \frac{1}{4} = (71.3454 \div 1.98184) \times \frac{1}{4} = 36 \times \frac{1}{4} = 9$, *the central distance* OG.

Also, $\overline{9 + \frac{12}{2}} \times 2 = \overline{9 + 6} \times 2 = 15 \times 2 = 30$, *the conjugate diameter* CH.

By Problem I., Case 4, Part VII., we have $30 - 6 = 24 =$ GH, *the greater absciss;* and $\sqrt{\overline{24 \times 6}}\ \sqrt{144} = 12$; *then, as* 12 : 20 (AG) :: 30 (CH) : 50, *the transverse diameter* KL

Again, by Problem 22, Part II., we have $\frac{6}{30} = .2$, *the tabular height; and the corresponding Area Seg. is* 111823; *then* $.111823 \times 50 \times 30 = 5.59115 \times 30 = 167.7345$, *the area of the generating segment* ACB.

Now, $\left(\frac{167.7315 \times 3}{40} - 12\right) \times 9 \times 4 = \left(\frac{503.2035}{40} - 12\right) \times 36 = (12.5800875 - 12) \times 36 = .5800875 \times 36 = 20.88315$; *then* $(12^2 - 20.88315) \times 40 \times .5236 = (144 - 20.88315) \times 40 \times .5236 = 123.11685 \times 40 \times .5236 = 4928.674 \times .5236 = 2578.5593064$, *the solidity required.*

2. The length of an elliptic spindle is 80, the middle diameter 24, and the diameter at $\frac{1}{4}$ of the length 18.99094; what is the solidity?

Ans. 20628.4744512.

PROBLEM XV.

To find the solidity of a parabolic conoid.

RULE.

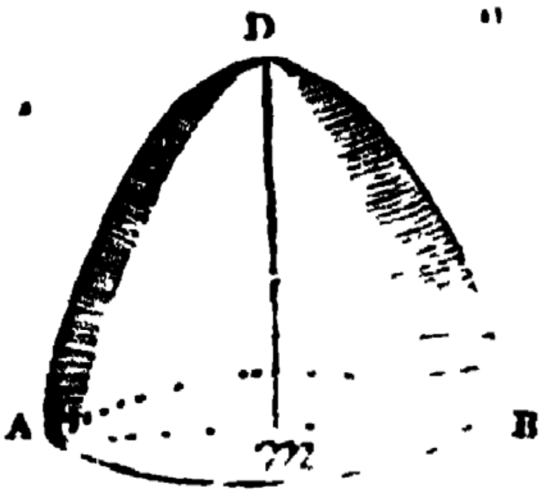
Multiply the square of the diameter of the base by the altitude, and the product thence arising by .3927, (one half of .7854,) and the last product will be the solidity.

Note 1. This Rule will give the solidity of any segment of a paraboloid, whose base is a circle.

2 A paraboloid is equal to half of its circumscribing cylinder

EXAMPLES.

1. What is the solidity of the paraboloid ADB, the height D*m* being 42, and the diameter of the base AB 24?



Here $24^2 \times 42 \times .3927 = 576 \times 42 \times .3927 = 24192 \times .3927 = 9500.1984$, *the solidity required.*

2. What is the solidity of a paraboloid, the height of which is 30, and the diameter of its base 50?

Ans. 29452.5.

3. Required the solidity of the segment of a paraboloid, the height of which is 9, and the diameter of its base 12. *Ans.* 508.9392.

PROBLEM XVI.

To find the solidity of the frustum of a paraboloid, when its ends are perpendicular to the axis of the solid.

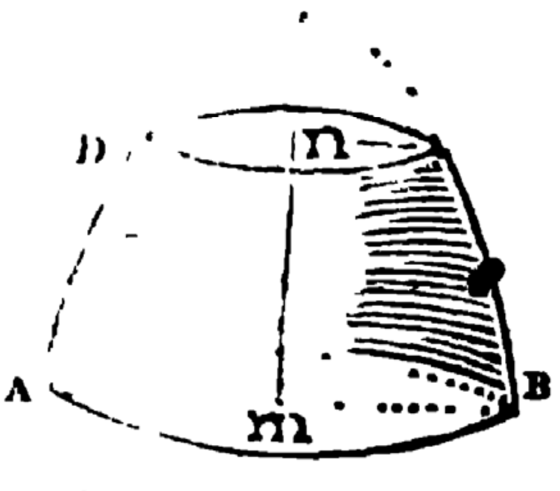
RULE.

Multiply the sum of the squares of the diameters of the two ends by the height of the frustum, and the product thus obtained by .3927; and the last product will be the solidity.

Note. If the lower frustums of two equal paraboloids be joined together at their greater ends, they will form a figure which, by Gaugers, is called a cask of the *third variety*; hence this Rule may be applied in Gauging.

EXAMPLES.

1. What is the content of the parabolic frustum ABCD; the diameter AB of the greater end being 30, that of the less end DC 24, and the height mn 9?



Here $(30^2 + 24^2) \times 9 \times .3927 = (900 + 576) \times 9 \times .3927 = 1476 \times 9 \times .3927 = 13284 \times .3927 = 5216.6268$, *the content of the frustum.*

2. What is the solidity of the frustum of a paraboloid; the diameter of the greater end being 80, that of the less end 40, and the altitude 45? *Ans.* 141372.

3. The length of a cask, in the form of two equal frustums of a parabolic conoid, is 30, the bung diameter 24, and the head diameter 18 inches; what is its content in imperial gallons?

Ans. 38.2397 *gallons.*

PROBLEM XVII.

To find the solidity of a parabolic spindle.

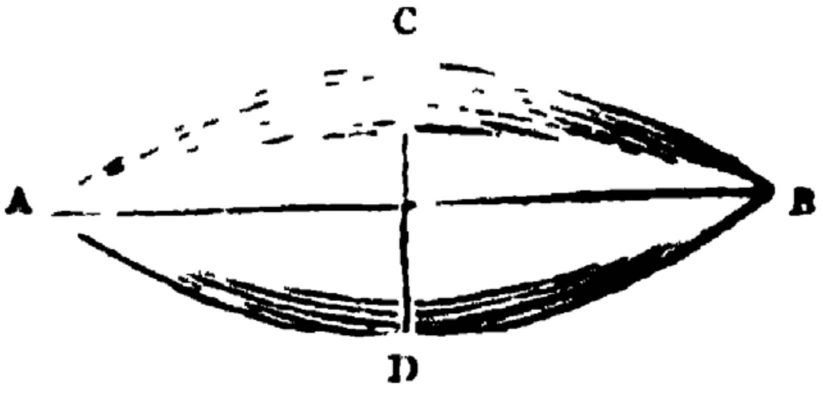
RULE.

Multiply the square of the middle diameter by the length of the spindle, and the product thence arising by .41888, and it will give the solidity.

Note A parabolic spindle is equal to $\frac{8}{15}$ of its circumscribing cylinder, hence we obtain the multiplier .41888, which is $\frac{8}{15}$ of .7854.

EXAMPLES.

1. The length AB of the parabolic spindle ACBD is 81, and the middle diameter CD 27, what is its solidity?



Here $27^2 \times 81 \times .41888 = 729 \times 81 \times .41888 = 59049 \times .41888 = 24734.44512$, *the solidity required.*

2. What is the solidity of a parabolic spindle, whose length is 20, and middle diameter 8?

Ans. 536.1664

PROBLEM XVIII.

To find the solidity of the middle frustum of a parabolic spindle.

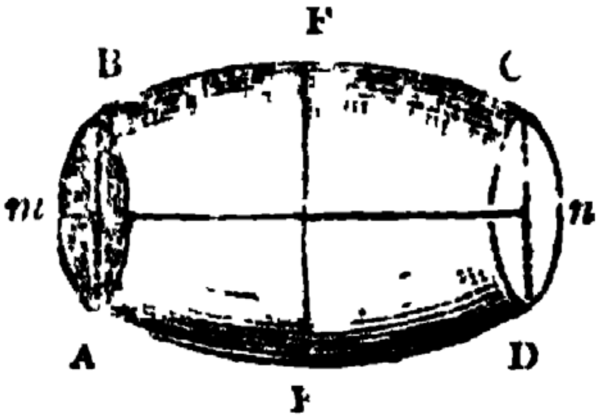
RULE.

Add 8 times the square of the middle diameter, 3 times the square of the end diameter, and 4 times the product of those diameters into one sum; then this sum being multiplied by the length, and the product thence arising by .05236, (one-fifteenth of .7854) will give the solidity.

Note. A cask in the form of the middle frustum of a parabolic spindle, is called a cask of the *second variety;* and is the most common of any of the varieties, hence the above Rule is very useful in cask gauging.

EXAMPLES.

1. What is the solidity of the middle frustum ABCD, of a parabolic spindle; the diameter of the end AB being 12, the middle diameter EF 16, and the length *mn* 20?



Here $16^2 \times 8 + 12^2 \times 3 + 16 \times 12 \times 4 = 256 \times 8 + 144 \times 3 + 192 \times 4 = 2048 + 432 + 768 = 3248$; *and* $3248 \times 20 \times .05236 = 64960 \times .05236 = 3401.3056$, *the solidity required.*

2. What is the solidity of the frustum of a parabolic spindle, whose length is 25, middle diameter 20, and end diameter 15? *Ans.* 6643.175.

3. The length of a cask, in the form of the middle frustum of a parabolic spindle, is 30, the bung diameter 24, and the head diameter 18 inches; what is its content in imperial gallons? *Ans.* 41.4009 *gallons.*

PROBLEM XIX.

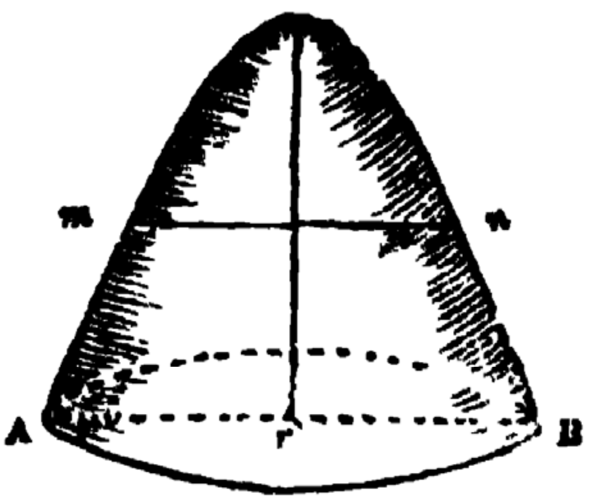
To find the solidity of a hyperboloid.

RULE.

To the square of the radius of the base, and the square of the diameter in the middle between the base and the vertex; then this sum being multiplied by the altitude, and the product thus obtained by .5236, will give the solidity.

EXAMPLES.

1. What is the solidity of the hyperboloid ACB, the altitude C*r* being 25, the radius A*r* of the base 26, and the middle diameter *mn* 34?



Here $26^2 + 34^2 = 676 + 1156 = 1832$; *and* $1832 \times 25 \times .5236 = 45800 \times .5236 = 23980.88$, *the solidity required.*

2. The altitude of a hyperboloid is 75, the radius of the base 78, and the middle diameter 102; what is the solidity? *Ans.* 647483.76.

PROBLEM XX.

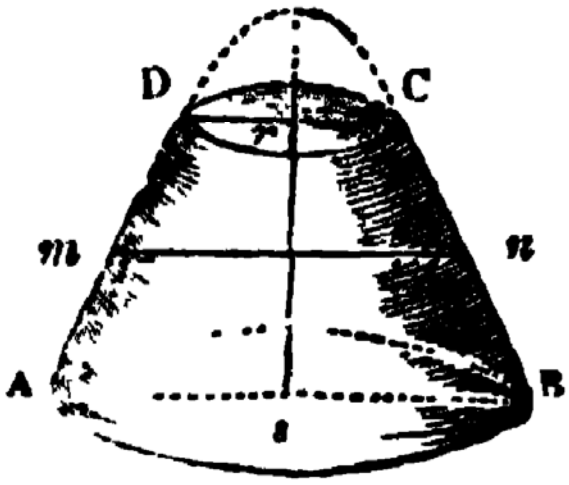
To find the solidity of the frustum of a hyperboloid

RULE.

To the sum of the squares of the greatest and least semi-diameters, add the square of the middle diameter; then this sum being multiplied by the altitude, and the product thence arising by .5236, will give the solidity.

EXAMPLES.

1. What is the solidity of the hyperbolic frustum ABCD, the diameter AB of the greater end being 20, the diameter CD of the less end 12, the middle diameter *mn* 17, and the altitude *rs* 24?



Here $10^2 + 6^2 + 17^2 = 100 + 36 + 289 = 425$; *then* $425 \times 24 \times .5236 = 10200 \times .5236 = 5340.72$, *the solidity of the frustum.*

2. The diameter of the greater end of a hyperbolic frustum is 40, the diameter of the less end 24, the middle diameter 34, and the altitude 48; what is the solidity? *Ans.* 42725.76.

PROBLEM XXI.

To find the solidity of a hyperbolic spindle.

RULE.

To the square of the greatest diameter, add the square of double the diameter taken at $\frac{1}{4}$ of the length; then this sum being multiplied by the length, and the product thus obtained by .1309, will give the solidity, *nearly.*

Note. When great accuracy is not required, this Rule may be used for any spindle formed by the revolution of a conic section, or part of a conic section, about its axis, as it will always give *nearly* the true solidity.

EXAMPLES.

1. What is the solidity of a hyperbolic spindle, whose length is 36, the greatest diameter 24, and the diameter at $\frac{1}{4}$ of the length 16.73338?

Here $24^2 + \overline{16.73338 \times 2}|^2 = 24^2 + 33.46676^2 = 576 + 1120.0240249 = 1696.0240249$; *then* $1696.0240249 \times 36 \times .1309 = 61056.8648964 \times .1309 = 7992.34361493876$, *the answer required.*

2. The length of a hyperbolic spindle is 30, the greatest diameter 20, and the diameter at ¼ of the length 13.94449; what is the solidity? *Ans.* 4625.20177.

PROBLEM XXII.

To find the solidity of the middle frustum of an elliptic or a hyperbolic spindle.

RULE.

To the sum of the squares of the greatest and least diameters, add the square of double the diameter taken exactly in the middle between them; and this sum being multiplied by the length, and the product again by .1309, will give the solidity.

Note This Rule will also give nearly the content of any frustum or segment formed by the revolution of a conic section, or part of a conic section, either about the axis of the section, or about any other line.

EXAMPLES.

1. The length of the middle frustum of an elliptic spindle is 60, the greatest diameter 48, the least diameter 36, and the diameter in the middle between them 45.23635; what is the solidity?

Here $48^2 + 36^2 + \overline{45.23635 \times 2}|^2 = 48^2 + 36^2 + 90.4727^2 = 2304 + 1296 + 8185.30944529 = 11785.30944529$; *then* $11785.30944529 \times 60 \times .1309 = 707118.5667174 \times .1309 = 92561.82038330766$, *the solidity required.*

2. The length of the middle frustum of a hyperbolic spindle is 40, the greatest diameter 32, the least diameter 24, and the diameter in the middle between them 29; what is the solidity? *Ans.* 25991.504.

3. What is the content of the middle frustum of any spindle; the length being 50, the greatest diameter 40, the least diameter 30, and the diameter in the middle between them 37.69696? *Ans.* 53565.871567.

4. What is the content of the segment of any spindle; the length being 15, the greatest diameter 12, and the middle diameter 9? *Ans.* 918.918.

PART VIII.

GAUGING.

GAUGING is the art of finding the capacities or contents of all kinds of vessels used by Maltsters, Brewers, Distillers, Wine-Merchants, Victuallers, &c. &c; such as cisterns, couches, coppers, coolers, tuns, vats, stills, casks, &c. &c.

REMARK.

The Art of Gauging is of such general utility, in the common affairs of life, that there are few persons who do not frequently want its assistance.—To Candidates for the Excise and Customs, it is indispensable; and to Victuallers, Common Brewers, Maltsters, Distillers, Rectifiers, Wine-Merchants, Spirit-Merchants, Oil-Merchants, Cyder and Perry Makers, Vinegar Makers, &c. &c. it is of considerable moment, for without its aid they could not ascertain, with accuracy, the quantity of those articles which they buy, sell, or manufacture.

NEW

IMPERIAL TABLES.

FEW ACTS of Parliament immediately interest so many individuals, as that passed on June 17th, 1824, for establishing uniformity of Weights and Measures. Every person that has any dealings, either in buying or selling, particularly in commodities that are measured, must feel desirous to know how he is affected by the change that has taken place; in order that he may neither be the object of intentional fraud himself, nor take, inadvertently, any undue advantage in his transactions with others.

The content of the New Imperial Standard Gallon is fixed at 277.274 cubic inches; and this is now the standard measure of capacity, to be used throughout the united Kingdom of Great Britain and Ireland, for both Spirits, Wine, Ale, Beer, and all other liquids; and also for every kind of grain, and all other dry commodities. But as the capacity of the gallon is changed, so likewise is the capacity of the tierce, the hogshead, the puncheon, the pipe, and the tun, in Wine and Spirit Measure; the firkin, the kilderkin, the barrel, the hogshead, the puncheon, the butt, and the tun, in Ale and Beer Measure; and the peck, the bushel, the strike, the coom, the quarter, the wey, and the last, in Dry Measure. This being the case, the author has calculated Tables, which exhibit the capacities of each of these denominations, in cubic inches.

TABLES OF WINE AND SPIRIT MEASURE

TABLE I. Exhibiting the different denominations in this measure.

4 gills	make	1 pint, *pt.*
2 pints		1 quart, *qt.*
4 quarts		1 gallon, *gal.*
42 gallons		1 tierce, *tier.*
63 gallons		1 hogshead, *hhd.*
84 gallons		1 puncheon, *pun.*
2 hogsheads, or 126 gallons		1 pipe or butt, *pipe.*
2 pipes, 4 hogsheads, or 252 gal.		1 tun, *tun.*

TABLE II. Exhibiting the number of cubic inches contained in each denomination according to the new standard gallon.

Cubic Inches.

277.274 = 1 gallon.
11645.508 = 42 = 1 tierce.
17468.262 = 63 = 1½ = 1 hogshead.
23291.016 = 84 = 2 = 1⅓ = 1 puncheon.
34936.524 = 126 = 3 = 2 = 1½ = 1 pipe.
69873.048 = 252 = 6 = 4 = 3 = 2 = 1 tun.

Note. As 277.274 cubic inches make one imperial gallon; 69.3185 make one quart; 34.65925 make one pint; and 8.6648125 make one gill.

TABLES OF ALE AND BEER MEASURE.

TABLE I. Exhibiting the different denominations in this measure.

2 pints .... make	1 quart, *qt.*
4 quarts..	1 gallon, *gal.*
9 gallons	1 firkin, *fir.*
2 firkins, or 18 gallons	1 kilderkin, *kil.*
2 kilderkins, or 36 gallons	1 barrel, *bar.*
1½ barrel, or 54 gallons.	1 hogshead, *hhd.*
2 barrels, or 72 gallons	1 puncheon, *pun.*
3 barrels, 2 hogsheads, or 108 gal. ..	1 butt, *butt.*
2 butts, 4 hogsheads, or 216 gallons	1 tun, *tun.*

TABLE II. Exhibiting the number of cubic inches contained in each denomination, according to the new standard gallon.

Cubic Inches.
277.274 = 1 gallon.
2495.466 = 9 = 1 firkin.
4990 932 = 18 = 2 = 1 kilderkin.
9981.864 = 35 = 4 = 2 = 1 barrel.
14972 796 = 54 = 6 = 3 = 1½ = 1 hogshead
19963.728 = 72 = 8 = 4 = 2 = 1⅓ = 1 puncheon.
29945.592 = 108 = 12 = 6 = 3 = 2 = 1½ = 1 butt.
59891.184 = 216 = 24 = 12 = 6 = 4 = 3 = 2 = 1 tun.

TABLES OF CORN OR DRY MEASURE.

TABLE I. Exhibiting the different denominations in this measure

2 pints .... make	1 quart, *qt.*
2 quarts, or 4 pints	1 pottle, *pot.*
2 pottles, or 4 quarts	1 gallon, *gal.*
2 gallons, or 8 quarts	1 peck, *pk.*
4 pecks, or 8 gallons	1 bushel, *bu.*
2 bushels, or 8 pecks	1 strike, *strike.*
4 bushels, or 2 strikes	1 coom, *coom.*
2 cooms, or 8 bushels	1 quarter, *qr.*
5 quarters, or 40 bushels	1 wey, *wey.*
2 weys, or 10 quarters	1 last, *last.*

TABLE II. Exhibiting the number of cubic inches contained in each denomination, according to the new standard gallon.

Cubic Inches.
277.274= 1 gallon.
554.548= 2= 1 peck.
2218.192= 8= 4= 1 bushel.
4436.384= 16= 8= 2= 1 strike.
8872.768= 32= 16= 4= 2= 1 coom.
17745.536= 64= 32= 8= 4= 2= 1 quarter.
88727.680=320=160=40=20=10= 5=1 wey.
177455.360=640=320=80=40=20=10=2=1 last.

Note 1. The number of cubic inches in the old wine gallon, is 231, in the old ale gallon, 282, in the old corn gallon, 268.8, and in the old corn bushel, 2150.4.

2. The old Scotch boll for wheat, beans, peas, rye, salt, and grass-seeds, contains 8789.34 cubic inches, equal to 4 bushels, 2 quarts, 1½ pint of our old measure.

3. The old Scotch boll for oats, barley, and malt, contains 12822.096 cubic inches, equal to 5 bushels, 3 pecks, 1 gallon, 2 quarts, 1½ pint of our old measure.

4. If the Scotch boll for wheat, &c. be still continued, it will contain, according to the new standard, 9066.942 cubic inches, equal to 4 bushels, 2 quarts, 1½ pint of our new measure, the same proportion as before.

5. If the Scotch boll for oats, &c. be still continued, it will contain according to the new standard, 13225.724 cubic inches, equal to 5 bushels, 3 pecks, 1 gallon, 2 quarts, 1½ pint of our new measure, the same as before.

6. In Ireland, the old malt and corn gallon contains 272¼ cubic inches, consequently, the content of the bushel is 2178 cubic inches.—The old Irish gallon, for all liquids, contains 217.6 cubic inches.

7. The old Scotch pint, for liquids, contains 103.4, the quart 206.8, and the gallon 827.23 cubic inches; consequently, 1 Scotch gallon is equal to 2.98343 imperial gallons = 2 gal. 3 qt. 1 pt. 3½ gills, or 3 gallons, *very nearly.*

8. In Scotland, 4 pecks make 1 firlot, and 4 firlots 1 boll, consequently, the wheat peck contains 549.33375, and the oat peck 801.381 cubic inches, the wheat firlot 2197.335, and the oat firlot 3205.524 cubic inches. *(See the 2nd and 3rd Notes.)*

RULES

For reducing any number of old gallons of wine, to the new standard; and vice versâ.

1. As 277.274 cubic inches, are to 231 cubic inches;

so are any number of old gallons, to the required number of new gallons.

2. As 231 cubic inches, are to 277.274 cubic inches; so are any number of new gallons, to the required number of old gallons.

REMARK.

The preceding rules will serve for Ale Measure, and Dry Measure; but 282 must be used for the old ale gallon; 268.8 for the old corn gallon; 2150.4 for the old corn bushel, and 2218.192 cubic inches, for the new corn bushel.

EXAMPLES.

1. How many new imperial gallons are contained in 1 hogshead, or 63 old gallons of wine?

As 277.274 : 231 :: 63 : 52.48598 *new gallons* = 52 *gal.* 1 *qt.* 1 *pt.* 3½ *gills, very nearly.*

2. How many old gallons of wine are contained in 1 hogshead, or 63 gallons, imperial measure?

As 231 : 277.274 :: 63 : 75.62018 *old gallons* = 75 *gal.* 2 *qt.* 0 *pt.* 3¾ *gills, very nearly.*

3. How many new gallons are contained in 1 hogshead, or 54 old gallons of ale?

As 277.274 : 282 :: 54 : 54.92640 *new gallons* = 54 *gal.* 3 *qt.* 1 *pt.* 1½ *gill, very nearly.*

4. How many old gallons of ale, are there in 1 hogshead, or 54 gallons of imperial measure?

As 282 : 277.274 :: 54 : 53.09502 *old gallons* = 53 *gal.* 0 *qt.* 0 *pt.* 3 *gills, very nearly.*

5. How many new bushels of corn, are there in 1 last, or 80 old bushels?

As 2218.192 : 2150.4 :: 80 : 77.55505 *new bushels* = 77 *bush.* 2 *pk.* 0 *gal.* 1 *qt.* 1½ *pint, very nearly.*

6. How many old bushels of corn, are there in 1 last, or 80 new bushels?

As 2150.4 : 2218.192 :: 80 : 82.52202 *old bushels* = 82 *bush.* 2 *pk.* 0 *gal.* 0 *qt.* 1 *pt.* 1½ *gill, very nearly.*

MULTIPLIERS,

By which old gallons and bushels may be reduced to imperial gallons and bushels.

Multiply .83311 by the content of any vessel, in old wine gallons; and the product will be imperial gallons. Multiply 1.01705 by old ale gallons; and the product will be imperial gallons. Multiply .96945 by old corn bushels; and the product will be imperial bushels.

EXAMPLES.

1. If a cask contain 1 hogshead, or 63 old gallons of wine; how many imperial gallons will it hold?

Here .83311 × 63 = 52.48593 *imperial gallons, the same as obtained in the first example, by the preceding rules.*

2. If the content of a vessel be 1 hogshead, or 54 old gallons of ale; what is its content in imperial gallons?

Here 1.01705 × 54 = 54.9207 *imperial gallons, the same as obtained in the third example, by the preceding rules.*

3. How many imperial bushels are contained in 1 last, or 80 old bushels?

Here .96945 × 80 = 77.556 *imperial bushels, the same as found in the fifth example, by the preceding rules.*

MULTIPLIERS,

By which imperial gallons and bushels may be reduced to old gallons and bushels.

Multiply 1.20033 by the content of any vessel, in imperial gallons; and the product will be old wine gallons. Multiply .98325 by imperial gallons; and the product will be old ale gallons. Multiply 1.03153 by imperial bushels; and the product will be old Winchester bushels.

EXAMPLES.

1. If a cask contain 52.48593 imperial gallons; how many old wine gallons will it hold?

Here 52.48593 × 1.20033 = 63 *old wine gallons.*

2. If the content of a vessel be 54.9207 imperial gallons; what is its content in old ale gallons?

Here 54.9207 × .98325 = 54 *old ale gallons.*

8. How many old Winchester bushels are contained in 77 556, imperial bushels?

Here 77.556 × 1.03153 = 80 *old bushels.* (*See the preceding Examples.*)

MULTIPLIERS,

For reducing Irish bushels and Scotch bolls to imperial bushels; and vice versâ.

Multiply .98188 by Irish bushels, and the product will be imperial bushels; or multiply 1.01846 by imperial bushels, and the product will be Irish bushels.

Multiply 3.96239 by Scotch wheat bolls, and the product will be imperial bushels; or multiply .25238 by imperial bushels, and the product will be Scotch wheat bolls.

Multiply 5.78043 by Scotch oat bolls, and the product will be imperial bushels; or multiply imperial bushels by .173, and the product will be Scotch oat bolls.

EXAMPLES.

1. How many imperial bushels are there in 1 last, or 80 Irish bushels?

Here .98188 × 80 = 78.5504 *imperial bushels.*

2. How many Irish bushels are there in 78.5504 imperial bushels?

Here 78.5504 × 1.01846 = 80 *Irish bushels.*

3. How many imperial bushels are there in 10 Scotch bolls of wheat?

Here 3.96239 × 100 = 396.239 *imperial bushels.*

4. How many Scotch bolls of wheat are there in 396.239 imperial bushels?

Here $396.239 \times .25238 = 100$ *Scotch bolls of wheat.*

5. How many imperial bushels are there in 100 Scotch bolls of oats?

Here $5.78043 \times 100 = 578.043$ *imperial bushels.*

6. How many Scotch bolls of oats are there in 578.043 imperial bushels?

Here $578.043 \times .173 = 100$ *Scotch bolls of oats.*

Note. Irish liquid gallons may be reduced to imperial gallons, by multiplying by .78478, and imperial gallons may be reduced to Irish liquid gallons, by multiplying by 1.27425

REMARKS.

1. The new imperial gallon is greater than the old wine gallon, by 46.274 cubic inches, which is nearly $\frac{1}{6}$ of 277.274, or $\frac{1}{5}$ of 231; hence, 6 old gallons are equal to 5 new gallons.

2. The new gallon is less than the old ale gallon, by 4.726 cubic inches, which is *nearly* $\frac{1}{60}$ of 282, or $\frac{1}{59}$ of 277.274; hence 60 new gallons are *nearly* equal to 59 old gallons.

3. The new gallon is greater than the old corn gallon, by 8.474 cubic inches, which is *nearly* equal to $\frac{1}{32}$ of 268.8; and the new bushel is greater than the old bushel, by 67.792 cubic inches; hence 32 new bushels are *nearly* equal to 33 old bushels.

4. Old wine gallons may be reduced into new gallons, by subtracting $\frac{1}{6}$; and new gallons may be brought into old wine gallons, by adding $\frac{1}{5}$.

5. Old ale gallons may be reduced into new gallons, by adding $\frac{1}{60}$; and new gallons may be brought into old ale gallons, by subtracting $\frac{1}{59}$.

6. Old bushels may be reduced into new bushels, by subtracting $\frac{1}{32}$; and new bushels may be brought into old bushels, by adding $\frac{1}{31}$.

OBSERVATION.

Those who wish to see more on the subject of the New Imperial Weights and Measures, are referred to the Author's Treatise on Practical Arithmetic, Second Edition, in which he has given extensive Tables, comparing the old and new measures; and also a copious abstract of the Act of Parliament, for establishing uniformity of Weights and Measures throughout the united kingdom of Great Britain and Ireland.

DESCRIPTION

OF THE

SLIDING RULE.

THIS instrument is in the form of a parallelopipedon; has four sliding pieces, which run in grooves, and is commonly made of box.

It was invented by Mr. Thomas Everand, about the year 1683, and is generally called *Everand's Sliding Rule;* but it has since been much improved by Mr. Verie; and is now adapted to the new imperial measure.

It is of various lengths, as 6, 9, 12, 18 inches, &c.; but 12 inches is the most common length.

This rule is much used in Gauging, in consequence of the ease and expedition with which calculations may be made by it.

Upon the first face of the Rule, is a line marked A, which is called *Gunter's Line,* from its inventor Mr.

Edmund Gunter; and is numbered from the left to the right with the figures 1, 2, 3, 4, 5, 6, 7, 8, 9, 10; and the space between each of these figures are graduated into subdivisions. At 2218.2, is a brass pin, marked IMB, signifying the cubic inches in the imperial bushel; and at 277.3, is another brass pin, marked IMG, denoting the number of cubic inches in the imperial gallon.

The second line on this face is upon the slide, and marked B. It is divided exactly in the same manner as that marked A. There is also another slide B, which is used with the former; the two brass ends are then placed together, in which position they form a double radius. At 277.3, on the second radius, is a brass pin marked G or IG, denoting the number of cubic inches in the imperial gallon.

By these lines, multiplication, division, proportion, &c. may be performed; and the manner of reading and using them is precisely the same as the lines A and B, upon the *Carpenter's Rule.* (See Section 2, Part IV.)

Upon the same face of the Rule, is another line marked MD, signifying malt-depth; and is numbered from the right to the left, with the figures, 3, 4, 5, 6, 7, 8, 9, 10, 2.

The 1, or 10 on this line is placed directly opposite to MB, on the line A; and if it be called 1000, the last division at the left end of the Rule, will denote 2218.2, the cubic inches in the imperial bushel.—This line is used with the lines A and B, in malt gauging.

Upon the second face of the Rule, or that opposite to the one already described, is a line marked D. This line begins on the upper edge of the Rule, and its numbered from the left to the right with the figures 1, 2, 3, 31, 32, which is at the right end of the Rule. The line is then continued from the left end of the other edge, 32, 4, 5, 6, 7, 8, 9, 10.

At 18.79 is a brass pin, marked IG, signifying the circular gauge-point for imperial gallons; at

47.1 is another brass pin, marked MS, which is the square gauge-point for imperial malt bushels; and at 53.1, is a third pin, marked MR, denoting the round or circular gauge-point for imperial malt bushels.

Upon this face of the Rule, is a line marked C. This line is upon the slide; and is numbered and divided in the same way as the lines A and B. Belonging to the Rule, is also another slide C, which is used with the former, in the same manner as the two slides marked B.

The line D, on this Rule, is similar to that marked D, on the *Carpenter's Rule;* and is used with the two slides C, just described; in finding the contents of vessels whose form is that of a cube, a parallelopipedon, a cylinder, &c. &c.

By these lines the square root of any number may also be readily extracted; for if 1 on C be set to 1 on D, we have against any proposed number on C, its root on D.

The back of the first slide, marked C, is divided, next the upper edge, into inches and tenths; and numbered from the left to the right, with 1, 2, 3, 4, 5, &c.; the second line is marked *spheroid,* and the third *second variety;* and both are numbered from the left to the right with the figures 1, 2, 3, 4, &c.; and the spaces between each of these figures are divided into ten equal parts.

These three lines are used for finding a mean diameter between the bung and the head of casks of the first and second varieties; which is performed in the following manner: Find the difference between the bung and head diameters, on the first line, or line of inches; then against it, for each variety, is a number, which being added to the head, will give the mean diameter sought; hence the cask is reduced to a cylinder, whose content may be found by Problem III.

The back of the second slide C, contains the gauge-points, divisors, and factors or multipliers, for imperial gallons and bushels.

The third face of the Rule contains a line marked Seg. St. or SS, signifying *segments standing*. This line is numbered from the left to the right, on the upper edge of the Rule, with 1, 2, 3, 4, 5, 6, 7, 8. It is then continued on the other edge, 8, 9, 10, 20, &c. to 100.

This line is used with the two slides, marked C, in finding the ullage of a standing cask, or the quantity of liquor it contains when it is not full.

Upon the fourth face of the Rule, or the one opposite to that last described, is a line marked Seg. Ly. or SL, denoting *segments lying*. This line is numbered nearly in the same manner as the last; and is used with the slides C, in ullaging lying casks.

Note 1. There are various kinds of Sliding Rules, some of which have a line marked E, for extracting the cube roots of numbers; and others a line for the *third variety* of casks, they are, however, all upon the same principle, and may be easily understood from the foregoing description.

2. Very neat and commodious Sliding Rules, have lately been made flat, with only two slides. These slides are the full thickness of the rule; and being divided on each side, answer every purpose of the four slides, and the rule is considerably reduced both in size and weight.

3. It may not be improper to observe, that in the preceding description of the Sliding Rule, the figures mentioned, refer to, the larger figures placed at the primary divisions of the rule; most of the subdivisions being marked with smaller figures.—For the method of estimating the values of the divisions and subdivisions, the reader is referred to the Description of the Carpenter's Rule, Section II. Part IV.

THE METHODS OF FINDING THE

MULTIPLIERS, DIVISORS, & GAUGE-POINTS,

Contained in the subsequent Table.

THE multipliers and divisors are chiefly obtained by division, in the following manner:

Divisors for squares *Multipliers for squares.*
277.274)1.0000 &c. (.003607 *imperial gallons.*
2218.192)1.0000 &c. (.000451 *imperial bushels.*

Divisors for squares *Multipliers for circles.*
277.274).785398 &c. (.002833 *imperial gallons.*
2218.192).785398 &c. (.000354 *imperial bushels.*

Divisors for circles.
.785398) 277.274(353.04 *imperial gallons.*
.785398)2218.192(2824.29 *imperial bushels.*

GAUGE-POINTS.

The gauge-points are found by extracting the square roots of the divisors.

Divisors for squares *Gauge-points for squares.*
277.274) *their square* (16 65 *imperial gallons.*
2218.192) *roots are* (47.10 *imperial bushels.*

Divisors for circles. *Gauge-points for circles.*
353.04) *their square* (18.79 *imperial gallons.*
2824.29) *roots are* (53.14 *imperial bushels.*

Thus all the numbers in the following table were obtained.

A Table of Multipliers, Divisors, and Gauge-Points for Squares and Circles.

Note The Areas, &c., are all in inches.	MULTIPLIERS FOR		DIVISORS FOR		GAUGE-POINTS FOR	
	SQUARES	CIRCLES.	SQUARES.	CIRCLES.	SQUARES	CIRCLES.
The side or diameter 1	1.	.785398	1.	1.27324	1.	1.128
A superficial foot	.006944	.005454	144.	183.34	12.	13.54
A solid foot	.000578	.000454	1728.	2200.16	41.57	46.91
Imperial gallon	.003607	.002833	277.274	353.04	16.65	18.79
Imperial bushel	.000451	.000354	2218.192	2824.29	47.10	53.14
A pound of hard soap cold ..	.036845	.028939	27.14	34.56	5.21	5.88
A pound of hot soap	.035714	.028050	28.	35 65	5.29	5.97
A pound of green soap	.038956	.0306	25.67	32.68	5 06	5.72
A pound of white soft soap ..	.039123	.030731	25 56	32.54	5.05	5.7
A pound of tallow net	.031844	.025101	31.4	39.98	5.6	6.32
A pound of green starch	.028736	.022565	34.8	44 32	5.9	6.66
A pound of dry starch	.024813	.019491	40.3	51.3	.35	7.16

Note The use of the Multipliers, Divisors, and Gauge-points, is shewn in the following Problems.

PROBLEM I.

The side of a vessel in the form of a cube, being given in inches; to find its content in imperial gallons and bushels.

RULE.

Multiply the side by itself, and that product again by the side; and the last product will be the content of the vessel in cubic inches; which being multiplied by .003607, and .000451; or divided by 277.3, and 2218.2, the respective products or quotients will be the content of the vessel in imperial gallons and bushels.

Note 1 The definitions of the cube, the parallelopipedon, the cylinder, &c. &c may be seen in Part IV., and if the content of any vessel be found in cubic inches, by the rules given in that Part, and then divided by 282, 231, and 2150.4, the respective quotients will be the content of the vessel in old ale and wine gallons, and malt bushels; but the Rules and Examples given in the following Problems will be found more particularly adapted to Gauging, than those in Part IV.

2. If the content of any vessel, in cubic inches, be divided by 277.3, and 2218.2; the respective quotients will be the content in *imperial* gallons and bushels

3. Gaugers always take their dimensions in inches or in inches and tenths, and when we say the side or diameter of a vessel measures so many inches, we mean the internal, not the external dimension.

EXAMPLES.

1. The side of a cubical vessel measures 46 inches; what is its content in imperial gallons and bushels?

Here $46 \times 46 \times 46 = 2116 \times 46 = 97336$, *the content in cubic inches; then* $97336 \times .003607 = 351.0909$ *imperial gallons; and* $97336 \times .000451 = 43.8985$ *imperial bushels.*

Or, $97336 \div 277.3 = 351.0133$ *imperial gallons; and* $97336 \div 2218.2 = 43.8806$ *imperial bushels.*

Note. The divisors are used in all the following solutions: the work, however, may be proved by the multipliers given in the preceding Table.

By the Sliding Rule.

In this operation, the square gauge-points 16.65, and 47.10, upon the line D, must be used.

$$\begin{array}{lcccc} & \textit{On}\ \text{D.} & \textit{On}\ \text{C.} & \textit{On}\ \text{D.} & \textit{On}\ \text{C.} \\ \text{As} & \left\{\begin{matrix}16.65\\47.10\end{matrix}\right\} : & 46 :: & 46 : & \left\{\begin{matrix}351.0\ \textit{imperial gallons.}\\44.0\ \textit{imperial bushels.}\end{matrix}\right. \end{array}$$

Note. New Sliding Rules have not always the square gauge-points marked upon them; but small brass pins may be easily inserted at those points.

2. The side of a cubical cistern is 134 inches; what is its content in imperial gallons and bushels?

Ans. 8676.8986 *imperial gallons, and* 1084.7101 *imperial bushels.*

3. The side of a cubical wine-vat measures 135.6 inches; how much wine will it contain at once?

Ans. 8991.4389 *gallons* = 8991 *gal.* 1 *qt.* $1\frac{1}{2}$ *pt.*

PROBLEM II.

The length, breadth, and depth of a vessel, in the form of a parallelopipedon, being given in inches; to find its content in imperial gallons and bushels.

RULE.

Multiply the length by the breadth, and the product thence arising by the depth; and the last product will be the content in cubic inches; which being divided by 277.3 and 2218.2, will give the content in imperial gallons and bushels.

Note. As the sides of vessels, in the form of a parallelopipedon, are seldom perfectly regular and parallel, it is best to take several lengths, in different places; and divide their sum by their number for a mean length. A mean breadth and depth may be found in the same manner.

EXAMPLES.

1. A cistern in the form of a parallelopipedon, measures 96 inches in length, 65 in breadth, and 48 in

depth; what is its content in imperial gallons and bushels?

Here 96 × 65 × 48 = 299520, *the content in cubic inches; then* 299520÷277.3=1080.1298 *gallons; and* 299520 ÷ 2218.2 = 135.0284 *bushels.*

By the Sliding Rule.

As 96 *on* C : 96 *on* D :: 65 *on* C : 79 *on* D, *which is a mean proportional between the length and breadth. (See Problem V, page* 178.*) Then,*

	On D.		*On* C.		*On* D.		*On* C.
As	16.65 47.10	:	48	::	79	:	1080.0 *gallons.* 135.0 *bushels.*

2. The length of a vessel in the form of a parallelopipedon, is 186, its breadth 94, and its depth 62 inches; what is its content in imperial gallons and bushels?

Ans. 2858.3050 *gallons, and* 357 3203 *bushels.*

3. A water-trough measures 85 3 inches in length, 54.7 in breadth, and 32.9 in depth; how many imperial gallons will it contain?

Ans. 553 5825 *gallons* = 553 *gal.* 2 *qt.* 0½ *pt.*

4. A maltster has a cistern whose length is 132, breadth 118, and depth 46 inches; how much barley can he steep at a time, admitting the water to occupy ⅗ of the cistern?

Ans. 193.8047 *bushels* = 24 *qr.* 1 *bush.* 3¼ *pk.*

PROBLEM III.

The diameter and depth of a vessel, in the form of a cylinder being given; to find its content in imperial gallons and bushels.

RULE.

Multiply the square of the diameter by the depth; and divide the product by 353, for imperial gallons; and by 2824.3, for imperial bushels.

Note 1. As cylindrical vessels are seldom perfectly round, it is best to measure cross diameters, in different parts; and divide their sum by their number, for a mean diameter.

2. If a cylindrical vessel be placed in an inclining position, so that the liquor forms an elliptical surface, the content may be found by the following Rule: Multiply the square of the diameter of the vessel by half the sum of the greatest and least depths of the liquor, and divide the product by 353, for imperial gallons. *(See Notes, Prob. V.)*

EXAMPLES.

1. The diameter of a cylindrical vessel is 34, and its depth 45 inches; what is its content in gallons and bushels?

Here $34 \times 34 \times 45 = 52020$; *then* $52020 \div 353 = 147.3654$ *gallons; and* $52020 \div 2824.3 = 18.4187$ *bushels.*

By the Sliding Rule.

Here the circular gauge-points 18.79, and 53.14, upon the line D, must be used.

$$\begin{array}{lcccccccc} & \textit{On D.} & & \textit{On C.} & & \textit{On D.} & & \textit{On C.} \\ \textit{As} & \left\{\begin{array}{c}18.79\\53.14\end{array}\right\} & : & 45 & :: & 34 & : & \left\{\begin{array}{l}147.4 \textit{ gallons.}\\18.4 \textit{ bushels.}\end{array}\right. \end{array}$$

2. The diameter of a cylindrical vessel, is 46.7, and its depth 68.4 inches; what is its content in gallons and bushels?

Ans. 422.5860 *gallons, and* 52.8176 *bushels.*

3. A cylindrical mash-tun measures 94 inches in diameter, and 82 in depth; how many bushels of malt will it contain at once? *Ans.* 256.5421 *bushels.*

4. At Heidelberg, in Germany, is a cylindrical wine cask, whose depth is 27, and diameter 21 feet; how many gallons will it contain, imperial measure?

Ans. 58286.9575 *gallons.*

Note. The convivial monument of ancient hospitality, mentioned in the 4th Example, was formerly kept full of the best Rhenish wine, and the electors have given many entertainments on its platform, but it now only serves as a melancholy instance of the extinction of that hospitality; for it is suffered to moulder in a damp vault, quite empty.

Although this vessel is of an extraordinary magnitude; yet it is much inferior in capacity, to many of the London porter-vats.

PROBLEM IV.

Given the dimensions of a vessel in the form of a prismoid, or the frustum of a square pyramid, or a cylindroid; to find its content in imperial gallons and bushels.

RULE.

To the sum of the areas of the two ends add four times the area of the middle section parallel to them; multiply this sum by the perpendicular depth, and $\frac{1}{6}$ of the product will be the content in cubic inches; which divide by 277.3 for imperial gallons, and 2218.2 for imperial bushels. *(See the Scholium, Prob. X. Sect. I. Part IV.)*

Note. 1. A cylindroid is a figure resembling the frustum of a cone, but having elliptical instead of circular ends. Sometimes one end is circular, and the other elliptical.

2. When the vessel is a prismoid, the length of the middle section is equal to half the sum of the lengths of the two ends; and its breadth is equal to half the sum of their breadths.

3. If the ends be elliptical, the transverse diameter of the middle section will be equal to half the sum of the transverse diameters of the two ends; and the conjugate diameter equal to half the sum of the conjugate diameters of the two ends.

4. If one end be an ellipse and the other a circle, add the transverse diameter of the elliptical end to the diameter of the circular end; and take half the sum for the transverse diameter of the middle section. The conjugate diameter of the middle section may be found in a similar manner; it is better, however, in all cases of *practice*, to take the real dimensions of the sections.

5. When the ends are rectangles, their areas may be found by Prob. 2; when they are circles, we may obtain their areas by Prob. 15; and when they are ellipses, we can find their areas by Prob. 21, *Part II.*

EXAMPLES.

1. The length and breadth of the bottom of a vessel in the form of a prismoid, measures 72 and 64, the length and breadth of the top 96 and 82, and the perpendicular depth 65 inches; what is its content in gallons and bushels?

Here $\overline{72 \times 64} + \overline{96 \times 82} = 4608 + 7872 = 12480$, *the area of the two ends.*

Also, $\frac{72 + 96}{2} = \frac{168}{2} = 84$, *the length of the middle section; and* $\frac{64 + 82}{2} = \frac{146}{2} = 73$, *the breadth of the middle section; then* $84 \times 73 \times 4 = 6132 \times 4 = 24528$, *four times the area of the middle section; whence* $(12480 + 24528) \times \frac{65}{6} = \frac{37008 \times 65}{6} = \frac{2405520}{6} = 400920$, *the content in cubic inches; then* $400920 \div 277.3 = 1445.7987$ *gallons; and* $400920 \div 2218.2 = 180.7411$ *bushels.*

Note This and some of the following Problems may be performed by the Sliding Rule, but as the operations are too tedious for *practice*, they are omitted.

2. Each side of the bottom of a cistern, in the form of the frustum of a square pyramid, measures 86, each side of the top 78, and the perpendicular depth 74 inches; what is its content in gallons and bushels?

Ans. 1795.7831 *gallons, and* 224.4931 *bushels.*

3. The perpendicular depth of a vessel, in the form of the frustum of an elliptical cone, is 46.8 inches; the transverse and conjugate diameters of the bottom measure 49.6 and 37.8 inches; the transverse and conjugate diameters of the top, 67.2 and 50.4 inches; required the content in gallons and bushels.

Ans. 343.8297 *gallons, and* 42.9825 *bushels.*

4. The perpendicular depth of a vessel with an elliptical base and a circular top, is 52.6 inches; the transverse and conjugate diameters of the bottom measure 61.6 and 46.2 inches; and the diameter of the top measures 42.6 inches; required the content in gallons and bushels.

Ans. 345.4747 *gallons, and* 43.1882 *bushels.*

PROBLEM V.

To find the content of a vessel in the form of the frustum of a cone, in imperial gallons and bushels.

RULE.

To three times the product of the top and bottom diameters, add the square of their difference; multiply the sum by the depth, and divide the product by 1059 for imperial gallons, and 8472.9 for imperial bushels.

Note 1. The general rule given in Prob. VI. Sect. I. Part IV. will give the content of any frustum, whatever may be the form of the two similar ends; that is, whether they be polygons, circles, or ellipses.

2. In taking the dimensions of circular vessels, it is best, in all cases, to measure cross diameters, as directed in the *second Note*, in Case I. of the next Problem.

3. When the frustum of a cone is cut by a diagonal plane passing through the opposite extremities of the ends; or when a vessel of that shape is placed in an inclining position, so that the liquor just touches the opposite extremities of the top and bottom, the two parts or sections thus formed are called *elliptic hoofs*, and their contents may be found by the following Rule: Multiply the product of the diameters of the ends of the frustum by a mean proportional between them; and cube the diameter of the hoof's base.

From the greater of the numbers thus found subtract the less; and divide the remainder by the difference of the diameters; then the quotient being multiplied by the perpendicular height of the hoof, and the product by .2618, will give the content in cubic inches, which divide by 277.3, and you will obtain the content in gallons.

4. If the greater end of the frustum be considered as the base of the hoof, the product of the diameters by a mean proportional between them, will be less than the cube of the diameter of the base; but the contrary will be the case, when the less end of the frustum represents the base of the hoof.

5. If a vessel be placed in such a position that the liquor just covers the bottom, and part of one side, measure the diameter at the bottom, the diameter at the upper extremity of the liquor, and also the liquor's perpendicular depth, or height of the hoof; then proceed with these dimensions as directed in Note 3, and you will obtain the content.

6. In Nesbit's and Little's *Practical* Gauging, there are Rules given for finding the contents of all cylindrical, pyramidical, and conical hoofs or ungulas, that can possibly be formed, by placing in various positions, vessels containing fluids.

EXAMPLES.

1. The bottom diameter of a vessel, in the form of the frustum of a cone, is 46, the top diameter 62, and the depth 60 inches; what is its content in gallons and bushels?

Here $62 \times 46 \times 3 = 2852 \times 3 = 8556$, *three times the product of the top and bottom diameters. Also,* $\overline{62 - 46}|^2 = 16 \times 16 = 256$, *the square of their difference; then* $(8556 + 256) \times 60 = 8812 \times 60 = 528720$; *and* $528720 \div 1059 = 499.2634$ *gallons; likewise,* $528720 \div 8472.9 = 62.4013$ *bushels.*

2. The bottom diameter of a guile-tun is 115, the top diameter 98, and the depth 75 inches; how many imperial gallons will it contain? *Ans.* 2414.9433 *gallons.*

3. The bottom diameter of a wine-vat is 78.6, the top diameter 64.3, and the depth 72.8 inches; what quantity of wine will it contain?

Ans. 1056.3513 *gallons.*

PROBLEM VI.

To find the content of a circular vessel, with curved sides.

CASE I.

When the sides are not much curved, as in the following figure.

RULE.

To the sum of the squares of the top and bottom diameters; add four times the square of the middle diameter; multiply this sum by the depth; divide the product by 2118.24, and 16945.74; and the respective quotients will be the content in imperial gallons and bushels.

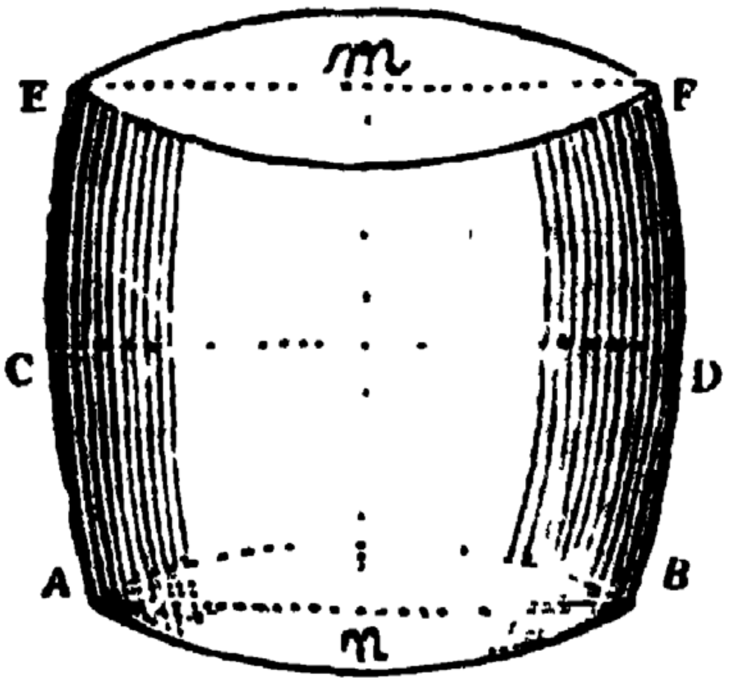
Note 1. The foregoing divisors are found by multiplying the circular divisors by 6.

2. As vessels of this kind are seldom perfectly round, it is best to measure two diameters at right angles to each other, and take half their sum for a mean diameter. *(See Problem VII.)*

3. The Rule given in this Problem is the same, in substance, as that given in Problem X. Sect. I. Part IV. *(See the Scholium in that Problem.)*

EXAMPLES.

1. The diameter AB, of the following vessel, measures 50, the diameter CD 56, the diameter EF 54, and the depth *mn* 46 inches; what is its content in gallons and bushels?



Here $50^2 + 54^2 = 2500 + 2916 = 5416$, *the sum of the squares of the top and bottom diameters; and* $56^2 \times 4 = 3136 \times 4 = 12544$, *four times the square of the middle diameter. Then,* $5416 + 12544 \times 46 = 17960 \times 46 = 826160$; *and* $826160 \div 2118.24 = 390.0219$ *gallons; also,* $826160 \div 16945.74 = 48.7532$ *bushels.*

2. The bottom diameter of a mash-tun is 75.3, the top diameter 81.6, the diameter taken in the middle between them 84.7, and the depth 56.9 inches; what is its content in imperial bushels?

Ans 137.7527 *bushels.*

3. If the length of a cask be 30, the bung diameter 24, and the head diameter 18 inches; what is its content in imperial gallons? *Ans.* 41.8082 *gallons.*

CASE II.

To find the content of a circular vessel, when its sides are much curved, by taking five diameters, at equal distances from each other, as in the following figure.

RULE.

Add into one sum, the squares of the top and bottom diameters, twice the square of the middle diameter, and four times the sum of the squares of the diameters taken at one-fourth and at three-fourths of the depth; multiply this sum by the depth; divide the product by 4236.48, and 33891.48; and the respective quotients will be the content in imperial gallons and bushels.

Note. The foregoing divisors are found by multiplying the circular divisors by twelve.

REMARK.

The Author has deduced the preceding Rule, from the method of equidistant ordinates, described in Problem 23, Part II.; and it may be used for all vessels of an ordinary size; but if a vessel be very deep, its sides very much curved, and great accuracy required, the content must be found by the following Rule: Find the areas of as many equidistant, parallel sections as you think necessary, by multiplying the square of the mean diameter of each section by .7854. Then to the sum of the areas of the two end sections, add four times the sum of the areas of all the even sections, and twice the sum of the areas of all the odd sections, not including the sections at the ends; multiply the sum by the common distance of the sections; divide the product by 3; and the quotient will be the content in cubic inches.—Divide the content thus found by 277.274, and 2218.192; and the respective quotients will be the content in imperial gallons and bushels.

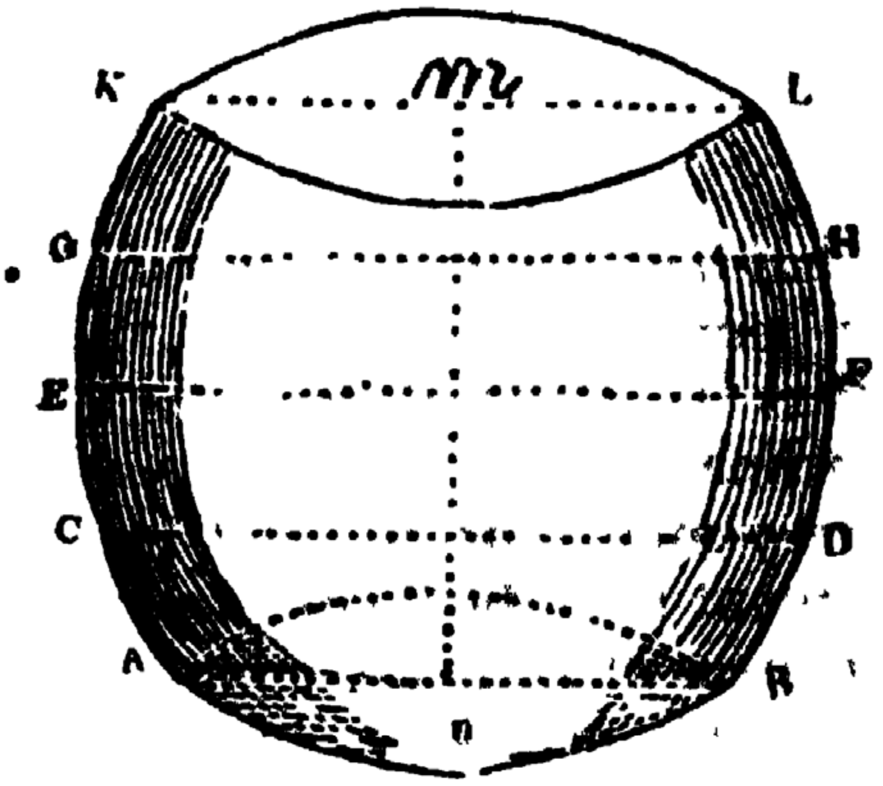
Note 1. Always make choice of an odd number of sections, in order that the number of parts into which the vessel is divided, may be equal. Seven or nine will, in general, be sufficient, except when the vessel is very deep, in which case it may be necessary to take eleven, thirteen, &c. as the case may require.

2. The perpendicular depth of the vessel must first be taken, in order to determine the common distance of the ordinates, which may be found by dividing the whole depth by the number of sections minus one.

3. Great care must be taken to obtain the diameters of the sections at equal perpendicular distances from each other; for if their distance be measured upon the side of the vessel, it is evident that the process will be incorrect. (*See Problem I. Case II. Part VI.*)

EXAMPLES.

1. The diameter AB, of the following vessel, measures 49, CD 62, EF 68, GH 65, KL 56, and the depth *mn* 48 inches; what is its content in imperial gallons and bushels?



Here $49^2 + 56^2 = 2401 + 3136 = 5537$, *the sum of the squares of the top and bottom diameters*; $68^2 \times 2 = 4624 \times 2 = 9248$, *twice the square of the middle diameter*; *and* $62^2 + 65^2 \times 4 = 3844 + 4225 \times 4 = 8069 \times 4 = 32276$, *four times the sum of the squares of the diameters taken at one-fourth and at three-fourths of the depth. Then,* $(5537 + 9248 + 32276) \times 48 = 47061 \times 48 = 2258928$; *and* $2258928 \div 4236.48 = 533.2087$, *the content in imperial gallons; also* $2258928 \div 33891.48 = 66.6517$, *the content in imperial bushels.*

2. The bottom diameter of a gyle-tun measures 74.2, and that of the top 84.3 inches; the diameter taken at $\frac{1}{4}$ of the depth from the bottom, is 93.6, and that at $\frac{3}{4}$ of the depth 98.7 inches; the middle diameter 102.4, and the depth 60.8 inches; how much ale will it contain at once? *Ans.* 1514.1428 *gallons.*

3. What is the content of a vessel in imperial gallons and bushels, the depth of which is 120 inches, and the mean diameters of seven equidistant, parallel sections, as follow: the diameter of the bottom or first section 124, the second 146, the third 161, the fourth

164, the fifth 166, the sixth 157, and the diameter of the top, or seventh section 144 inches?

Ans. The area of the first section is = 12076.3104, *the second* = 16741.5864, *the third* = 20358.3534, *the fourth* = 21124.1184, *the fifth* = 21642.4824, *the sixth* = 19359.3246, *and the seventh* = 16286.0544; *then by the method of equidistant ordinates, or parallel sections, we find the content* = 2275094.36 *cubic inches* = 8205.2206 *gallons, and* 1025.6525 *bushels, the answer required.*

REMARK.

In order to find the content of a copper with a convex or a concave bottom, generally called a *rising* or a *falling* crown, pour in as much water as will just cover the bottom; and make marks with chalk upon the sides of the copper, at the surface of the water; then draw it into a vessel of a known measure, or one that may be easily gauged; after which find the content of the remainder of the copper, as directed in this Problem, which being added to the number of gallons requisite to cover the bottom, will give the whole content.—The content of a still may also be found in the same manner.

PROBLEM VII.

To gauge and inch a round guile-tun according to the method practised in the Excise.

RULE.

Take cross diameters of the guile-tun, in the middle of every ten inches from the bottom upwards; that is, measure the first diameters at five inches from the bottom, the second at fifteen, the third at twenty-five, &c.; and take half the sum of each two for mean diameters. Divide the square of the first mean diameter by 353.04, multiply the quotient by 10; and the product will be the content of the first ten inches of the vessel. Find the content of every ten inches of the perpendicular depth in the same manner, which being added together, will give the content of the whole tun in imperial gallons.

Note 1 It must here be observed that the square of the diameter of any section divided by 353.04, will give the area of that section in imperial gallons, for in finding the areas of figures, Gaugers always consider their depth to be one inch.

2 Mash-tuns are used for the purpose of macerating malt in hot water, to extract from it the saccharine substance; and guile-tuns, sometimes called working-vats, are vessels in which wort is fermented, in order to convert it into ale or beer.

REMARK.

The most eligible method of taking the dimensions of circular vessels, is to quarter them, in order to obtain cross diameters as correctly as possible.—First, take the diameter of the top of the vessel, which multiply by .707, or .7; and the product will be the side of the inscribed square. *(See Part II. Prob. XV. Rule III. in the Notes.)*

Apply the side of the square thus found, four times to the top of the vessel, by your dimension cane, or tape; and mark, with chalk, each angle of the inscribed square; and thus will the top of the vessel be quartered.—With a steady hand, draw a chalk line, from each quartering point, down the inside of the vessel, to the bottom; taking care that these lines be equally distant from each other, not only at the bottom of the vessel, but also at every horizontal section; otherwise the vessel will not be truly quartered in every part.

Place your dimension cane, or other instrument, in a perpendicular direction, with its end resting upon the bottom of the vessel, about half-way between the centre of the bottom and the side; at the same time laying a rod, or holding a cord diametrically across the top, so as to come in contact with the dimension cane; and thus you will have the perpendicular depth of the vessel, upon the dimension cane, where it intersects the rod or cord at the top.

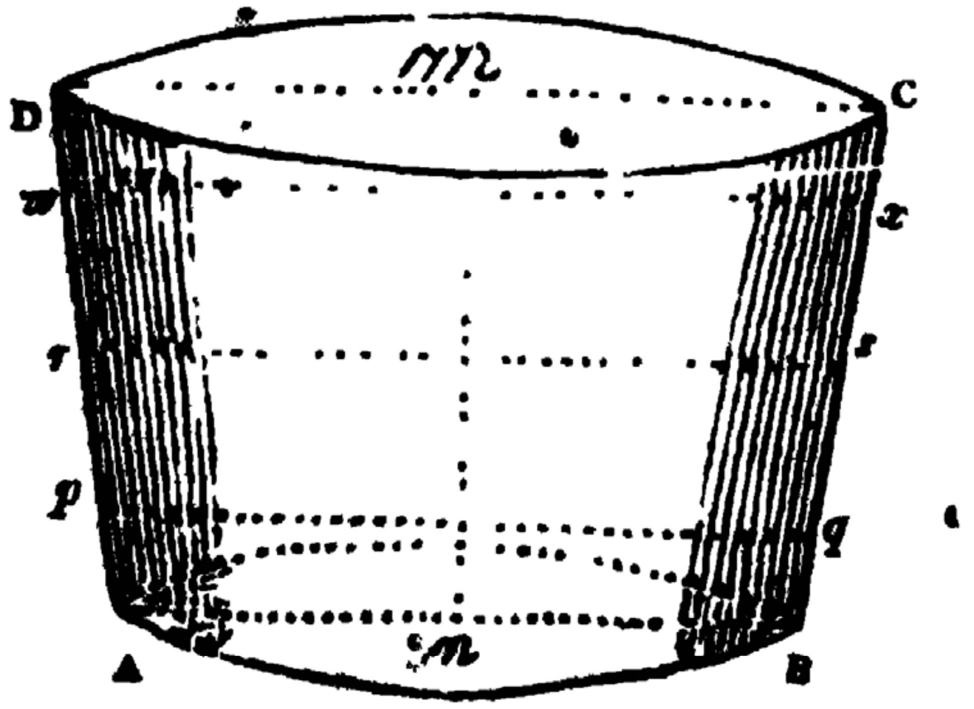
Now, in order to take cross diameters in the middle of every ten inches, mark the dimension cane with chalk, at 5, 15, 25, 35 inches &c.; and place that instrument in the same manner in which it stood in taking the depth; then bring a rod horizontally in contact with the cane, at 5 inches from the bottom, so that the rod and cane may form right angles; and let the end of the rod, at the same time, touch one of the quartering lines; and there make a mark with chalk, on the inside of the vessel; and you will have one point at which a diameter must be taken.

Do the same at 15, and 25 inches, &c. from the bottom; and you will obtain all the points in one of the quartering lines, at which diameters must be taken. These points may then be transferred to the other three quartering lines, by a pair of compasses or a dipping piece; and hence you may proceed to take cross diameters, as before directed; always beginning at the bottom.

Note. When vessels are small, cross diameters may be most easily taken by an instrument called the *Diameter Rod, or Rule.* When they are large, they must be measured by the *Dimension Cane.*

EXAMPLES.

1. Let the following figure ABCD represent a guiletun, in the form of the frustum of a cone, whose perpendicular depth *mn* is 30, the mean diameters *pq* 42.2, *rs* 45.3, and *wx* 48.6 inches; what is its content in imperial gallons?



Here $\frac{42.2 \times 42.2}{353.04} = \frac{1780.84}{353.04} = 5.044$ *gallons, the area of the segment at pq; and* $5.044 \times 10 = 50.44$ *gallons, the content of the first* 10 *inches of the perpendicular depth.*

Again, $\frac{45.3 \times 45.3}{353.04} = \frac{2052.09}{353.04} = 5.812$ *gallons, the area of the section at rs; and* $5.812 \times 10 =$ 58[illegible] *gallons, the content of the second* 10 *inches of the depth.*

Also, $\frac{48.6 \times 48.6}{353.04} = \frac{2361.96}{353.04} = 6.690$ *gallons, the area of the segment at wx; and* $6.690 \times 10 = 66.90$ *gallons, the content of the third and last* 10 *inches of the depth.*

Lastly, $50.44 + 58.12 + 66.90 = 175.46$ *gallons* = 4 *barrels,* 3 *firkins, and* 4.46 *gallons, the whole content of the guile-tun.*

Note. In Practice, when the diameters are taken in the middle of every ten inches, it is not necessary to multiply by 10; for if the decimal dot, in the area, be removed one figure towards the right-hand, you will obtain the content the same as if you actually multiplied the area by 10; but in the foregoing Example I have multiplied the area of each section by 10, in order to shew the learner how to proceed when diameters are taken in the middle of every 6 or 8 inches; or when the vessel is not full of liquor.

2. If the depth of the liquor in the foregoing guile-tun be 26.8 inches; how many gallons does it contain?

Here $50.44 + 58.12 = 108\ 56$ *gallons, the content of the first* 20 *inches of the depth; and* $6.69 \times 6.8 = 45.492$ *gallons, the content of the remaining* 6.8 *inches; then* $108.56 + 45.492 = 154.052$ *gallons, the content required.*

3. If the depth of the liquor in the preceding guile-tun be 18.4 inches; how many gallons does it contain? *Ans.* 99.2608 *gallons.*

REMARKS.

1. In order to facilitate the Practice of Gauging, it is necessary to be furnished with a measuring-tape, divided into inches, a gauging-rod, divided into inches and tenths, a chalk-line, plummet, &c. as the case may require; and also a dimension-book ruled in columns proper for the dimensions intended to be taken.

2. The following Table, in which are entered the dimensions, areas, and contents of the foregoing guile-tun, may serve as a specimen for mash-tuns, guile-tuns, coppers, stills, &c.

3. By the solution to the first Example, we find the content of the first 10 inches from the bottom, to be 50.44 gallons; and as 9 gallons make 1 firkin, and 4 firkins 1 barrel, we have only to divide 50.44 gallons, by 9 and 4, successively; and we obtain 1 barrel, 1 firkin, and 5.44 gallons, for the reduced content of the first 10 inches.—In the same manner we have 58.12 gallons = 1 barrel, 2 firkins, and 4.12 gallons, for the reduced content of the second 10 inches; and likewise 66.90 gallons = 1 barrel, 3 firkins, and 3.90 gallons, for the reduced content of the third 10 inches of the depth; the whole being equal to 4 barrels, 3 firkins, and 4.46 gallons, as in the preceding Solution and following Table.

A. B.'s Guiletun, gauged March 10th, 1827.

Depths	*Cross Diameters.*		*Mean Diam.*	*Areas in Gallons.*	*Areas in*			*Contents in*		
					B.	*F.*	*G.*	*B.*	*F.*	*G.*
10	41.8	42.6	42.2	5.044	0	0	5.044	1	1	5.44
10	44.7	45.9	45.3	5.812	0	0	5.812	1	2	4.12
10	48.5	48.7	48.6	6.690	0	0	6.690	1	3	3.90
30	*Whole depth and content*							4	3	4.46

REMARK.

In order to find the content of the foregoing guile-tun, at every inch from the bottom, proceed thus: Add the area of the first section to itself, and the sum will be the content at two inches of the depth; to this add the same area, and the sum will be the content at three inches; and thus continue adding the area of the first section, until you come to the tenth inch; and you will have the content at every inch of the first ten inches. To this content add the area of the second section, and you will obtain the content at eleven inches from the bottom; and thus proceed until you come to the 30th inch, changing the area at every ten inches; and you will have the content at every inch of the depth, as in the following Table.

A TABLE OF THE FOREGOING GUILE-TUN.

Wet Inches	Contents in B.	F.	G.	Wet Inches.	Contents in B.	F.	G.	Wet Inches.	Contents in B.	F.	G.
1	0	0	5.044 5.044	11	1	2	2.252 5.812	21	3	0	7.250 6.690
2	0	1	1.088 5.044	12	1	2	8.064 5.812	22	3	1	4.940 6.690
3	0	1	6.132 5.044	13	1	3	4.876 5.812	23	3	2	2.630 6.690
4	0	2	2.176 5.044	14	2	0	1.688 5.812	24	3	3	0.320 6.690
5	0	2	7.220 5.044	15	2	0	7.500 5.812	25	3	3	7.010 6.690
6	0	3	3.264 5.044	16	2	1	4.312 5.812	26	4	0	4.700 6.690
7	0	3	8.308 5.044	17	2	2	1.124 5.812	27	4	1	2.390 6.690
8	1	0	4.352 5.044	18	2	2	6.936 5.812	28	4	2	0.080 6.690
9	1	1	0.396 5.044	19	2	3	3.748 5.812	29	4	2	6.770 6.690
10 *	1	1	5.440 5.812	20 *	3	0	0.560 6.690	30	4	3	4.460

Note 1. The *stars* or *asterisks*, in the preceding Table, denote the changes of the areas; and their use will tend to prevent mistakes, in *tabulating* other vessels.

2. The use of the foregoing Table is evident; for when a vessel is once *gauged* and *inched*, we have only to take the depth of the liquor, and against it we find the content required: thus if the depth be 23 inches, we have 3 B. 2 F. 2.63 G. for the content. If the depth be 23.5 inches, we take one-half of the area of the third section; namely one-half of 6.69 G. = 3.345 G, which being added to the content at 23 inches, we obtain 3 B. 2 F. 5.975 G. for the content required. Again, if the depth be 23.8 inches, we take $\frac{8}{10}$ of 6.69 G. = 5.352 G; and this being added to the content at 23 inches, we have 3 B. 2 F. 7.982 G. for the content at 23.8 inches of the depth.

3. Any mash-tun, guile-tun, copper, or still, whose sides are curved, may be gauged and inched in the same manner; but if the sides be much curved, cross diameters must be taken in the middle of every 4, 6, or 8 inches of the perpendicular depth, as the case may require; and the areas of the different sections, must be used, as before directed, in *tabling* the vessels.

4. In order to gauge and inch a mash-tun, or a guile-tun, in the form of a prismoid, or the frustum of a square pyramid, take lengths and breadths in the middle of every 6, 8, or 10 inches of the perpendicular depth, as the case may require; and divide the product of each length and breadth by 277.3, and the quotients will be the areas of the sections in imperial gallons, then enter the dimensions, areas, and contents, in Tables as before directed.

5. Large mash-tuns, and guile-tuns are generally fixed in an inclining position, called the *drip* or *fall* of the tun, in order that the liquor may be conveniently drawn off; when this is the case, pour in as many gallons of water as will just cover the bottom; and make marks upon the sides of the tun, at the surface of the water; then draw it off, and proceed with the remainder of the tun, as before directed.

6 The water that is used to cover the bottoms of guile-tuns, and of coppers with *rising* or *falling* crowns, must either be measured with a gallon measure, or drawn *off*, into a vessel of a known capacity, or into one that may be easily *gauged*; and the quantity must be added to the content of the other part of the guile-tun or copper.

PROBLEM VIII.

To gauge a back or cooler.

Backs or coolers are vessels which receive the wort, when let out of the copper, in order to be cooled. They are commonly of a rectangular form, and seldom exceed nine or ten inches in depth.

RULE.

Multiply the mean length by the mean breadth; divide the product by 277.3, and the quotient will be the area in imperial gallons; which being multiplied by the mean depth, will give the quantity of wort contained in the cooler.

Note 1. In taking the dimensions of a cooler, find the middle of both ends, and also that of the sides, at which take the length and breadth; but as coolers are never fixed in a horizontal position, but inclined a little, in order that the wort may run off, and as their bottoms are frequently warped, it is necessary to take the depth of the liquor in several places, and divide the sum of these depths by their number, for a mean depth.

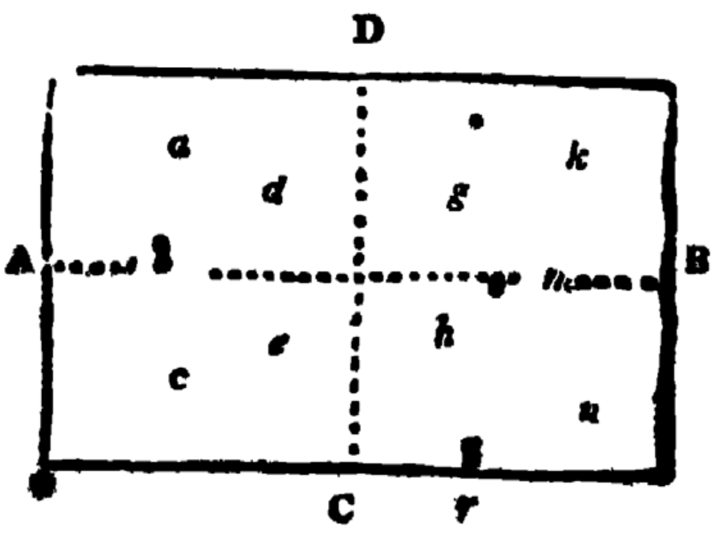
2. In order to find a constant dipping-place, which will save much trouble, proceed thus: Find a mean depth, as directed in the last note; then try in different parts of the cooler, until you find a place of the same depth as your mean depth; at which make a mark upon the side of the cooler, for a constant dipping-place.

3. It has been found by experiments that 10 gallons of hot wort will only measure to 9 gallons when the wort is cold, therefore, an allowance of one gallon in every ten must be made, when wort is gauged hot. In order to do this, multiply the number of warm gallons by .9, and the product will be the number of gallons when cold.

EXAMPLES.

1. The length AB of a cooler is 125.6 inches, the breadth CD 73.4 inches, and the depth of the wort taken in ten different places, as below: how many gallons are contained in the cooler?

Depths.
Inches.
At a = 4.3
b = 4.2
c = 4.2
d = 4.5
e = 4.4
g = 4.6
h = 4.5
k = 4.9
m = 4.8
n = 4.6
10)45.0
4.5 *mean.*



Inches.
125.6 *length.*
73.4 *breadth.*
5024
3768
8792
277.3)9219.04(33.24 *gallons, the area for one inch deep.*
8319
9000
8319
6814
5546
12680
11092
1588

Gallons
33.24 *area.*
4.5 *mean depth.*
16620
13296
149.580 *the content of the wort in the cooler.*

By the Sliding Rule.

Divisor.	*Length.*	*Breadth.*	*Area.*
As 277.3 *on* A :	125.6 *on* B ::	73.4 *on* A :	33.24 *on* B.

And,

Unity.	*Area.*	*Depth.*	*Content.*
As 1 *on* A :	33.24 *on* B ::	4.5 *on* A :	149.58 *on* B.

Note. The constant dipping-place may be either at *d* or *h*; but it is preferable to find one nearer the side of the cooler, as at *r*.

2. If the mean depth of warm wort, in the foregoing cooler, be 5.8 inches; how many gallons will there be when the wort is cold?

$$\begin{array}{rl}
\text{Gallons.} & \\
33.24 & \textit{area.} \\
5.8 & \textit{depth.} \\
\hline
26592 & \\
16620 & \\
\hline
192.792 & \textit{gallons of warm wort.} \\
.9 & \textit{multiplier.} \\
\hline
173.5128 & \textit{gallons when cold.} \\
\hline\hline
\end{array}$$

3. If the mean depth of warm wort, in the preceding cooler be 6.4 inches; for how many gallons must the Excise officer charge the duty?

Ans. 191.4624 *gallons.*

Note 1 The depth of the liquor in a cooler is always taken to the tenth of an inch; and in order to facilitate the practical part of Gauging, Excise officers generally make a Table, exhibiting the content in barrels, firkins, and gallons, at every tenth of an inch. This is called *tenthing a cooler;* and may be performed in the following manner· Find the area of the cooler, and reduce it to barrels, firkins, and gallons; which being divided by ten, will give the content at one-tenth of an inch in depth. Add this content to itself, and the sum will be the content at two tenths. Again, to this content add that of the first tenth, and you will obtain the content at three tenths of the depth. Continue this operation until you arrive at six or seven inches of the depth, which will generally be found sufficient; as the wort in coolers is not often deeper.

2. It has been before observed that coolers are generally rectangular; but should you meet with one of any other shape, its area may be found by the Rules given in Part II.

PROBLEM IX.

To gauge a cistern, couch, or floor of malt.

RULE.

Take the dimensions as directed in the last Problem; then multiply the mean length by the mean breadth; divide the product by 2218.2, and the quotient will be the area; which being multiplied by the mean depth, will give the content in bushels.

Note 1. According to Act of Parliament, barley must lie under water in the cistern, forty hours; in which time it is supposed to swell or increase to one-fourth more; so that four bushels in twenty are allowed for this increase.

From the cistern the barley is removed to the couch; and after having lain there twenty-six hours, it is deemed a floor. The same allowance is made in the couch as in the cistern; but when the corn has been thrown out of the couch into the floor, and there grown according to the usual custom, it is supposed to increase one-half; consequently an allowance is made of ten bushels in every twenty.

2. If cistern or couch bushels be multiplied by .8, the product will be neat bushels; but floor-bushels must be multiplied by .5, in order to reduce them to neat bushels.

3. The duty is always charged upon the best gauge of the cistern, couch, or floor; and in order to find from which the charge will arise, without reducing them to neat bushels, proceed thus: Multiply the best gauge of the cistern or couch by 1.6; and if the product exceed the floor bushels, the charge must be made from the cistern or couch; but if not, the charge must be made from the floor.

This multiplier is found by dividing eight-tenths by five-tenths. (*See the last Note.*)

EXAMPLES.

1. The mean length of a cistern is 96, the mean breadth 64, and the mean depth 32 inches; what is the area and content in malt bushels?

To find the area.

```
           Inches.
            96 length.
            64 breadth.
           ---
           384
          576
        ------
2218.2)6144.0(2.769 area in bushels.
       44364
       ------
       170760
       155274
       ------
        154860
        133092
        ------
         217680
         199638
         ------
          18042
```

By the Sliding Rule.

M. B. — *Length.* — *Breadth.* — *Area.*
As 2218.2 *on* A : 96 *on* B :: 64 *on* A : 2.77 *on* B.

To find the Content.

Bushels.
2.769 *area.*
32 *depth.*
5538
8307
88.608 *content.*

By the Sliding Rule.

Unity. — *Area.* — *Depth.* — *Content.*
As 1 *on* A : 2.77 *on* B :: 32 *on* A : 88.6 *on* B.

Or,

The content may be readily found by means of the line M D, on the Sliding Rule, without knowing the area: thus,

Length. — *Depth.* — *Breadth.* — *Content.*
As 96 *on* B : 32 *on* M D :: 64 *on* A : 88.6 *on* B.

2. The mean length of a floor of malt is 115, the mean breadth 112, and the mean depth 4.6 inches; what is its content in floor bushels?

Inches.
115 *length.*
112 *breadth.*
230
115
115
2218.2)1288.0(5.806 *area in bushels.*
110910
178900
177456
144400
133092
11308

Bushels.
5.806 *area.*
4.6 *depth.*
34836
23224
26.7076 *content.*

By the Sliding Rule.

Length. *Depth.* *Breadth.* *Content*
As 115 *on* B : 4.6 *on* M D :: 112 *on* A : 26.7 *on* B.

3. The mean length of a cistern is 126.4, the mean breadth 62.6, and the mean depth of the barley 32.8 inches; how many neat bushels are contained in the cistern?

Inches.
126.4 *length.*
62.6 *breadth.*
7584
2528
7584
2218.2)7912.64(3.567 *area in bushels.*
66546
125804
110910
148940
133092
158480
155274
3206

Bushels.
3.567 *area.*
32.8 *depth.*
28536
7134
10701
116.9976 *content in cistern bushels.*
.8 *multiplier in Note 2.*
93.59808 *content in neat bushels.*

4. If the mean depth of the barley in the foregoing cistern be 38.6 inches; how many neat bushels does it contain? *Ans.* 110.14896 *bushels.*

5. The length of a couch is 136.2, the breadth 72.6, and the depth of the barley 42.8 inches; what is its content in neat bushels? *Ans.* 152.60768 *bushels.*

6. If the depth of the barley in the foregoing couch be 46.3 inches; how many neat bushels does it contain? *Ans* 165.08728 *bushels.*

7. The length of a floor of malt is 236, the breadth, 212, and the depth 5.2 inches; what is its content in neat bushels? *Ans.* 58.643 *bushels.*

8. If the best cistern-gauge be 68.4, the best couch-gauge 69.8, and the best floor-gauge 109.5 bushels; from which will the charge of the duty arise?

Ans. From the couch.

Note In this Problem, we have supposed the cistern, couch, and floor to be in the form of a parallelopipedon, which is most commonly their shape, but their contents may be obtained by the Rules given in Part IV., Sect. I., whatever form they may assume.

PROBLEM X.

Cask Gauging.

The performance of this part of Gauging is the most difficult that occurs; as no Rules can be given by which the exact form of casks may be ascertained.

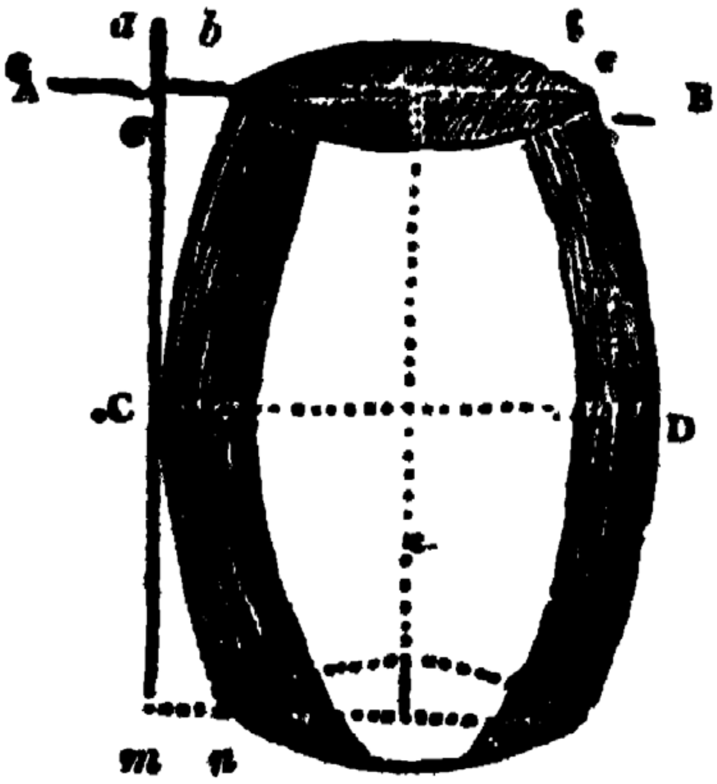
There are commonly reckoned four forms or varieties of casks, viz.

1. The middle frustum of a spheroid.
2. The middle frustum of a parabolic spindle.
3. The lower frustums of two equal paraboloids.
4. The lower frustums of two equal cones.

The Rule for finding the content of the 1st variety, is given in Prob. 13, Part VII.; that for the 2nd in Prob. 18, Part VII.; that for the 3rd in Prob. 16, Part VII.; and the content of a cask of the 4th variety may be obtained by Prob. 8, Sect. I., Part IV.; but it is very probable that there never was a cask that agreed exactly with any of the varieties; for very few casks are to be met with that will contain so much as the first form, or so little as the third or fourth; so that the second variety is the most general form of casks.

Note Excise officers generally consider all casks as belonging to the first variety, and gauge them as such; but this practice ought to be abolished, as being injurious to the Trader. *(See Nesbit's and Little's Practical Gauging.)*

To take the dimensions of a standing cask.



Measure the distance between the inside of the chimb, close to the head, and the outermost sloped edge of the opposite staff, which will be the head diameter within the cask, *very nearly.*

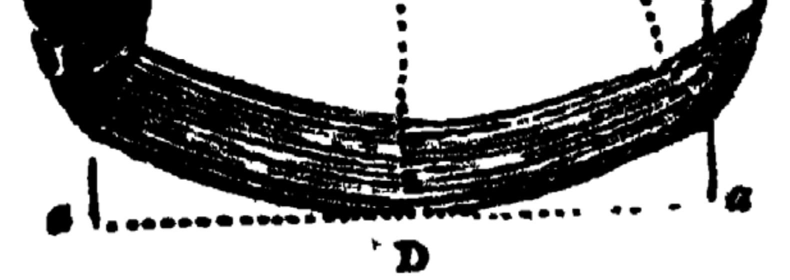
In order to find the bung diameter, lay a straight rod AB, across the centre of the head, and perpendicular to it, place another straight rod *am*, so as to touch the bulge of the cask at C; measure the distance between the outer edge of each chimb at *b* and *c*; also measure *ab*, which should be equal to *mn*; then *bc* being added to twice *ab*, will give the bung diameter CD, including the thickness of the staff on each side of the cask. From this take twice the thickness of the staff, at the bulge, as nearly as your judgment directs, having regard to the size of the cask; and you will obtain the internal bung diameter.

It is unnecessary to give any directions for taking the length of a cask in this position, admitting a hole to be in the upper head, which is most commonly the case.

Note 1. The external bung diameter of a standing cask may also be found by dividing the circumference by 3.1416. (*See Prob. XIII. Part II.*)

2. The staves of casks in general, are thicker at the bulge than at the head; London-made casks, however, have their staves commonly much thicker at the head than at the bulge. The best method of forming a correct judgment, is to examine empty casks of the same size and make, as those you are about to gauge. By this means, you will come to tolerably correct conclusions relating to the deductions necessary to be made for the thickness of the heads, staves, &c (*See the Notes in the next Article.*)

To take the dimensions of a lying cask.



Measure the head diameter in the same manner as directed for a standing cask; and in taking the bung diameter CD, make an allowance for the thickness of the staff at the bung-hole.

The length may be most expeditiously found by a pair of callipers, allowing for the thickness of the heads, according to the size of the cask; but as it cannot be expected that every person concerned in Gauging, is furnished with this instrument, the length may be obtained in the following manner: Apply a straight rod AB, to the bulge of the cask; and at right angles to it, place two others *ma*, and *na*, touching the chimbs at each end, and making *mr* equal to *nr*; then measure the distance *mn*, from which subtract the depths of the chimbs, together with the thickness of the heads, as nearly as you can judge, and the remainder will be the internal length of the cask.

Note 1. In taking the dimensions of a cask, the Gauger ought carefully to observe that the bung-hole be in the middle; that the bung-staff be regular and even within, and that the staff opposite the bung-hole be neither thicker nor thinner than the rest, which he may easily ascertain by the gauging-rod; and if any impropriety be discovered, a proper allowance must be made for it, in the dimensions.

2. It is also necessary to observe that the heads of the cask be equal, and truly circular; if not, take cross diameters of each head, and divide their sum by four for a mean diameter.

To determine the content of a cask of any of the four varieties, by finding a mean diameter, or reducing it to a cylinder.

RULE.

Multiply the difference between the bung and head diameters, when it is 6 inches, or less,

$$\text{By}\left\{\begin{matrix}.68\\.62\\.55\\.5\end{matrix}\right\}\text{for the}\left\{\begin{matrix}1\\2\\3\\4\end{matrix}\right\}\text{Variety.}$$

Or, if the difference between the bung and head diameters exceed 6 inches,

$$\text{By}\left\{\begin{matrix}.7\\.64\\.57\\.52\end{matrix}\right\}\text{for the}\left\{\begin{matrix}1\\2\\3\\4\end{matrix}\right\}\text{Variety.}$$

Add the product to the head diameter, and the sum will be a mean diameter. Square the mean diameter, and multiply that square by the length of the cask; then if the product thence arising be divided by 353.04, you will obtain the content of the cask, in imperial gallons.

By the Sliding Rule.

Find the difference between the bung and the head diameters, on the inside of the slide marked C; and opposite to it, for each variety, is a number, which being added to the head, will give the mean diameter. Then, as the gauge-point on D, is to the length of the cask on C, so is the mean diameter on D, to the content on C.

Note. Neither of the above methods of finding a mean diameter, is strictly true; but in consequence of their simplicity, they are generally adopted by Officers of the Excise. (See Moss's Gauging, Sect. X.)

EXAMPLES.

1. If the length of a cask be 30, the bung diameter 24, and the head diameter 18 inches; what is its content in imperial gallons, for the first, second, and third varieties?

For the middle frustum of a spheroid, or the First Variety.

Bung diameter	24 inches.
Head diameter	18 inches.
Difference......	6
Multiplier	.68
	48
	36
Product	4.08
Head diameter	18.
Mean diameter	22.08
Ditto............	22.08
	17664
	4416
	4416
	487.5264 square.
	30 length.
Divisor 353.04)	14625.7920(41.428 *gallons.*

By the Sliding Rule.

The difference between the bung and head diameters is 6 inches, against which, on the line of inches, we find 4.16 on the line marked *spheroid*, which being added to 18, the head diameter, gives 22.16 inches, for the mean diameter. Then,

	On D.		*On* C.		*On* D.		*On* C.
As	18.79	:	30	::	22.16	:	41.5 *gallons.*

Note. The content given by the Rule on page 335, is 41.8088 gallons.

For the middle frustum of a parabolic spindle, or the Second Variety.

```
         Bung diameter    24 inches.
         Head diameter    18 inches.
                          ——
         Difference......  6
         Multiplier......  .62
                          ———
                           12
                          36
                          ———
      Product.........    3.72
      Head diameter      18.
                         —————
      Mean diameter      21.72
      Ditto...........   21.72
                         —————
                          4344
                        15204
                        2172
                       4344
                       ————————
                       471.7584 square.
                             30 length.
                       ————————————
          353.04)14152.7520(40.088 gallons.
```

By the Sliding Rule.

Against 6, the difference of the diameters, on the line of inches. we find 3.8 on the line marked second variety, which being added to the head diameter 18 we obtain 21.8 for the mean diameter. Then,

On D. *On* C. *On* D. *On* C.
As 18.79 : 30 :: 21.8 : 40.15 *gallons.*

Note. The content found by the Rule on page 341, is 41.4009 gallons.

For the lower frustums of two equal paraboloids, or the Third Variety.

Bung diameter	24 inches.
Head diameter	18 inches.
Difference......	6
Multiplier......	.55
	30
	30
Product	3.30
Head diameter.	18.
Mean diameter	21.30
Ditto...........	21.30
	639
	213
	426
	453.6900(square.
	30 length.
353 04)	13610.7000(38.552 *gallons.*

By the Sliding Rule.

The mean diameter, found as above, is 21.3 inches. Then,

On D. *On* C. *On* D. *On* C.
As 18.79 : 30 :: 21.3 : 38.6 *gallons.*

Note. The content found by the Rule on page 339, is 38.2397 gallons.

2. The length of a cask is 45, the bung diameter 36, and the head diameter 27 inches; what is its con-

tent in imperial gallons, for the first, second, and third varieties?

Ans. For the first variety, 141.343; *for the second,* 136.796; *and for the third,* 131.586 *imperial gallons.*

Note. By the following General Rules, the content of this cask is 133.188 gallons, which is between the second and third varieties. *(See the Second Example.)*

GENERAL RULES

For finding the contents of casks, from the head diameter, bung diameter, and length; without paying any regard to their variety.

RULE I.

Add into one sum, 39 times the square of the bung diameter, 25 times the square of the head diameter, and 26 times the product of those diameters; and multiply the sum by the length of the cask, Then, if the product thus obtained, be multiplied by .000031473, or divided by 31773 244: the result will be the content of the cask, in imperial gallons.

Note. This Rule is taken from Dr. Hutton's excellent Mathematical and Philosophical Dictionary, Vol. 1, page 528, and has now been adapted, by the Author, to the New Imperial Gallon. It gives the contents of casks less than the Rule for the second variety, and more than that for the third, being nearly in the middle between them, but the Dr observes that, "It agrees well with the real contents of casks, as hath been proved by several casks which have actually been filled with a true gallon-measure, after their contents had been computed by this method."

RULE II.

'Divide the head diameter by the bung diameter, to two places of decimals: and find that number, in the column of quotients, in the following Table, against which we have a multiplier or factor, for imperial gallons. Multiply this factor by the square of the bung diameter, and the product thence arising by the length of the cask; and the last result will be the content of the cask, in imperial gallons.

Note 1. If the quotient of the head by the bung diameter do not terminate in two places of figures, without a fractional remainder, find the multiplier answering to the first two decimals of the quotient, and subtract it from the next greater multiplier; then if the remainder be multiplied by the fractional part of the quotient, the product will be the corresponding proportional part to be added to the first multiplier. This method ought always to be used when the fractional remainder is large, or accuracy required.

2. This Rule gives the content very nearly the same as Rule I.; and will be found much easier in practice, as it requires a great deal fewer figures in the operation; particularly when the fractional remainder is rejected.

A NEW TABLE

Of multipliers or factors, for finding the content of any cask, in imperial gallons.

Quotient of the head divided by the bung diameter.	*Multipliers for Imperial Gallons.*	*Quotient of the head divided by the bung diameter.*	*Multipliers for Imperial Gallons.*
.50	.0018333	.76	.0023038
.51	.0018494	.77	.0023240
.52	.0018657	.78	.0023443
.53	.0018822	.79	.0023649
.54	.0018988	.80	.0023856
.55	.0019155	.81	.0024064
.56	.0019324	.82	.0024274
.57	.0019495	.83	.0024486
.58	.0019667	.84	.0024698
.59	.0019842	.85	.0024913
.60	.0020017	.86	.0025131
.61	.0020193	.87	.0025348
.62	.0020372	.88	.0025568
.63	.0020552	.89	.0025788
.64	.0020734	.90	.0026012
.65	.0020917	.91	.0026236
.66	.0021102	.92	.0026462
.67	.0021288	.93	0026689
.68	.0021476	.94	.0026918
.69	.0021666	.95	.0027149
.70	.0021858	.96	.0027381
.71	.0022050	.97	.0027615
.72	.0022244	.98	.0027850
.73	.0022440	.99	.0028086
.74	.0022638	1.00	.0028325
.75	.0022837		

EXAMPLES.

1. The length of a cask is 30, the bung diameter 24, and the head diameter 18 inches; what is its content in imperial gallons?

By Rule I.

Here 24 × 24 × 39 = 576 × 39 = 22464, *thirty-nine times the square of the bung diameter;* 18 × 18 × 25 = 324 × 25 = 8100, *twenty-five times the square of the head diameter; and* 24 × 18 × 26 = 432 × 26 = 11232, *twenty-six times the product of the diameters. Then,* (22464 + 8100 + 11232) × 30 × .000031473 = 41796 × 30 × .000031473 = 1253880 × .000031473 = 39.46336 *imperial gallons. Or,* 1253880 ÷ 31773.244 = 39.46339 *imperial gallons, the same as before.*

By Rule II.

Here 18 ÷ 24 = .75, *the quotient of the head divided by the bung diameter. Opposite to this in the preceding Table, we have the multiplier* .0022837; *then,* .0022837 × 24^2 × 30 = .0022837 × 576 × 30 = 1.3154112 × 30 = 39.462336 *imperial gallons, the same as by the first Rule.*

Note. The content of this cask, found by the foregoing Rules, for the First Variety, is 41.428 gallons; for the Second Variety, 40.086 gallons, and for the Third Variety, 38.852 gallons: hence, we see that the content given by the General Rules, is between the Second and Third Varieties.

2. The length of a cask is 45, the bung diameter 36, and the head diameter 27 inches, what is its content in imperial gallons?

Ans. By the first Rule, 133.188; *and by the second Rule,* 133.185 *imperial gallons.*

3. What is the content of a cask whose bung and head diameters are 32 and 24, and length 40 inches?

Ans. By the first Rule, 93.5427 *gallons.*

4. The bung diameter of a cask is 45, the head diameter 34.2, and the length 56.4 inches; what is its content in imperial gallons?

Ans. By the second Rule, 263.1169 *gallons.*

5. What is the content of a cask whose bung and head diameters are 37.3 and 28.6, and length 52.8 inches? *Ans. By the second Rule,* 170.2364 *gallons.*

DIAGONAL ROD.

To find the content of a cask by the diagonal or gauging rod.

This rod has two lines of numbers graduated upon it. One of them is divided into inches and tenths, for the purpose of taking dimensions; and the other is called a diagonal line; and expresses the content of any cask, in Imperial Gallons, corresponding to the cask's diagonal in inches and tenths.

When the rod is put into the bung-hole of a cask, so as to meet the head, where it intersects the staff opposite the bung-hole, the content of the cask is exhibited, in imperial gallons, on the diagonal line; reckoning from the end of the rod to the centre of the bung-hole.

The diagonal rod is very much used in gauging, in consequence of the ease and expedition with which the contents of casks may be obtained by it; and it commonly gives nearly the same as the General Rules.

Its construction is founded upon Theo. 20, Part I.; viz. that similar solids are to each other as the cubes of their like dimensions.

Note 1. Those who have not a diagonal rod, may nevertheless find the content of a cask by that method, in the following manner: Measure the diagonal of the cask, in inches and tenths; then multiply the cube of this diagonal by .002266, and the product will be the content of the cask in imperial gallons.

2. If the bung and head diameters, and length of a cask be given, the diagonal may be found by the following Rule: To the square of half the sum of the diameters, add the square of half the length; and the square root of the sum thence arising will be the diagonal, or distance between the centre of the bung-hole and the point where the middle of the opposite staff and head intersect each other.

EXAMPLES.

1. The diagonal of a cask is found to be 20 inches; what is its content in imperial gallons?

By the Diagonal Line.

Opposite to 20 inches, the given diagonal of the cask we find on the diagonal line, 18 *gallons, the content required.*

By Note I.

Here $20 \times 20 \times 20 = 8000$, *the cube of the diagonal; and* $.002266 \times 8000 = 18.128$ *gallons, the same as by the diagonal line.*

2. The length of a cask is 30, the bung diameter 24, and the head diameter 18 inches; required the diagonal and content.

By Note 2, *we have* $\frac{24 + 18}{2} = \frac{42}{2} = 21$, *half the sum of the diameters; and* $\frac{30}{2} = 15$, *half the length of the cask; then* $\sqrt{21^2 + 15^2} = \sqrt{441 + 225} = \sqrt{666} =$ 25.8 *inches, the diagonal.*

Again, by Note 1, *we have* $666 \times 25.8 \times .002266 = 17182.8 \times .002266 = 38.9362248$ gallons; which is nearly the same as given by the General Rules. (See the first Example.)

3. The length of a cask is 45, the bung diameter 36, and the head diameter 27 inches; required the diagonal and content.

Ans. The diagonal is 38.71 *inches, and the content* 131.4437 *gallons; which is the same as found for the third variety; and differs only* 1.74 *gallons from the content given by the General Rules. (See the second Example.)*

PROBLEM XI.

To ullage a standing cask.

The liquor contained in a cask when it is not full, is called the *wet ullage;* the vacuity or space not occupied by the liquor, is termed the *dry ullage;* and the method of finding the content of the liquor, is called ullaging a cask.

RULE.

Divide the wet inches by the length of the cask, to three places of decimals; and if the quotient exceed .500, add to the said quotient one-tenth of the excess; but if it be less than .500, subtract one-tenth of the deficiency: then the whole content of the cask being multiplied by the sum in the first case, or the remainder in the second, will give the ullage required.

Note When the whole content of the cask is not known, it must be found from proper dimensions, before the above Rule can be applied.

By the Sliding Rule.

Set the length of the cask upon C to 100 on the line marked *Segt. St.* or SS; then against the wet inches on C, you will have a *segment* on the line SS, which call a fourth number.

Set 100 on the line marked A, to the content of the cask upon B; then against the fourth number, before found, on A, is the quantity of liquor in the cask, upon B.

EXAMPLES.

1. The length of a cask is 30, the bung diameter 24, the head diameter 18, and the wet inches 22; what is the ullage, in imperial gallons?

Here 22 ÷ 30 = .733, which exceeds .500, by .233, one-tenth part of which is .0233; then .733 + .0233 = .7563, the multiplier.

The whole content of the cask, as found in the General Rules, in the last Problem, is 39.46 *imperial gallons; then,* 39.46 × .7563 = 29.843598 *imperial gallons, the ullage required.*

By the Sliding Rule.

On C.		*On* SS.		*On* C.		*On* SS.
As 30	:	100	::	22	:	75.5, *a fourth number.*

And,

On A.		*On* B.		*On* A.		*On* B.
As 100	:	39.46	::	75.5	:	29.6 *gallons.*

2. Let the dimensions and content be the same as in the last example: what is the ullage in imperial gallons, for 8 wet inches?

Here 8 ÷ 30 = .266, *which is less than* .500 *by* .234, *one-tenth part of which is* .0234; *then* .266 − .0234 = .2426, *the multiplier; hence,* 39.46 × .2426 = 9.572996 *gallons, the ullage required.*

Note. If we add 29.84, the ullage of the first example, to .57, the ullage of the second, we obtain 39.41 gallons, the whole content of the cask, *nearly*

By the Sliding Rule.

On C.		*On* SS.		*On* C.		*On* SS.
As 30	:	100	::	8	:	24.5, *a fourth number.*

And,

On A.		*On* B.		*On* A.		*On* B.
As 100	:	39.46	::	24.5	:	9.7 *gallons.*

3. The length of a cask is 50, the bung diameter 40, the head diameter 30, and the wet inches 20; what are the *wet* and *dry* ullages, in imperial gallons?

Ans. By Rule II. Prob. X. the content is found to be 182.696; *and hence the wet ullage* 71.25144, *and dry ullage* 111.44456 *imperial gallons.*

PROBLEM XII.

To ullage a lying cask.

RULE.

Divide the wet inches by the bung diameter, to three places of decimals; and if the quotient exceed .500, add to the said quotient one-fourth of the excess; but if it be less than .500, subtract one-quarter of the deficiency: then multiply the whole content of the cask by the sum in the first case, or the remainder, in the second, and the product will be the ullage required.

By the Sliding Rule.

Set the bung diameter upon C to 100 on the line marked *Seg. Ly.* or SL; then against the wet inches on C, you will have a *segment* on the line SL, which call a fourth number.

Set 100 on the line marked A, to the content of the cask upon B; then against the fourth number on A, is the quantity of liquor in the cask, upon B.

EXAMPLES.

1 The length of a cask is 30, the bung diameter 24, the head diameter 18, and the wet inches 15; what is the ullage, in imperial gallons?

Here $15 \div 24 = .625$, *which exceeds* .500 *by* .125, *one-fourth part of which is* .03125; *then* $.625 + .03125 = .65625$, *the multiplier.*

The whole content of the cask, as found by the General Rules, in the tenth Problem, is 39.46 *imperial gallons; then,* $39.46 \times .65625 = 25.895625$ *imperial gallons, the ullage required.*

By the Sliding Rule.

On C. *On* SL. *On* C. *On* SL.
As 24 : 100 :: 15 : 67, *a fourth number.*

And,

On A. *On* B. *On* A. *On* B.
As 100 : 39.46 :: 67 : 26.4 *gallons.*

2. Let the dimensions and content be the same as in the last example; what is the ullage, in imperial gallons, for 9 wet inches?

Here 9 ÷ 24 = .375, *which is less than* .500, *by* .125, *one-fourth part of which is* .03125; *then* .375 − .03125 = .34375, *the multiplier; hence,* 39.46 × .34375 = 13.564375 *imperial gallons, the ullage required.*

Note. If we add 25.89, the ullage of the first example, to 13.56, the ullage of the second, we obtain 39.45 gallons, the whole content of the cask, *nearly*

By the Sliding Rule.

On C. *On* SL. *On* C. *On* SL.
As 24 : 100 :: 9 : 32.5, *a fourth number.*

And,

On A. *On* B. *On* A. *On* B.
As 100 : 39.46 :: 32.5 : 12.8 *gallons.*

3. The length of a cask is 32, the bung diameter 25.6, the head diameter 19.2, and the wet inches 15.8; what are the *wet* and *dry* ullages, in imperial gallons?

Ans. By Rule II. Prob. X. the content is found to be 47.892660224; *and hence the wet ullage* 30.9506, *and the dry ullage* 16.942 *imperial gallons.*

MISCELLANEOUS QUESTIONS

CONCERNING

GAUGING.

1. The perpendicular depth of a vessel in the form of a parallelopipedon is 52, its breadth 75, and the diagonal of its bottom 125 inches; what is its content in imperial gallons? *Ans.* 1406.419 *gallons.*

2. A vessel in the form of a parallelopipedon contains 675 imperial gallons; its length is 85, and its breadth 64 inches; what is its depth?

Ans. 34.4044 *inches.*

3. The diagonal of a cylindrical vessel is 45, and its diameter 27 inches; what is its content in imperial gallons? *Ans.* 74.3456 *gallons.*

4. The greatest diameter of a vessel in the form of the frustum of a cone, is 96, the least diameter 48, and its slant height 51 inches; how many imperial gallons will it contain? *Ans.* 685.3257 *gallons.*

5. A reservoir measures 144 inches in length, 122 in breadth, and 85 in depth; how long will a person be in filling it with water, by means of a pump; supposing he makes 30 strokes in a minute, and lifts 3 pints of water at each stroke? *Ans.* 7 *hours,* 58.717 *minutes.*

6. The slant height of a cistern in the form of the frustum of a square pyramid is 153, the perpendicular height 135, and the side of the less end 92 inches; what is its content in imperial gallons?

Ans. 13935.2326 *gallons.*

7. The altitude of a vessel in the form of a hexagonal prism is 60, and the side of its base 30 inches; what is its content in imperial bushels?

Ans. 63.2477 *bushels.*

8. The greatest depth of the liquor in a cylindrical vessel, placed upon an inclined plane, is 38 inches, the least depth 32 inches, and the diameter of the vessel 36 inches; how many imperial gallons does it contain? *Ans.* 128.4985 *gallons.*

9. The transverse diameter of an elliptical bath measures 144, the conjugate 112, and the depth 60 inches; how many imperial gallons will it contain?

Ans. 2740.7712 *gallons.*

10. If the internal diameter of a hollow sphere be 100 inches; how many bushels of corn, imperial measure, will it hold? *Ans.* 236.0472 *bushels.*

11. The top diameter of a conical vessel measures 32, and its slant height 34 inches; how many imperial gallons of wine will it contain? *Ans.* 29.002 *gallons.*

12. The top diameter and depth of a vessel, in the form of the greater segment of a globe, are 32 inches each; how many imperial gallons will it hold?

Ans. 108.2773 *gallons.*

13. If the linear side of each Platonic body be 30 inches; required their respective contents in imperial gallons.

Ans. The content of the tetraedron is 11.4748, *the hexaedron* 97.3674, *the octaedron* 45.8994, *the dodecaedron* 746.1385, *and the icosaedron* 212.4261 *imperial gallons.*

14. Two porters agreed to drink off a pot of strong beer at two pulls, or a draught each; now, the first having given it a black eye, as it is called, or drunk till the surface of the liquor just touched the opposite edge of the bottom, gave the remaining part to the other; what was the difference of their shares, supposing the pot was the frustum of a cone, whose top diameter was 3.7, bottom diameter 4.23, and perpendicular depth 5.7 inches? *Ans.* 7.06511 *cubic inches.*

15. At Konigstein, near Dresden, in Germany, is a cask whose head diameter is 25, bung diameter 26, and perpendicular altitude 28 feet; how many gallons of wine, imperial measure, will it contain?

Ans. The content found by Rule II., Problem X., is 89674.383 *gallons, which exceeds the content of the cask at Heidelberg, by* 31387.4255 *imperial gallons. (See Example* 4, *Problem III.)*

Note. Those who wish to see this subject more fully treated, are referred to Nesbit's and Little's Treatise on Practical Gauging.

REMARK.

The Konigstein cask was begun in the year 1722, and finished in 1725, under the direction of General Kyaw; and is considered to be the largest cask in the world.

It consists of 157 staves, each 8 inches in thickness; and one of its heads is composed of 26, and the other of 28 boards.

The top or upper head of this enormous cask is railed round, and affords sufficient room for twenty persons to regale themselves; and there are several sorts of large goblets, called "Welcome Cups," offered to strangers, who are invited to drink by a Latin inscription, which in English is as follows: "Welcome, Traveller, and admire this Monument, dedicated to Festivity, in order to exhilarate the Mind with a cheerful Glass, in the year 1725, by FREDERIC AUGUSTUS, King of *Poland*, and Elector of *Saxony*, the Father of his Country, the TITUS of his Age, the Delight of Mankind: Drink, therefore, to the Health of the Sovereign, the Country, the Electoral Family, and Baron KYAW, Governor of *Konigstein*; and if thou be able, according to the Dignity of this cask, the most capacious of all Casks, drink to the Prosperity of the whole Universe; so farewell."

THE METHOD OF

COMPUTING DISTANCES

BY THE

VELOCITY OF SOUND.

THE velocity of sound, or the space through which it is propagated in a given time, has been very differently estimated by authors who have written upon this subject. Roberval states it at the rate of 560 feet in a second of time; Gassendus at 1473; Musenne at 1474; Duhamel, in the History of the Academy of Sciences at Paris, at 1338; Newton at 968; Derham, in whose measure Flamstead and Halley acquiesce, at 1142.

The reason of this variety is ascribed, by Derham, partly to some of those gentlemen using strings and plummets instead of regular pendulums; partly to the too small distance between the sonorous body and the place of observation; and partly to no regard being paid to the winds.

By the account since published by M. Cassini de Thury, in the Memoirs of the Royal Academy of Science at Paris, 1738, where cannon were fired at various, as well as great distances, under many varieties of wind, weather, and other circumstances, and where the measures of the different places had been settled with the utmost exactness, it was found that sound was propagated, on a medium, at the rate of 1038 French feet, in a second of time; and as the French foot is to the English, in the proportion of 15 to 16, it follows that 1038 French feet are equal to 1107 English feet. Therefore, the difference of the measures of Derham and Cassini is 35 English feet, or 33 French feet in a second.

The medium velocity of sound, therefore, is nearly at the rate of 1 mile, or 5280 feet, in 4⅔ seconds, or 1 league in 14 seconds, or 13 miles in 1 minute. But sea miles are to land miles nearly as 7 to 6; and, therefore, sound moves over 1 sea mile in nearly 5⅓ seconds, or 1 sea league in 16 seconds.

It is also commonly observed, that persons in health, have about 75 pulsations, or beats of the artery at the wrist, in 1 minute; consequently, in 75 pulsations, sound flies about 13 land miles, or 11¼ sea miles, which is nearly 1 land mile in 6 pulsations, and 1 sea mile in 7 pulsations, or 1 league in about 20 pulsations; and hence the distance of objects may be found by knowing the time occupied by sound, in passing from those objects to an observer.

Having given the time, in seconds or pulsations, that sound is in passing from an object to an observer, to find the distance of the object.

RULES.

1. As 1 second is to 1107 feet, so is the time in seconds, to the distance of the object in feet, which

being divided by 3, will give the distance in yards; and this again divided by 1760, will give the distance in miles.

2. As 75 pulsations is to 22140 yards, so is the time, in pulsations, to the distance in yards.

3. When accuracy is not required, say, as 6 pulsations is to 1 mile, or 1760 yards; so is the time in pulsations, to the distance in miles, or yards. Or take $\frac{3}{14}$ of the time in seconds, for the distance in miles.

Note. In 1½ pulsations, sound flies a quarter of a mile; in 3 pulsations, half a mile, in 4½ pulsations, three quarters of a mile, in 6 pulsations, one mile, &c. &c.. hence the distance of an object from which sound proceeds, may be easily ascertained without the trouble of calculation.

EXAMPLES.

1. After observing a flash of lightning, I found by my watch that it was 16 seconds before I heard the thunder; how far was I from the cloud whence it came?

```
      ″       ft.       ″
As 1  :  1107  ::  16
           16
         ----
         6642
        1107
        -----
      3)17712 yards.
   1760)5904(3 miles, 624 yards.   Ans.
        5280
        ----
         624 yards.
         ====
```

2. If the report of the Tower-guns be heard at Shooter's Hill, 38 seconds after they are fired; what distance are the two places from each other?

Seconds.
38
3
14)114(8.142 miles. *Ans.*
112
20
14
60
56
40
28
12 rem.

3. After seeing the flash of a gun at sea, I counted 58 pulsations at the wrist; what was the distance between the ship from which the gun was fired, and the place where I stood? *Ans.* 9.728 *miles.*

4. Perceiving a man at a distance felling a tree, I remarked that 5 of my pulsations passed between seeing him strike with the axe, and hearing the report of the blow; what was the distance between us?
Ans. 1476 *yards.*

5. The report of a clap of thunder was heard 4 pulsations after seeing the flash of lightning; what was the distance of the cloud from which the thunder issued? *Ans.* 1180.8 *yards.*

Note 1. Here it may not be improper to remark, that when the report of thunder is heard nearly at the same instant the lightning is seen, the observer is in great danger; as it is evident that the thunder cloud is very near him.

2. It may also be remarked that persons should never take shelter, in a thunder storm, under a tree; as the branches will attract the lightning, if it comes within the sphere of their attraction. Many have met premature deaths by taking shelter under trees, during thunder storms.

METHODS
OF ASCERTAINING THE
TONNAGE OF SHIPS.

PRELIMINARY OBSERVATIONS.

If the number of cubic feet of water which a ship displaces, in sinking from the *light water-mark* to the *load water-mark*, be divided by 35, the number of cubic feet of sea-water in 1 ton, the quotient will be the number of tons which the vessel is capable of carrying, or the vessel's true burthen.

Now the number of cubic feet of water so displaced, is exactly equal to the solid content of so much of the body of the ship, as is contained between the two water-lines or marks; but in consequence of the great variety of forms given to this part of the vessel, no exact practical Rule can be given that will apply in all cases. If, indeed, the areas of three or five horizontal sections of that part of the hull contained between the said two water-lines, could be obtained by means of equi-distant ordinates, then the content might be correctly found by Prob. VI., Part VIII.; but as taking of the dimensions of these sections would be attended with great difficulty, we shall give the following Parliamentary Rules for ascertaining the tonnage of Merchants' and King's ships.

CASE I.

When the vessel is laid dry.

RULE.

Measure the length on a straight line along the rabbit of the keel of the ship, from the back of the main stern-post, to a perpendicular line let fall from the fore-part of the main-stem, under the bow-sprit; from this length, subtract $\frac{3}{5}$ of the extreme breadth; and the remainder will be the length of the keel, for tonnage. The breadth must be taken from outside to outside of the plank, in the broadest part of the ship, whether above or below the main-wales, exclusive of all manner of doubling planks, or sheathing, that may be wrought upon the sides of the vessel; then multiply the length of the keel, in feet, by the breadth; divide the last product by 94; and the quotient will be the tonnage required.

EXAMPLES.

1. The length from the back of the stern-post to a line let fall from the fore-part of the main-stem, is 88 feet 6 inches; the extreme breadth from outside to outside of the plank, 26 feet 6 inches; required the tonnage of the ship.

Feet.
Gross length 88.5 *of the keel.*
$26.5 \times \frac{3}{5} = 15.9$ *the deduction.*

True length 72.6 *difference.*
26.5 *breadth of the beam.*

3630
4356
1452

1923.90 *first product.*
13.25 *half the breadth.*

961950
384780
577170
192390

25491.6750 *second product.*

Then 25491.675 ÷ 94 = 271.188 *tons, the burthen or tonnage required.*

2. The length from the back of the stern-post to a line let fall from the fore-part of the main-stem, is 108 feet 9 inches; the extreme breadth from outside to outside of the plank, 29 feet 6 inches; required the tonnage of the vessel.

Ans. 421.469 *tons.*

CASE II.

When the vessel is afloat.

RULE.

Let fall a plumb-line over the stern of the ship, and measure the distance between this line and the aft-part of the stern-post, at the land water-mark; then measure from the top of the said plumb-line, in a parallel direction with the water, to a perpendicular point immediately over the load water-mark, at the fore-part of the main-stem; from the last measured distance subtract the former; and the remainder will be the ship's extreme length. From this length deduct 3 inches for every foot of load-draught of water, for the rake abaft; and also $\frac{3}{5}$ of the ship's extreme breadth, for the rake of the stem; and the remainder will be the true length of the keel, for tonnage. The extreme breadth must be measured, and the tonnage found as directed in the first Case.

EXAMPLE.

The true length of an eighty-gun ship, after all deductions are made, in taking the dimensions, is 150 feet 9 inches; and the extreme breadth 50 feet 6 inches; required the tonnage of the vessel.

Ans. 2044.9478 *tons.*

Note 1. It is found by experience, that ships of war carry less; and most merchant-ships carry considerably more tonnage than they are rated at, by the preceding Rules.

2. Some writers on this subject, divide by 100, instead of 94, for King's ships. On the same principles, the divisors for Merchants' ships should be decreased, perhaps to 90 or 92.

3. Solutions to the last two Questions may be found in the Key to Nesbit's and Little's Practical Gauging.

THE

BUILDER'S DICTIONARY,

CONTAINING

An Explanation of the most general Terms made use of in

ARCHITECTURE.

A.

Abacus. A square table, list, or plinth, in the upper part of the chapiters of columns, especially those of the Corinthian order.

Abrevoirs. The seams or joints between stones and bricks, in laying them.

Acanthus. The herb whose leaves are represented in the capital of the Corinthian column.

Acroters. Sharp and spiry battlements or pinnacles, that stand in ranges, with rails and balusters, upon flat buildings. Also pedestals upon the corners and middle of pediments, to support statues.

Alcove. A recess in a room for a bed of state; also an arched building in a garden or pleasure-ground.

Amphitheatre. An edifice of an oval or circular form, with rows of seats rising one above another, in order to accommodate the people who attend in it, to see public diversions.

Antichamber. The room that leads to the chief apartment; also a room in a nobleman's house, in which strangers stay till the party to be spoken with is at leisure.

Aqueduct. A constru[illegible] of timber or stone, made on uneven ground to preserve the level of the water, and convey it, as by a canal, from one place to another.

The *Romans* were extremely sumptuous and magnificent in their Aqueducts, some of which extended 100 miles. Frontinus and Blasius inform us that the city of Rome was supplied with 500,000 hogsheads of water every twenty-four hours, from 9 Aqueducts, which emptied themselves through 13,514 pipes of an inch in diameter.

Arch. A curved line; a vault. See page 238.

Architect. A person who professes the science of Architecture; and whose business it is, if required, to give draughts or designs of intended buildings, with estimates of the expenses necessary to complete them; to superintend the workmen, and to give directions concerning the manner and method of executing the whole.

Architecture. The science which teaches the method of building, being a skill obtained by the art of designing, aided by the precepts of Geometry. By it, Rules are given for planning and raising all sorts of structures, according to Geometry and proportion.

The scheme or projection of a building, is easily laid down in three several draughts or designs. The first is a plan, which exhibits the extent, division, and distribution of the ground into apartments, and other conveniences. The second shews the heights of the different stories, and the outward appearance of the whole building; and is called the *design or elevation.* The third, called the *section or profile,* shews the inside of each apartment.

From these three draughts or designs, the Architect or Undertaker makes a computation of the probable expense of the building, and the time required to complete it.

Architrave. The moulding or ornament immediately above the capital of a column; it being always the next member below the frieze. Also the chief or principal beam of a building.

Doors and windows which have architraves on the jambs, and over the cap-pieces, are also denominated *architrave-doors*, and *architrave-windows*.

Ashlar. Freestone, as it comes rough out of the quarry, before it receives form and shape from the tool of the stone-cutter.

Ashlering. Quartering in garrets, from the floor to the under side of the rafters. It is generally perpendicular to the floor; and 2½ or 3 feet in height.

Astragal. A little round moulding in the form of a ring, serving as an ornament at the top or bottom of a column. The shaft of a column always terminates at the top with an astragal, and at the bottom with a fillet.

Attic. A little order, placed above another much greater; for instead of pillars, this order has nothing but pilasters, with a cornice architraved, for an entablement. This order is sometimes used in conjunction with the Ionic and Doric orders.

The word is also used to denote the uppermost part of a building, or the highest story.

B.

Balcony. A projection beyond the naked front of a wall or building, supported by pillars, and encompassed with a balustrade. Also a kind of open gallery.

Balk. A large piece of timber.

Ballustrade. A term used to signify a row of turned pillars, called balusters, made of marble, iron, wood, or stone, with a hand-rail upon the top; and fixed upon a terrace, the top of a building, a balcony, a stair-case, &c. &c.

Band. Any flat member that is broad, but not very deep. A list, a fillet.

Barge-course. A term used for that part of the tiling which projects over the gable-end of a building.

Base. A rest, a support, a foundation; the bottom of any thing. It is used to signify any body which bears up another; but it is particularly applied to the bottoms of columns and pedestals.

Batten. A name given by workmen, to a piece of wooden *stuff*, from two to four inches broad, and about one inch in thickness.

Batten-door. A door having the panels nailed upon the boards of which it is composed; whereas, in a wainscot-door, the panels are grooved into the framing.

Batter. A term used by workmen, to signify that a wall, a piece of timber, &c. does not stand upright; but inclines from you. If it inclines towards you, they say it *over-hangs* or *hangs-over*.

Battlements. Indented notches in the top of a wall or other building, made for the purpose of looking through, &c.

Bead. A little round moulding, commonly made upon the edge of a member, in the Corinthian and Composite orders; and other works of ornament. It is cut or curved in short embossments, resembling beads in a neck-lace.

Beam. A large piece of timber lying across a building, into which the feet of the principal rafters are framed. It is generally called the *tie-beam*. Also, any large piece of timber.

Beam-filling. The method of filling up the vacant space between the raising-plate and the roof of a building. The materials with which the space is filled up.

Bearing. The distance that the ends of tie-beams, girders, joists, &c. are let into the walls of buildings. Also, the distance between the fixed extremities of beams, girders, joists, &c. is sometimes called their *bearings*.

Bed-moulding. All the parts of a cornice that are placed under the corona. It commonly consists of these four members; viz. (1.) an OG; (2.) a list; (3.) a large boultine; and (4.) another list under the coronet.

Bevil. A kind of square used by Masons, Joiners, &c. Its two legs are moveable on a point or centre; consequently they may be set to any angle.

Bevil-angle. An angle greater or less than a right-angle.

Binding-joists. Those joists in any floor, into which the trimmers of well-holes and chimney-ways are framed. Also, those joists which are framed into the girders, to support the bridging-joists.

Block. A large piece of marble or stone, as it comes out of the quarry, before it assumes any form from the hand of the workman.

Blocking-course, or simply *Blocking*, in Masonry, a course of stones laid on the top of the cornice, crowning the walls.

Blocking-courses were used by the ancients to terminate the walls of a building as well as attics; and are made sometimes equal to the breadth of the pilasters. They serve as an ornament, and prevent the water from falling to the front of the building, turning it either way into lead gutters or spouts made for the purpose.

Boast. A term used among workmen, to signify the taking off the superfluous parts of door-posts, window-jambs, mouldings, &c &c.

Border. Those pieces of deal which are put round the slab of a chimney-hearth. Also, the outer part or edge of any thing.

Boultine. A moulding whose convexity is just one-fourth of a circle; being the member next below the plinth, in the Tuscan and Doric capitals.

Brace. A piece of timber framed in with bevel-joints, in order to keep the roof of a building, &c. from swerving. When braces are framed into king-posts and principal rafters, they are, by some workmen, called *struts*.

Brackets. Pieces of wood that support shelves, the steps of stairs, &c.

Brest-summer. A large piece of timber, in the internal part of a building, into which the girders of the naked floor are framed. In the ground-floor, it is called a sill; and in a garret-floor, a beam. See *summer-tree*.

Brick. A square mass of clay burnt hard, for the use of building.

Bridging-joists. Those joists which lie upon, and are supported by, the binding-joists; and upon which the boarded-floor is laid.

When those joists are supported by the walls of the building, or framed into the girders, without any binding-joists, they are generally called *common-joists.*

Butment. That which supports the foot of an arch. A shoulder.

Buttress. A pier made against the outside of a wall or building, to support and strengthen it.

C.

Camber-beam. A piece of timber cut in an arching direction, or with an obtuse angle in the middle.

Capital. The upper part of a column. Such as have no ornaments, are the Tuscan and Doric, which are called *capitals with mouldings,* and the rest which have leaves with other ornaments, are denominated *capitals with sculptures.* Also, the head or top of any pillar or pilaster.

Cap, or cap-piece. The highest or uppermost part of any thing.

Cartouses. A kind of modillions.

Casement. A hollow moulding, which some Architects make one-sixth of a circle, and others one-fourth. Also, that part of a window which opens.

Casings. The boards put round door-ways, window-openings, &c.

Cavetto. A concave moulding, which has quite a contrary effect to the quarter-round; the workmen call it *mouth* when in its natural position, and *throat* when turned upside down.

Caulicoll. The carved scrolls, under the abacus, in the Corinthian capital.

Ceiling. The lathing and plastering at the top of a room, upon the under-side of the joists of the next room.

Ceiling-joists. The joists to which the ceiling-laths are nailed, when there is no chamber above.

Cement. A strong, glutinous, binding composition, A good cement for uniting bricks together, for carving, scrolls, capitals, &c. &c. may be made of resin, bees-wax, brick-dust, and chalk boiled together. The bricks must be rubbed even; then heated, and rubbed together, with the cement between them.

A strong and useful cement for joining marble, and all other hard stones, may be made as follows: Melt two pounds of bees-wax, and one pound of resin together; then add one pound and a half of the same kind of matter, burnt and well pulverized, as the body to be cemented is composed of; let the whole mass be kneaded with water, and applied warm, to the parts of the body to be cemented, which must also be heated.

An exceedingly strong cement that will become as hard as stone, and last for ages, may be made in the following manner: take lime, well slacked, and sand in equal proportions; and temper them with linseed oil to the consistency of mortar: then beat it well in a trough or upon a floor, and it will be fit for use.

When an old stone or brick wall is to be covered with this cement or plaster, let the face be chipped a little with a bricklayer's hammer, or a mallet and chisel; then drench it with linseed oil and white lead, until it will drink no more.

Dr. Higgins had a patent in 1779, now expired, for his invention of a water cement, or stucco, as follows: fifty-six pounds of pure coarse sand, forty-two pounds of pure fine sand, mix them together, and moisten them thoroughly with lime-water; to the wetted sand add fourteen pounds of pure fresh burnt lime, and while beating them up together, fourteen pounds of bone-ash; the quicker and more perfectly these materials are beaten together, and the sooner used, the better will the cement be. Fine sand alone, or coarse sand alone, will do for some works; but the finer the sand, the more lime must be used.

Cankers of a forge well beaten with lime and sand, make a good cement for furnaces.

Chambers. The rooms between the ground story and the garrets.

Channel. That part of the Ionic capital which is under the abacus, and lies open upon the echinus or eggs; and has its centres or turnings on every side, to form the volutes. Also, the ornamental part of the Doric triglyphs; a hollow bed of running waters.

Chapiter. The top or head of a pillar or column.

Chaplet. A small ornament carved into round pearls or beads.

Cheek. Any piece of metal or timber which answers to another of the same sort, opposite to it, in any utensil, machine, or building.

Cheek-boards. The boards which form the sides of door-cases, window-cases, &c.

Chimney. The passage through which the smoke ascends from the fire.

Chimney-jambs. The sides of a chimney, on the extremities of which the mantle-tree rests.

Chimney-piece. An ornamental piece of wood or stone, standing on the foreside of the jambs, and projecting over the mantle-tree.

Cima-recta. A wave, called by the English workmen *ogee* or OG, which is of two kinds; viz. *cima-recta* or *OG*, and *cima reversa*, or *the back OG*, whose beauty consists in having its height and projection equal to each other.

Circle. A round figure comprehended under one curved line, called the circumference. The diameter is a right line passing through the centre, and terminating in the circumference on each side.

Cistern. A receptacle for water.

Clamp. A kiln built above the ground, for the purpose of burning bricks. Also, pieces on the ends of tables, window-shutters, &c.

Cleansed-work. Any stone-work that is polished or made smooth, by scouring or rubbing it. It is frequently called polished-work.

Clinkers. Those bricks which, by the violence of the fire, are run and glazed over.

Coins or quoins. The corners of brick or stone walls.

Also, the stones in the angles of buildings, whether plain, rustic, or otherwise.

Collar-beam. A beam framed crosswise between two principal rafters.

Column. A round pillar, composed of a base, a shaft, and a capital; and serves to support the entablement. Also, any round pillar.

Conduit. A canal of pipes for the conveyance of water.

Console. A projecting ornament cut upon the key of an arch, or other member; and which occasionally serves to support little cornices, busts, vases, &c.

Coping. The top or covering of a wall, made sloping to carry off the water.

Corbel. An ornament representing a basket; a short piece of timber or stone jutting out of a wall for a supporter.

Corbel-holes. Arches in the walls to receive images or statues.

Cornice. The third and uppermost member of the entablement, which is different in the several orders. The word *cornice* is likewise applied to every prominent or jutting member that crowns any body. Cornices are also placed on the top of the wainscot, under the eaves of houses, &c.

Corona or crown. Any thing that finishes an ornament. A large flat member which crowns, not only the cornice, but the entablement. The larmier.

Coves or covings. Thin stones placed against the jambs and back of a chimney. They are of two kinds; viz. side-coves, and back-coves.

Coussinet. The stone which crowns a pier, or that lies immediately over the capital of the impost and under the sweep. The bed of it is level below, and curved above, receiving the first rise or spring of the arch or vault.

Cupola. See *dome.*

D.

Dado or die. The plain parts of the pedestal of a column, between the base and the cornice.

DENTICLES or DENTILS. Ornaments in a cornice, resembling teeth. They are frequently made in the Ionic and Corinthian orders.

DOME or CUPOLA. A spherical roof, resembling the bell of a clock, raised over the middle of a building, as a church, hall, pavilion, vestibule, stair-case, &c. by way of crowning. Domes are generally supported by columns, both for the sake of ornament and strength.

Sometimes domes are of a polygonal form.

DOOR-CASE. The frame in which a door is inclosed.

DORMAN-TREE. See *summer-tree.*

DORMANT-WINDOW. A window made in the roof of a house; being raised upon the rafters.

DOVE-TAIL. A kind of joint, made by joiners, resembling the tail of a dove or pigeon.

DOUCINE. See *cima-recta.*

DRAG. A door is said to *drag*, when in opening or shutting, it stops or grates upon the floor.

DRAUGHT or *design.* The figure of an intended building described upon paper; in which are laid down, by a scale, in due proportion to the whole building, the several devised apartments, with doors, windows, passages, and other conveniences.

DRESS. To prepare stones, timber, &c. for any purpose.

DROPS. Ornaments in the Doric entablature, representing drops or little bells, immediately under the triglyphs.

E.

EAVES. The margin of the roof of a building, which projects beyond the wall, to throw off the water.

ECHINUS. A member or ornament near the bottom of the Ionic, Corinthian, and Composite capitals. It is, by some workmen, called *eggs and anchors;* and by others, *quarter-round.*

EMBOSSING. A kind of sculpture or engraving, in which the figures project from the plane upon which they are engraved. It is called *bass-relief, mean-relief,* or *high-relief,* according as the figures are more or less protuberant.

Enriched-mouldings. Cornices or mouldings ornamented with modillions, beads, denticles, leaves, and other desirable additions.

Entablement or *entablature.* The architrave, the frieze, and the cornice taken together.

Entry. A passage leading from the outer door into the house. Any door, gate, or passage.

Eye. The middle of an Ionic volute or scroll, cut in the form of a rose. A round window made in a pediment, or at the top of a dome, &c.

F

Fabric. A church, a house, or any other building.

Face. Any member that has a considerable breadth, and but a small projection; as the architrave in the front of a building, &c. The front of any thing.

Facia. Any flat member that is not very broad. It is, by some workmen, called a *string,* and by others a *riband.*

Feather-edged. Thinner on one edge than the other.

Fell. To hew down; to cut down.

Festoon. An ornament of carved work, in the form of a wreath, or garland of flowers, fruits, and leaves intermixed or twisted together; being thickest in the middle, and suspended by the two extremities, whence it hangs down with a graceful sweep.

These ornaments were formerly much used on triumphal arches, &c.

Fillet. A little square ornament or moulding, which accompanies or crowns a larger. A list, a band.

Fire-place. That part of a room in which the fire is made.

Fire-stone. Stone that will bear the fire well, without being soon consumed.

There are also fire-bricks, which have the same property, and are much used about ovens, stoves, &c.

Floating. A term used by Plasterers, for their best ceilings.

Floor. The bottom part of a room on which we walk. Floors are of different kinds; some are of earth, some of brick, some of stone, and some of wood.

The framed timbers of a floor, viz. the summers, girders, and joists, are, by Carpenters and Joiners, denominated *naked flooring*, in order to distinguish it from the boarded-flooring.

FLUSH. A term used by Carpenters and Joiners, when the work is even or smooth. It is also used by Masons, to signify the breaking off of any part of a stone.

FLUTES or *flutings*. Hollows or channels made in the shaft of a column, from the base to the capital. The Doric, Ionic, Corinthian, and Composite columns are commonly fluted; but the Tuscan column scarcely ever.

FLYERS. Stairs which do not wind, but go straight; and whose steps are all in the form of rectangles.

FOLIAGE. Ornaments wrought in the form of branches and leaves.

FOOT-PACE or *half-pace*. A part of a pair of stairs, whereon you arrive, after having ascended four, six, or more steps. Here two or three paces may generally be taken before you ascend another step, by which means your legs are eased. A kind of landing.

FOUNDATION. The lowest part of a building, upon which the walls of the superstructure are raised. It is generally laid under ground.

FRAME. Any thing constructed so as to inclose or admit something else.

FRIEZE or *frize*. A large flat member, which separates the architrave from the cornice.

FRET-WORK. Plain bordering around ceilings, &c. It is made with divers fillets or bands, which form a great variety of figures by their turnings.

FRONT. The principal face or side of a building; or that which presents the chief aspect or view. The face or forepart of any thing.

FRONTISPIECE. The decorations on the forepart of any thing.

FUNNEL. The narrow part of a chimney.

FURRING. A piece of timber put upon a joist or rafter, to strengthen or repair it.

FUST. The shaft, trunk, or body of a column, extending from the base to the capital.

G.

Gable-end. The upright triangular end of a house, from the eaves to the top of the roof.

Gallery. A kind of immoveable scaffold, in the inside of a room, accommodated with seats; as in a place of worship, a theatre, &c.

Garland. An ornament in imitation of branches, leaves, fruits, and flowers, twisted and involved into each other. It is sometimes called a wreath.

Girders, Large pieces of timber into which the ends of the joists of a naked floor are framed. The ends of the girders are sometimes framed into the summers and brest-summers: and sometimes they are supported by the walls of the building.

Gothic-architecture. That species of architecture which is far removed from the manner and proportions of the antique; having its ornaments wild and chimerical, and its profiles generally very incorrect. It is frequently very heavy, solid, and massive; but sometimes, on the contrary, exceedingly light, delicate, and rich. Almost all the ancient cathedrals are built after the gothic manner.

Gothic-roof. A roof formed by two circular arches, struck from different centres, and meeting in a point over the middle of the span.

Granary. A store-house for corn.

Grange. A farm, with all the appendages; as a dwelling-house, barns, stables, cow-houses, granaries, and other necessary places for husbandry, at a distance from neighbours.

Grate. The range of bars within which fires are made.

Groin. A kind of arch or roof formed by the intersection of vaults.

Groove. A term used by Joiners, to signify the channel made by their plough, in the edge of a moulding, or stile, to receive the panels, in wainscotting.

Grotesque. Distorted, unnatural; formed without due proportion.

Grotto. A cave, a cavern, made for coolness and pleasure.

Ground-floor. The lower story of the house.

Ground-plot. The ground upon which a building is to be erected.

Gutter. A channel for the conveyance of water.

Guttering. The boarding and bearers upon which the lead is laid that forms the gutters, in roofing.

H.

Hall. A manor-house. Also, the first large room at the entrance of a house.

Hand-rail. See balustrade.

Haunches, or *hances*. The ends or bases of elliptical arches. The word is frequently used to denote the ends of any arch.

Head. The top or upper part of any thing.

Headers. A term used among Bricklayers, when bricks are laid endwise in a wall; but when they are laid lengthwise, they call them stretchers.

Hearth. The floor or pavement of a chimney, or fire-place.

Heel. An inverted Ogee. A talon.

Hew. To cut with an edged instrument.

Hinges. Those appendages, by which doors, the lids of boxes, &c. make their motion.

Hips. In Carpentry, are those pieces of timbers which are placed at the corners of a roof. They are much longer than the rafters, by reason of their oblique position.

Hollow. A concave moulding, by some called a casement. Any excavation.

House. A habitation; a place of human abode.

Housing. A term used by Bricklayers, when a tile or brick is cast crooked or hollow, or has become so by burning; they then say, such a tile or brick is "housing."

J.

Jambs. The posts of a door. Also the posts or piers on the sides of windows, fire-places, &c.

Imposts. The capitals of pilasters, that support arches. That part of a pier that supports the arch.

Intercolumniation. The space between two columns.

Joint. The place where any two pieces of matter form a junction.

Joists. Those pieces of timber in a naked-floor, that are framed into the girders and summers. They receive various names, according to the positions in which they are laid; as trimming-joists, binding-joists, bridging-joists, and ceiling-joists.

K.

Kerf. The aperture or slit made in a piece of timber or board, by a saw.

Key-stone. The principal stone in the crown or middle of an arch.

King-post, or *king-piece*. A piece of timber placed in a perpendicular direction, upon the middle of the tie-beam, and extending to the top of the principal rafters, in order to support them. The rafters, in general, are also supported by braces or struts, proceeding from the king-post.

Kitchen. A room in a house where the provisions are cooked, &c.

Knee-piece. A piece of timber cut crooked, or with an angle.

L.

Landing. The top of a stair-case.

Larmier. The flat square member that forms the utmost projection of a cornice. The corona.

Laths. Long, thin, narrow slips of wood used in tiling and ceiling.

Lead. A soft heavy metal, used for roofs, gutters, pipes, cisterns, sinks, &c.

Leaves. Well-known ornaments with which ceilings, cornices, &c. are enriched.

Ledgers. Pieces of timber used by Masons and Bricklayers, in making scaffolds to work upon. The ledgers are laid parallel to the wall of the building; and made fast, by ropes, to the scaffold-poles.

Lime. Chalk or stone burnt, of which mortar, cement, &c. is made.

Lining-boards. See casings.

Lintels. Pieces of timber that lie horizontally over the top of the door-posts, window-jambs, &c. to bear the weight of the walls. The upper part or head of a door-frame is sometimes called the lintel.

List. A little square moulding, serving to crown or accompany a larger one. The space which separates the flutings of a column. It is sometimes called a fillet, and sometimes a band.

Lobby. An antichamber.

M.

Mallet. A wooden hammer used by Joiners, Masons, Stonecutters, &c.

Mantle, or Mantle-tree. The head-piece which rests upon the tops of chimney-jambs.

Marble. A fine, hard stone, of various colours, used in statues and elegant buildings. See page 219.

Member. Any part of an integral.

Metope, or Metopa. The space between every triglyph of the Doric frieze.

Model. An artificial pattern made of wood, stone, clay, or other matter, with all its parts and proportions, in order to give an idea of the effect that will be produced, when the work is executed on a larger scale.

Modillions. Small consoles or brackets, set as ornaments under Ionic, Corinthian, and Composite cornices.

Module. A kind of a measure used to regulate the proportions of the several members of columns.

Monument, A building erected to preserve the memory of some illustrious personage, remarkable event, or great achievement.

Mortar. A cement or composition formed of lime, sand, and water; and used by Masons and Bricklayers, in building.

Mr. John Phillips says, that equal parts of lime and sand, make exceedingly good mortar; especially if sharp river sand be used; as fine sand makes much weaker mortar than coarse. (See Crosby's Builder's Price Book, page 45.

Mortise-hole. A hole cut into a piece of timber, that another piece, called the tenon, may be put into it, and form a joint.

Mosaic-work. Curious kind of work, consisting of small inlaid pebbles, cockles, and shells of various colours.

Mouldings. Under this name are comprehended all those juttings or projections beyond the naked of walls, columns, &c. which serve only for ornaments; whether they be square, round, straight, crooked, concave, or convex. There are seven kinds of mouldings more considerable than the rest; viz. the *doucine* or *cima-recta;* the *talon* or *heel;* the *ovolo* or *quarter-round;* the *plinth;* the *astragal;* the *denticle;* and the *cavetto.*

Munnions. The upright posts that divide the lights in a window-frame.

N.

Naked of a wall. The surface or plain from which the mouldings and other projections arise.

Nave. The middle part or body of a church, distinct from the aisles or wings.

Newel-post. The upright post in a winding staircase, round which the steps are set.

Niches. Hollows or cavities in walls, in which statues are sometimes placed.

Nosings. The edges or front parts of steps; part of the sides of marble chimney-pieces; also, pieces put on at the ends or front of any thing.

O.

Obelisk. A quadrangular pyramid, generally erected in some public place, to serve as a monument of some memorable action, or event, &c.

Ogee or **o g.** A sort of moulding, consisting of a round and a hollow, something resembling an S.

Openings. The breaks or apertures that are left in the walls of buildings, for the doors, windows, &c.

Orders. The different forms and proportions of columns, &c. There are commonly reckoned five orders; viz. the Tuscan, the Doric, the Ionic, the Corinthian, and the Composite.

The *Tuscan* is the most simple of the five orders; having neither triglyphs, dentils, nor modillions. It is sometimes called the Rustic Order; being very strong and massive. The height of the Tuscan column is seven diameters; and the height of the entablement one-fourth of the column's height.

The *Doric* order seems the most natural, and best proportioned of all the orders. The frieze is adorned with triglyphs, drops, and metopes; and the height of the column is eight diameters; and that of the entablement one-fourth of the column's height.

The *Ionic*, or third order, is a kind of mean between the strong and delicate orders. The height of the column is nine diameters; and that of the entablement one-fifth of the column's height. The capital is ornamented with volutes; and the cornice with denticles. The shaft of this column is generally fluted.

The *Corinthian* order is the noblest, richest, and most delicate of all the orders. The capital of this column is adorned with two rows of acanthus leaves; and the entablement with modillions, and sometimes with both modillions and dentils.

The height of the column is ten diameters; and that of the entablement one-fifth of the column's height.

The *Composite* or fifth order, is so called, because it is formed out of the other orders. It borrows a quarter-round from the Tuscan and Doric; a row of leaves from the Corinthian; and volutes from the Ionic. Its cornice has simple modillions or dentils. The height of the column and entablement is the same as that of the Corinthian order.

The Doric, Ionic, and Corinthian orders were invented by the Greeks; and the Tuscan and Composite by the Romans.

Those who wish to make themselves acquainted with the method of drawing and working the five orders, are referred to Nicholson's Student's Instructor.

Orlo. A fillet under the ovolo, or quarter-round.

Ornaments. All the sculpture or carved work with which a piece of architecture is enriched.

Ovolo. A round moulding, whose profile or sweep is about one-fourth of a circle. It is commonly enriched with figures resembling eggs and anchors, or arrow-heads placed alternately.

P.

Panel. A square or rectangular piece of wainscotting, sometimes framed or grooved in between thicker pieces; as in doors, window-shutters, &c. Also the face of a hewn stone.

Pantry. A room in which victuals are kept.

Parapet. A wall breast high, serving either as a rest for the arm, or an inclosure about a key, bridge, terrace, &c.

Parlour. A lower room designed for the entertainment of company.

Partition. That which divides one apartment from another.

Passage. An entry or narrow place, serving for a thorough-fare into other rooms, &c.

Pavement. Stones or bricks laid upon the ground, in order to make the passage easy.

Pedestal. The lower member of a column, or that upon which it stands. The base of a statue.

Pediment. An ornament or crowning, which finishes the front of a building; and serves as a decoration over doors, windows, niches, &c. It is generally of a triangular shape; but sometimes forms the arch of a circle. The ridges of houses first gave Architects the idea of this noble part.

Perquetting. A floor composed of divers small figures, as triangles, squares, rhombuses, &c.

Perron. A flight of steps leading to the front door of a great house, when the floor of the lower story is raised a little above the ground.

Persian Order. That which has figures of Persian slaves, instead of columns, to support the entablement.

Piazza. A portico.

Pier. The column which supports the arch of a bridge. The space between two windows in a room.

Pilaster. A square column, sometimes insulated, but oftener set within a wall; and only shewing a fourth or a fifth part of its thickness. Pilasters frequently have bases and capitals the same as columns.

Pile. A strong piece of wood driven into the ground, to make a good foundation.

Pillar. A round, square, or pentagonal column.

Pinnacle. A turret or elevation above the rest of the building. A high spiring top.

Pitch. The elevation of a roof. See page 222, note 4.

Plaster. A composition made of lime, sand, hair, and water, with which walls, ceilings, &c. are overlaid.

Plastic. The art of forming figures or ornaments in plaster.

Plat-band. Any flat, square moulding, having less projection than height.

Platford. The ceiling of a chamber, or other room; the soffit.

Platform. The ichnography or ground-plot of a house. The roof or covering of a building that is flat at the top. Any horizontal plane.

Plinth. A flat square member under the mouldings of the bases of columns, pedestals, &c. A square projection near the bottom of a wall.

Pole. A long slender piece of timber.

Polished-work. Any stone-work that is made smooth, by scouring or rubbing it. In some places it is called cleansed-work.

Porch. A roof supported with pillars before a door. An entrance; a portico; a covered walk.

Portails. The decorations of a front-gate; or in the front of a church, &c.

Portal. A gate; an entrance; the arch under which a gate or door opens.

Portico. A kind of gallery, raised upon arches, where people walk under shelter. It has sometimes a soffit or ceiling, but is more commonly vaulted. Also, a roof, gallery, or other projection before a door, in the front of a hall, church, &c. supported by columns detached from the building.

Post. A piece of timber placed in a perpendicular direction.

Prickpost. A post placed between two principal posts.

Principal. Capital; essential; chief; of the first rate.

Profile. The figure or draught of a building, &c. in which are expressed the several heights, breadths, and thicknesses, such as they would appear if the building were cut perpendicularly from the top of the roof to the foundation; hence it is frequently called the section.

This word is used to denote the outline of any member of a building; as that of the base, a cornice, &c. It also denotes the prospect of a building, or any piece of architecture, viewed sideways.

Project. All mouldings, &c. are said to project, when they jut out, or are beyond the adjacent surface.

Projecture. The prominency or embossment which mouldings and other members have beyond a column, the naked of a wall, &c.

Proportion. The justness of the members in each part of a building, and the relation they bear to the whole.

Pulvinata. A frieze swelling or bulging out like a pillow.

Punchin. A short piece of timber so placed as to bear some considerable weight.

Purlings. Those pieces of timber that lie in a horizontal position; and support the inferior rafters. They are sometimes called side-pieces or side-wavers.

Putlogs. Pieces of timber used by Masons and Bricklayers in making scaffolds. They are laid at right-angles to the wall of the building, with their ends in the scaffold-holes and upon the ledgers, by which means they are supported.

Q.

Quadrant. The fourth part of a circle.

Quarters. Those slight upright pieces of timber, placed between the punchins and posts, against which the weather-boarding and laths are nailed, in making partitions.

Quarter-round. Any moulding whose contour is one-fourth of a circle, or nearly so.

Quirk. A piece taken off the corner of a ground-plot, to make a court-yard.

Quoins. See *Coins.*

R.

Rafters. Those pieces of timber which extend from the ridge to the eaves of a roof. The feet of the principal rafters are always framed into the tie-beam.

Rails. The horizontal pieces of timber between the panels of wainscotting. A number of cross beams supported by upright posts.

Range. A grate or fire-place. Also, when work runs straight, without breaking into angles, it is said *to range.*

Regula. See orlo.

Return. The continuation of a moulding, &c. after having turned a corner. The side that reclines from the front of any straight work.

Ridge. The top of a roof, where the ends of the rafters meet each other.

Ridge-stones. The stones which cover the top of a roof.

Riser. A board placed edge-ways to support the upper part of a step called *the tread.* The perpendicular face of a step.

Rising-pieces. Pieces of timber laid under the ends of tie-beams, girders, &c.

Roof. The covering of a house; but particularly the timber-work.

Rose. An ornament cut in imitation of a rose. It is used in friezes, abacuses, cornices, the ceiling of churches, &c. and sometimes it is made between the modillions, in the Corinthian and Composite orders.

Rustic. A manner of building quite rude, rather in imitation of nature, than according to the rules of art.

Rustic-work. Any stone-work, the face of which is hacked or picked into holes, with the point of a hammer, &c.

S.

Saloon. A lofty, spacious hall, the side-walls of which are connected to a flat roof or ceiling, in the middle, by arches on each side. See page 238.

Sand. Small particles of stone not conjoined. Sand is one of the principal ingredients in making mortar.

Scaffold. A temporary gallery or stage, raised by the side of a building, for the workmen to stand upon.

Scaffold-holes. Holes left in the sides of a building, for the end of the putlogs.

Scantling. The breadth and thickness of a piece of timber.

Scotia. A concave member, whose contour is the arc of a semicircle, or nearly so. It is commonly placed in the base of a column, between the torus and the astragal.

Scribe. To fit one irregular piece of *stuff* to another, by marking them with compasses, &c. Also the mark made upon a piece of timber with any pointed instrument.

Scribe-iron. An instrument with which timber is marked.

Scroll. See volute.

Sculpture. The art of carving wood, stone, &c. into images and other ornaments.

Section. See profile.

Semicircle. A half-round; half of a circle.

Sewer. A passage for water to run through.

Sill. The bottom part of a window-frame.

Shaft. The trunk or body of a column. See *fust*.

Shaky or shaken. Such stuff as is cracked by the heat of the sun, &c. is called *shaky* or *shaken* stuff.

Shingles. Thin pieces of timber used to cover houses, instead of tiles or slates.

Shoulder. The butments of a tenon. A prominence; a support.

Skirting-boards. The narrow boards fitted round the under-side of wainscotting, next the floor.

Slabs. The sappy planks cut off the outsides of timber. Also the foot-pace to a fire-place is called a slab.

Slates. Thin stones with which houses are covered. They are chiefly of two kinds, blue and grey.

Slating. The covering of a house with slates.

Sleeper. The oblique rafter that lies in the gutter of a roof.

Sluice. A water-gate: a flood-gate; a vent for water.

Soffit. In great edifices, it signifies the ceiling or wainscot of any apartment, formed by cross beams or flying cornices, and having the square panels of its compartments enriched with sculpture, painting, gilding, &c. In common buildings, it denotes the board at the top of a window-opening, opposite to the window-board.

Sole. The stone upon which window-jambs are placed. The bottom part of any thing.

Span. The chord line of an arch, or the direct distance between its extremities. It is particularly applied to bridges and other buildings with arched roofs, to denote the distance that the feet of the arches are from each other.

Spar. A small beam of timber. The inferior rafters are sometimes called spars.

Stairs or Stair-case. The passage by which we ascend and descend from one story of a house to another. It generally consists of steps, landing-places, and a balustrade.

Statues. Embossed figures or images, either of wood, stone, or metal, representing some person distinguished by his birth, merits, or achievements, &c. They serve to perpetuate the memory of the person they represent.

Stiles. The upright pieces that extend from the bottom to the top in wainscot. A set of steps or rails to pass from one inclosure to another, is also called a stile.

Stone. A hard substance generated in the earth; and much used in building. See page 219.

Stove. A hot-house for preserving exotic plants. A close place in which fire is made.

Strike. To form by a sweep,—as "to strike a circle." To put in motion; to stamp; to impress.

Strings. Pieces of timber which support the steps of wooden stairs.

Strut. A piece of timber framed into the king-post and principal rafters. See *brace*.

Stuff. A general term for all kinds of timber upon which Joiners work.

Summer-tree. A large piece of timber, in the internal part of a building, having a row of mortise-holes on each side, into which the ends of the girders are framed.

Brest-summers are laid adjoining the walls of a building, and consequently have mortise-holes only on one side. See Brest-summer.

Sweep. The compass of any continued motion. The direction of motion in compassing any thing.

Systyle. That manner of placing columns, so that the space between them consists of two diameters.

T.

Tabling. Flat stones laid upon the gable-ends of a house, from the eaves to the ridge, to keep the slates fast.

Talon. A moulding consisting of square fillets, which sometimes terminates the ornaments of Joiner's Work. A heel.

Tassels. Pieces of timber that lie under the ends of mantle-trees, beams, &c.

Tenon. That part of a piece of timber which is put into a mortise-hole.

Terrace. An open walk or gallery; a flat roof of a house; also a kind of strong mortar, chiefly used in making basins, cisterns, wells, and other reservoirs for water.

Thorough-lighted. Having windows at both sides, or both ends.

Threshold. The stone or step under a door, upon which the door-posts are placed. The bottom part of [illegible] door-frame.

Tie-beam. The beam into which the feet of the principal rafters are framed.

Tiles. Thin plates of burnt clay, used in covering houses.

Timber. All kinds of wood used by Carpenters, Joiners, Turners, &c. &c. For a description of Timber Trees, see page 197.

Tooled-work. Implies the manner of finishing; hence any stone struck with parallel strokes, by either a broad or narrow chisel, is so called.

The door-posts, window-jambs, chimney-jambs, &c. of common buildings, are always tooled; but those of elegant buildings are most commonly polished. In buildings of mediocrity, the door-posts, window-jambs, &c. are frequently tooled; and the chimney-jambs, coves, slabs, &c. polished. See polished-work.

Top-rail. The upper part of a window-frame.

Torus. A large round moulding in the base of a column, between the plinth and the list.

Transom. A piece that is framed across a double-light window.

Traverse. To plane a board, &c. across the grain.

Tread. The upper or horizontal part of a step.

Triglyph. A member of the Doric frieze, placed between the metopes.

Trim. When workmen fit a piece of timber, stone, &c. into some other work, they say, they *trim in* a piece.

Trimmers. Pieces of timber framed into the binding-joists, against well-holes, and chimney-ways.

Trunk. The shaft or a fust of a column. The body of a tree.

Truss. That part of a roof which supports the purlins and inferior rafters. It generally consists of the tie-beam, principal rafters, king-posts, braces, and punchins. See page 226.

Tusk. A bevil shoulder, made to strengthen the tenon of a joist, where it is let into the girder.

V.

Valleys. The gutters over the sleepers, in the roof of a building.

Vault. A cellar; a cave; a cavern; a continued arch. See page 238.

Vestibule. A large, open space before the door, or at the entry of a house. A porch.

Volute. A kind of scroll, wreath, or spiral contortion, used in the Ionic and Composite capitals, of which it is the chief ornament.

It is supposed, by some, to represent the bark of trees, twisted into spiral bines; and by others, the head-dresses of virgins in their long hair.

The volute is also used in the Corinthian capital; and as it is an ornament that contributes very greatly to the beauty of columns, Architects have invented various ways of delineating it.

Consoles, modillions, and other ornaments, have likewise their volutes.

Urn or Vase. A vessel with a mouth narrower than the body; and serving as an ornament or crowning over balustrades, chimney-pieces, columns, pyramids, funeral monuments, the tops of buildings, &c.

It is also considered as an attribute to rivers, fountains, cascades, jets, &c.

W.

Wainscot or Wainscotting. The inner woodenc overing of a wall. It commonly consists of panels, stiles, rails, and a cornice.

Water-board. A board nailed over the bottom of an outer-door, to carry off the water.

Water-spout. A trough placed immediately under the eaves of a house, to receive the water that falls upon the roof. It always has a communication with the water-trunk, or conductor.

Water-table. A kind of ledge left in a stone or brick wall, about 18 or 20 inches from the ground, at which place the wall is decreased in thickness.

Water-trunk. A pipe by which water is conveyed from the top or eaves of a building. It is sometimes called a conductor.

Weather-boarding. The boards nailed against the quarters, in making timber partitions.

Well-hole. The space left in a floor for a stair-case.

Wreath. An ornament resembling branches, leaves, fruits, and flowers, interwoven and entwined one into another. It is frequently called a garland.

QUESTIONS

FOR THE

EXAMINATION OF THE PUPIL,

TO BE

ANSWERED VERBALLY.

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